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Page 284, middle; *for* February *read* January.

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OF THE

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An Analysis of the Profit from Endowment Assurances. By JAMES CHATHAM, F.F.A., F.I.A., *Secretary and Actuary of the Scottish Life Assurance Company.*

[Read before the Faculty, 11 December 1905.]

A great deal has been written about Endowment Assurances, which now form so large a proportion of the new business of life assurance offices, but their effect upon the business has been only partially investigated, and much yet remains to be done. It is generally admitted that the mortality in this class is better than that prevailing in other classes, and recently Mr. R. P. Hardy brought before the Congress of Actuaries in New York an interesting paper, in which he stated that "while probably the results would vary in their incidence according to the tables brought into contrast, it appears to me that one effect of a better mortality is to show a gradually reduced surplus. And if this be the case, some care is necessary in relying upon future profits according to their past experience." He appended a table showing the quinquennial surplus or deficiency arising when the premiums are charged, and the reserves made, according to the American table as adjusted by Mr. Arthur Hunter, while

the mortality experienced is that of the O^M table; and the results show that, for endowment assurances of 15, 20 and 25 years, the surplus at each quinquennium, as a rule, diminishes very rapidly. It should be noted that these calculations were made at 3 per cent interest throughout.

Still more recently Mr. N. B. Gunn, in his Presidential Address (session 1903-04) to the Faculty of Actuaries, referring to Mr. Hardy's paper, said that he had made a trial calculation of the profit earned, according to various rates of mortality, on a group of assurers paying an average office premium. The assumptions made were (1) that the premiums are accumulated at the average rate of interest realised by an office, (2) that the expenses of management at an average office rate are annually deducted from them, and (3) that the claims arise at the rate in the O^M, O^{HM}, and $H^M H^{M(5)}$ tables respectively. All these, he says, gave practically the same result, namely, that the bonus in the second quinquennium was approximately double that of the fifth. These statements are different from what was to be expected from the results given by the late Mr. Sunderland in his paper "On Bonuses on Endowment Assurance Policies" (*J. I. A.*, xxviii, 257); but it appears that 4 per cent interest was used by Mr. Gunn in valuing the policies as well as in accumulating the fund, so that here also the same rate of interest was adopted throughout the calculations.

As the subject is one of great importance to assurance offices generally, I thought it desirable to make an investigation on strictly practical lines to determine the surplus and rate of bonus at successive divisions of profits, avoiding on the one hand a low rate of interest for the accumulation of the fund, much below that realised by any office, and on the other hand a high rate of interest for the valuation of the policies, such as no office employs. It is the object of this paper to lay the results before you; but before doing so, it is necessary to state the basis upon which the calculations have been made.

As is well known, the results of endowment assurances depend more upon the duration than upon the age at entry; and as I have found that, in an office doing a considerable proportion of its business under endowment assurances, the average age at entry for that class is 31, and the average duration 27 years, I have taken the age at entry as 30, and the duration as 25 years. I have also found that the average office premium for a policy of this kind, with profits, is £4, 0s. 7d. per cent, and I have therefore adopted that rate. In Appendix A, however, I give

the results for ages at entry 20 and 40, the durations being 30 and 20 years, and the office premiums £3, 4s. 0d. and £5, 6s. 1d. per cent respectively.

With regard to expenses, a common method is to take the cost on the new premiums as ten times as much per cent as the cost on renewal premiums, and a moderate percentage to take in a well-managed office is 66⅔ of the new premiums and 6⅔ of renewal premiums. Another method is to take the cost of the new business as £2 per cent on the sum assured, and the cost of renewal premiums as 7½ per cent. I may mention, however, that in this case the one is practically the actuarial equivalent of the other; that is to say, the larger initial cost of the former is counterbalanced by the lower renewal cost afterwards; and as the first method corresponds perhaps more closely with actual conditions, I have adopted it.

The proper rate of interest to employ is, I think, the effective one, free of income tax, deduced from Mr. G. F. Hardy's well-known formula; and as I find that the average rate for British Offices so calculated is £3, 15s. 8d. per cent, I have taken it as 3¾ per cent.

The question of the mortality likely to be experienced among endowment assurances is rather a difficult one to determine, inasmuch as it is feared that the light mortality shown in the recent joint investigation by the Institute and the Faculty may not be continued, on account of this form of policy having become so popular with the public generally; but, as I have shown in my previous papers, the rule that the higher the rate of premium the lower the rate of mortality, is so general, that I have no doubt it will be found to be applicable to this class also. I think if it is taken as 80 per cent of the $O^{[M]}$ table this will be a fair allowance, a select table being used to give effect to the medical examination at the outset.

I have assumed that claims are payable immediately upon proof of death and title, and I have taken the date of payment as being on the average two months after death.

The O^{M} table is becoming very generally adopted for valuations, and I have therefore employed it. The rate of interest almost invariably used is 3 per cent, and that is the rate I have adopted. I have assumed that the interim bonus is 27s. 6d. per cent per annum, that being about the average rate given by offices; and to allow for the large number of cases in which the bonuses do not vest for some years, I have also assumed that none is given should death occur during the first five years.

It will be convenient briefly to recapitulate the assumptions made here. They are as follows:—

Age at Entry,		30.
Duration of Assurance,		25 years.
Office Premium,		£4, 0s. 7d. per cent.
Expenses,		$66\frac{2}{3}$ per cent of first premium, and $6\frac{2}{3}$ per cent of renewal premiums.
Interest	Assumed,	3 per cent.
	Realised,	$3\frac{3}{4}$ per cent.
Mortality	Assumed,	O^{M}.
	Experienced,	80 per cent of $O^{[M]}$.

Claims paid two months after death.
Interim Bonus after first five years, 27s. 6d.

As already mentioned, the object is to determine the surplus and rate of bonus at successive divisions of profits under these conditions.

Suppose 10,000 policies to be effected for £100 each on the above assumptions, then the following table shows the number of them in force at the beginning of each year, also the number of actual deaths during each year, and the number expected according to the O^{M} table used in the valuation.

TABLE 1.

Year.	Policies at beginning of Year.	Actual Deaths during Year.	Expected Deaths during Year.	Year.	Policies at beginning of Year.	Actual Deaths during Year.	Expected Deaths during Year.
1	10,000	25	60	14	9,233	81	97
2	9,975	40	62	15	9,152	86	101
3	9,935	46	64	16	9,066	86	105
4	9,889	50	67	17	8,980	91	109
5	9,839	54	69	18	8,889	94	114
6	9,785	57	72	19	8,795	98	118
7	9,728	60	75	20	8,697	101	124
8	9,668	64	78	21	8,596	107	129
9	9,604	68	80	22	8,489	111	135
10	9,536	72	84	23	8,378	116	142
11	9,464	75	87	24	8,262	121	149
12	9,389	76	90	25	8,141	128	156
13	9,313	80	93				

The next table shows the progress of the Fund on the assumptions mentioned above, adding in the case of the claims the total reversionary bonus as shown in Table 4, and any interim bonus. I have not considered it necessary to divide the claims into sums assured and bonus, as the former in any year are easily obtained by multiplying the number of deaths in the above table opposite the year by 100.

TABLE 2.

Year.	Fund at beginning of Year.	Premiums less Expenses.	Claims by Death.	Interest at 3¾ per cent.	Fund at end of Year.
	£	£	£	£	£
1	...	13,431	2,500	473	11,404
2	11,404	37,512	4,000	1,784	46,700
3	46,700	37,361	4,600	3,094	82,555
4	82,555	37,188	5,000	4,427	119,170
5	119,170	37,000	5,400	5,788	156,558
6	156,558	36,797	5,957	7,177	194,575
7	194,575	36,583	6,353	8,589	233,394
8	233,394	36,357	6,865	10,030	272,916
9	272,916	36,116	7,387	11,497	313,142
10	313,142	35,861	7,921	12,989	354,071
11	354,071	35,590	8,385	14,507	395,783
12	395,783	35,308	8,601	16,058	438,548
13	438,548	35,022	9,164	17,644	482,050
14	482,050	34,721	9,390	19,262	526,643
15	526,643	34,417	10,085	20,914	571,889
16	571,889	34,093	10,256	22,596	618,322
17	618,322	33,770	10,977	24,316	665,431
18	665,431	33,428	11,469	26,064	713,454
19	713,454	33,074	12,092	27,844	762,280
20	762,280	32,706	12,600	29,654	812,040
21	812,040	32,326	13,595	31,494	862,265
22	862,265	31,923	14,255	33,354	913,287
23	913,287	31,506	15,058	35,242	964,977
24	964,977	31,070	15,873	37,154	1,017,328
25	1,017,328	30,615	16,967	39,086	1,070,062

The results of the valuation of the policies in force at the end of the several quinquenniums, and the bonuses obtained, are shown in the following tables :—

TABLE 3. $3\frac{3}{4}$ *per cent.*

End of Year.	Reserve.			Fund.	Surplus.
	Sums Assured.	Existing Bonuses.	Total.		
	£	£	£	£	£
5	138,495	...	138,495	156,558	18,063
10	288,320	19,791	308,111	354,071	45,960
15	448,975	71,614	520,589	571,889	51,300
20	620,710	133,256	753,966	812,040	58,074
25	801,300	205,761	1,007,061	1,070,062	63,001

TABLE 4. $3\frac{3}{4}$ *per cent.*

End of Year.	Bonus.		Rate of Bonus.			Annual Rate of Bonus.	
	Cash.	Revy.	Cash.	Revy.	Total Revy.	Cash.	Revy.
	£	£	£	£	£	£	£
5	18,063	30,690	1·846	3·136	3·136	·369	·627
10	45,960	68,932	4·856	7·284	10·420	·971	1·457
15	51,300	67,671	5·658	7·464	17·884	1·132	1·493
20	58,074	66,998	6·756	7·794	25·678	1·351	1·559
25	63,001	63,001	7·862	7·862	33·540	1·572	1·572

Here it will be seen that there is no diminution either in the surplus or in the rate of bonus. On the contrary, they both increase throughout the whole duration of the policies.

I thought it would be interesting to analyse the surplus in order to show the extent to which the various sources of profit had contributed to the bonus, and thus ascertain what has produced the increase. The results are shown in the table below. I should mention that I have inserted in it the full profits earned during any quinquennium, and consequently it has been necessary to deduct the interim bonus paid out of them during the quinquennium; also that all interest earned above 3 per cent has been treated as profit from interest, although a small portion of it is interest on profit from loading and mortality. I have also inserted in the table the amounts which the various sources of profit have contributed to the annual rate of bonus, both cash and reversionary.

TABLE 5. $3\frac{3}{4}$ *per cent.*

	QUINQUENNIUM.				
	1st	2nd	3rd	4th	5th
	£	£	£	£	£
Quinquennial Profit—					
Mortality, . .	10,474	5,275	4,002	3,557	931
Interest, . .	3,220	10,589	18,671	27,599	37,331
Loading, . .	4,369	31,539	30,387	29,005	27,338
	18,063	47,403	53,060	60,161	65,600
Less Interim Bonus,	...	1,443	1,760	2,087	2,599
Net Profit, . .	18,063	45,960	51,300	58,074	63,001
Cash Bonus—	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
Annual Rate.					
Mortality, . .	0 4 3	0 2 3	0 1 9	0 1 8	0 0 5
Interest, . .	0 1 4	0 4 6	0 8 3	0 12 10	0 18 7
Loading, . .	0 1 9	0 13 4	0 13 5	0 13 6	0 13 8
	0 7 4	1 0 1	1 3 5	1 8 0	1 12 8
Less Interim Bonus,	...	0 0 7	0 0 9	0 1 0	0 1 3
	0 7 4	0 19 6	1 2 8	1 7 0	1 11 5
Reversionary Bonus—					
Annual Rate.					
Mortality, . .	0 7 4	0 3 4	0 2 4	0 1 11	0 0 5
Interest, . .	0 2 3	0 6 9	0 10 10	0 14 10	0 18 7
Loading, . .	0 3 0	1 0 0	0 17 8	0 15 7	0 13 8
	0 12 7	1 10 1	1 10 10	1 12 4	1 12 8
Less Interim Bonus,	...	0 0 11	0 1 0	0 1 2	0 1 3
	0 12 7	1 9 2	1 9 10	1 11 2	1 11 5

These results are, I think, very interesting. The profit from mortality during the first quinquennium, £10,474, is large, owing to selection, and falls to about one-half in the next quinquennium; and thereafter gradually diminishes until in the fifth quinquennium it is very small indeed, only £931. The profit from interest is the converse of this. It is very small at first, only £3,220, and rapidly increases until in the fifth quinquennium it amounts to £37,331, or more than ten times what it is in the first quinquennium. The profit from loading, on the other hand, is, with the exception of the first quinquennium, very constant all through at about £30,000. In the first quinquennium the profit from loading is very small, owing to the heavy initial cost of acquiring the business. Some offices make an allowance for this by neglecting the first premium when allotting the bonus. The small cost of the interim bonus may be noted.

These features are reflected in the annual rates of bonus, cash and reversionary, where they are brought out even more clearly. With the exception of the first quinquennium, the contribution from mortality is very small, while on the other hand that from interest is very important; indeed, in the last quinquennium it is the most important of all. Nor is the reason far to seek, for in the last quinquennium the reserve value does not differ greatly from the sum assured (including bonus), and the excess of interest may be calculated, roughly speaking, on the latter. The contribution from loading is also important, although in the first quinquennium it is much smaller than in the others.

It is easy now to see what has produced the increase in the surplus and the rate of bonus. As previously mentioned, both Mr. Hardy and Mr. Gunn used the same rate in valuing the policies as in accumulating the funds, the former stating expressly that he had done so as he was ascertaining the profit from mortality only, and therefore they did not take into account in their calculations the profit from interest.

The item of interest is of special importance in the case of endowment assurances on account of their large reserves as compared with whole life policies. The reserves at O^M 3 per cent for whole life and 25 year endowment assurance policies are given for age at entry 30 and various durations, in the following table, a column being added to show the percentage by which the latter is greater than the former:—

TABLE 6.

End of Year.	Whole Life.	Endowment Assurance 25 Years.	Percentage of Excess.
5	6·485	14·154	118·3
10	13·497	30·465	125·7
15	21·079	49·523	134·9
20	29·205	72·209	147·2
25	37·740	100·000	165·0

It will be seen that the reserves of the endowment assurances are considerably more than double those of whole life policies.

The figures relating to the profit from interest showed this item was so important, that I thought it would be interesting to see what the difference would be if 4 per cent interest were adopted instead of $3\frac{3}{4}$ per cent, as the former rate is realised by a number of offices; and the results are given in the following tables. I do not think it necessary to give all the figures in detail, as I have done for $3\frac{3}{4}$ per cent, but merely the results corresponding to those in Tables 3, 4 and 5.

TABLE 7. *4 per cent.*

End of Year.	Reserve.			Fund.	Surplus.
	Sums Assured.	Existing Bonuses.	Total.		
	£	£	£	£	£
5	138,495	...	138,495	157,648	19,153
10	288,320	20,985	309,305	358,918	49,613
15	448,975	76,894	525,869	583,792	57,923
20	620,710	146,159	766,869	835,005	68,136
25	801,300	230,459	1,031,759	1,108,720	76,961

TABLE 8. *4 per cent.*

End of Year.	Bonus.		Rate of Bonus.			Annual Rate of Bonus.	
	Cash.	Revy.	Cash.	Revy.	Total Revy.	Cash.	Revy.
	£	£	£	£	£	£	£
5	19,153	32,542	1·957	3·326	3·326	·391	·665
10	49,613	74,411	5·242	7·863	11·189	1·048	1·573
15	57,923	76,407	6·389	8·428	19·617	1·278	1·686
20	68,136	78,606	7·926	9·144	28·761	1·585	1·829
25	76,961	76,961	9·605	9·605	38·366	1·921	1·921

TABLE 9. *4 per cent.*

	Quinquennium.				
	1st	2nd	3rd	4th	5th
Quinquennial Profit—	£	£	£	£	£
Mortality, . .	10,474	5,279	4,016	3,584	940
Interest, . .	4,310	14,238	25,280	37,634	51,282
Loading, . .	4,369	31,539	30,387	29,005	27,338
	19,153	51,056	59,683	70,223	79,560
Less Interim Bonus,	...	1,443	1,760	2,087	2,599
Net Profit, . .	19,153	49,613	57,923	68,136	76,961
Cash Bonus—					
Annual Rate.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
Mortality, . .	0 4 3	0 2 3	0 1 9	0 1 8	0 0 5
Interest, . .	0 1 9	0 6 0	0 11 2	0 17 6	1 5 7
Loading, . .	0 1 9	0 13 4	0 13 5	0 13 6	0 13 8
	0 7 9	1 1 7	1 6 4	1 12 8	1 19 8
Less Interim Bonus,	...	0 0 7	0 0 9	0 1 0	0 1 3
	0 7 9	1 1 0	1 5 7	1 11 8	1 18 5
Reversionary Bonus—					
Annual Rate.					
Mortality, . .	0 7 4	0 3 4	0 2 4	0 1 11	0 0 5
Interest, . .	0 3 0	0 9 0	0 14 9	1 0 3	1 5 7
Loading, . .	0 3 0	1 0 0	0 17 8	0 15 7	0 13 8
	0 13 4	1 12 4	1 14 9	1 17 9	1 19 8
Less Interim Bonus,	...	0 0 11	0 1 0	0 1 2	0 1 3
	0 13 4	1 11 5	1 13 9	1 16 7	1 18 5

The profit from loading is of course the same as in the former Table, and that from mortality practically the same, but the profit from interest is very much greater. I annex a table showing the excess of the annual reversionary bonus due to an increase of ¼ per cent in the rate of interest, and the percentage which this excess bears to the interest profit at 3¾ per cent, and also to the total profit at that rate:

TABLE 10.

	Excess of Reversionary Bonus.	Percentage of	
		Interest Profit.	Total Profit.
	s. *d.*		
1st Quinqn.	0 9	33·9	6·0
2nd ,,	2 3	34·5	7·9
3rd ,,	3 11	35·3	12·9
4th ,,	5 5	36·4	17·3
5th ,,	7 0	37·4	22·2

The importance of the rate of interest earned by the funds is brought out here very clearly. In the fifth quinquennium the excess due to only ¼ per cent in the rate of interest represents no less than 22·2 per cent of the whole bonus.

I have taken into account the three principal sources of profit, namely, mortality, interest, and loading; but there are others, such as profit from surrenders and lapses, from investments, and from non-participating and annuity business. These, however, with the exception of the first mentioned, vary considerably, and it is difficult, if not impossible, to make allowance for them in an investigation of this kind. I have therefore left them out of account; but I thought it would be desirable to see how the profit from surrenders and lapses would affect the calculations, especially in the first quinquennium; and with this object in view I got out the rate of withdrawal for endowment assurances in the manner explained in Appendix B.

The rates so obtained for grouped ages at entry 28 to 32, that is, central age 30, are as follows. It is only practicable to give them for the first nine years of assurance. The rates for the whole life with profits class are placed alongside for comparison.

TABLE 11.

Years Elapsed.	Rate of Withdrawal.	
	Endowment Assurance.	Whole Life.
1	·08935	·08150
2	·04676	·05111
3	·03586	·03870
4	·03361	·03101
5	·02664	·02652
6	·02239	·02367
7	·02033	·02102
8	·01853	·01901
9	·01722	·01668

The endowment assurance rates run so regularly, and agree so closely with the whole life, that I took them just as they stand. It is necessary to join them on at age 40 to the ultimate rate, excluding the first ten years, and the following are the rates deduced from the aggregate table:

TABLE 12.

Age.	Endowment Assurance.		Whole Life.
	Unadjusted.	Adjusted.	Unadjusted.
40	·01344	·0156	·01257
41	·01621	·0142	·01191
42	·01169	·0130	·01161
43	·01020	·0120	·01187
44	·01240	·0112	·01166
45	·01067	·0112	·01128
46	·01045	·0112	·01110
47	·01197	·0112	·01052
48	·00949	·0112	·01075
49	·01507	·0112	·01063
50	·01152	·0112	·00962
51	·01102	·0112	·00936
52	·00989	·0112	·00948
53	·01091	·0112	·00860
54	·01125	·0112	·00929

It will be seen that the rate of withdrawal in endowment assurances is after the first few ages fairly constant, whereas the rate of withdrawal in the whole life class steadily diminishes. I examined the figures of the endowment assurance experience;

and as it seems to be a feature of it, I kept the rate constant throughout from age 44, and I give above a column showing the rates actually used. It is somewhat difficult to explain this feature, but probably it may to some extent be accounted for by the favourable terms upon which endowment assurances can be surrendered shortly before they mature, and they would thus count as "withdrawals" instead of as "terminations."

We are now in a position to ascertain the combined effect of deaths and withdrawals on the policies taken out, and the following table shows, on the above assumptions, the number of policies at the beginning of each year, and the number of deaths and withdrawals during each year. The number of withdrawals is obtained by multiplying the number at the beginning of the year, less the deaths, by the rate of withdrawal deduced above.

TABLE 13.

Year.	Policies at beginning of Year.	Deaths during Year.	Withdrawals at end of Year.	Year.	Policies at beginning of Year.	Deaths during Year.	Withdrawals at end of Year.
1	10,000	25	891	14	6357	56	71
2	9,084	37	423	15	6230	57	69
3	8,624	40	308	16	6104	59	68
4	8,276	42	277	17	5977	60	66
5	7,957	43	211	18	5851	62	65
6	7,703	45	171	19	5724	64	63
7	7,487	47	151	20	5597	66	62
8	7,289	48	134	21	5469	68	60
9	7,107	50	122	22	5341	70	59
10	6,935	52	107	23	5212	72	58
11	6,776	53	95	24	5082	75	56
12	6,628	54	85	25	4951	77	...
13	6,489	55	77				

The next step is to determine the surrender value to be allowed. A number of offices allow a surrender value after two premiums have been paid; but even in those offices where three premiums require to be paid before a surrender value is guaranteed, an allowance would probably be made, in the case of an endowment assurance of the kind we are dealing with, after two premiums had been paid. The following table shows the reserve value at O^M 3 per cent, assuming the bonus to be as in Table 16; also, on the same assumption, the surrender value, taking it as (1) the O^M 3 per cent reserve less one-third, (2) the O^M $4\frac{1}{2}$ per cent reserve excluding the first year, and (3) the value at O^M $4\frac{1}{2}$ per cent of the paid-up policy for the proportion of the sum assured

which the number of premiums paid bears to the original number payable, together with the value of the existing bonuses. Another way of calculating the surrender value in the early years of assurance is to take the value of a corresponding whole life policy, say one half of the premiums paid after the first, and add $\frac{9}{10}$ths of the difference between the whole life and endowment assurance premiums. A column has been added showing the surrender value according to this method for the first ten years.

TABLE 14.

End of Year.	O^M 3 per cent Reserve.	O^M 3 per cent less one-third.	O^M 4½ per cent ex. 1st Year.	P. U. P. at O^M 4½ per cent.	Life Policy plus excess.
	£	£	£	£	£
2	5·433	3·622	2·359	3·291	4·049
3	8·259	5·506	4·809	5·114	6·690
4	11·164	7·443	7·367	7·065	9·330
5	14·154	9·436	10·031	9·152	11·971
10	33·407	22·271	27·569	24·372	25·174
15	59·695	39·797	52·961	48·664	...
20	91·564	61·043	86·263	82·721	...
24	124·353	82·902	122·850	121·860	...

The 3 per cent reserve, less one-third, is frequently used in calculating values in the early years of assurance, and it will be seen that for the first ten years the O^M 3 per cent reserve, less one third, agrees very closely with the value of the paid-up policy at O^M 4½ per cent; also that these values are not too great, having regard to the figures in the last column of the table. After ten years it is known that the deduction of one-third from the reserve gives too small surrender values; but on the other hand the 4½ per cent value excluding the first year, which does not work well in the early years of assurance, gives very fair values. It will be seen that after ten years the O^M 4½ per cent value, excluding the first year, agrees very closely with the value of the paid-up policy at O^M 4½ per cent, and there is no doubt that, taken all over, the value of the paid-up policy at O^M 4½ per cent best answers the purpose. I have therefore adopted it.

A fund was then made up similar to that already given, the only difference being that surrenders and lapses, or surrenders, as I shall designate them for shortness in future, were brought into the calculation. I need not give the details, but the results for 3¾ per cent interest are shown in the following tables:—

TABLE 15. $3\frac{3}{4}$ *per cent, with surrenders.*

End of Year.	Reserve.			Fund.	Surplus.
	Sums Assured.	Existing Bonuses.	Total.		
	£	£	£	£	£
5	109,027	...	109,027	129,036	20,009
10	206,430	19,939	226,369	267,053	40,684
15	302,288	62,092	364,380	405,614	41,234
20	394,912	105,854	500,766	543,492	42,726
25	487,400	152,764	640,164	681,915	41,751

TABLE 16. $3\frac{3}{4}$ *per cent, with surrenders.*

End of Year.	Bonus.		Rate of Bonus.			Annual Rate of Bonus.	
	Cash.	Revy.	Cash.	Revy.	Total Revy.	Cash.	Revy.
	£	£	£	£	£	£	£
5	20,009	33,996	2·598	4·413	4·413	·520	·883
10	40,684	61,019	6·004	9·005	13·418	1·201	1·801
15	41,234	54,393	6·755	8·911	22·329	1·351	1·782
20	42,726	49,292	7·812	9·013	31·342	1·562	1·803
25	41,751	41,751	8·566	8·566	39·908	1·713	1·713

These results will be better appreciated if they are placed side by side with those in Table 3, where, of course, there are no surrenders. This is done in the following table :—

TABLE 17. $3\frac{3}{4}$ *per cent.*

End of Year.	With Surrenders.			Without Surrenders.		
	Reserve for Sum Assured and Bonus.	Fund.	Surplus.	Reserve for Sum Assured and Bonus.	Fund.	Surplus.
	£	£	£	£	£	£
5	109,027	129,036	20,009	138,495	156,558	18,063
10	226,369	267,053	40,684	308,111	354,071	45,960
15	364,380	405,614	41,234	520,589	571,889	51,300
20	500,766	543,492	42,726	753,966	812,040	58,074
25	640,164	681,915	41,751	1,007,061	1,070,062	63,001

The effect of surrenders upon life business is at once apparent here. They make substantial reductions upon the reserve, the fund, and the surplus, with the exception of the first quinquennium in the case of the last mentioned, which they slightly increase. It is curious that the surplus after the first quinquennium is practically stationary; but of course it has to be distributed over a smaller number of policies, and consequently the true effect cannot be seen until the rates of bonus are compared. This is done for the annual rate of bonus, both cash and reversionary, in the following table:—

TABLE 18. $3\frac{3}{4}$ *per cent.*

End of Year.	With Surrenders.		Without Surrenders.	
	Cash.	Revy.	Cash.	Revy.
	£	£	£	£
5	·520	·883	·369	·627
10	1·201	1·801	·971	1·457
15	1·351	1·782	1·132	1·493
20	1·562	1·803	1·351	1·559
25	1·713	1·713	1·572	1·572

It will be seen that surrenders have a very marked effect upon the rate of bonus, increasing it at the end of the first quinquennium by over 40 per cent, and this helps to counteract the heavy initial cost of acquiring the business. The actual percentages by which the rate of bonus is increased through their influence, are as follows:—

TABLE 19.

End of Year.	Increase in rate of Bonus.
5	40·8 per cent.
10	23·6 do.
15	19·4 do.
20	15·6 do.
25	9·0 do.

An analysis of the rates of bonus is given in the following table:—

TABLE 20. $3\frac{3}{4}$ *per cent, with surrenders.*

	QUINQUENNIUM.				
	1st	2nd	3rd	4th	5th
	£	£	£	£	£
Quinquennial Profit—					
Mortality, . .	9,537	4,020	2,975	2,312	593
Interest, . .	2,803	8,300	13,644	19,062	24,341
Loading, . .	739	23,865	21,217	19,110	17,026
Surrenders, . .	6,930	5,572	4,604	3,617	1,393
	20,009	41,757	42,440	44,101	43,353
Less Interim Bonus,	...	1,073	1,206	1,375	1,602
Net Profit, . .	20,009	40,684	41,234	42,726	41,751
Cash Bonus—	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
Annual Rate.					
Mortality, . .	0 5 0	0 2 4	0 1 11	0 1 8	0 0 6
Interest, . .	0 1 5	0 4 11	0 8 11	0 13 11	0 19 11
Loading, . .	0 0 5	0 14 1	0 13 11	0 14 0	0 14 0
Surrenders, . .	0 3 7	0 3 4	0 3 0	0 2 8	0 1 2
	0 10 5	1 4 8	1 7 9	1 12 3	1 15 7
Less Interim Bonus,	...	0 0 8	0 0 9	0 1 0	0 1 4
	0 10 5	1 4 0	1 7 0	1 11 3	1 14 3
Reversionary Bonus—					
Annual Rate.					
Mortality, . .	0 8 5	0 3 7	0 2 7	0 1 11	0 0 6
Interest, . .	0 2 6	0 7 4	0 11 9	0 16 1	0 19 11
Loading, . .	0 0 8	1 1 2	0 18 4	0 16 2	0 14 0
Surrenders, . .	0 6 1	0 4 11	0 4 0	0 3 1	0 1 2
	0 17 8	1 17 0	1 16 8	1 17 3	1 15 7
Less Interim Bonus,	...	0 1 0	0 1 1	0 1 2	0 1 4
	0 17 8	1 16 0	1 15 7	1 16 1	1 14 3

The only feature calling for special remark in the above table is the small profit from loading in the first quinquennium, due of course to the large number of surrenders and consequent loss of loading. The profit from surrenders takes its place, and the two together are much larger than the profit from loading only.

Similar tables with surrenders are given below at 4 per cent interest, and do not call for any special remark.

TABLE 21. 4 *per cent, with surrenders.*

End of Year.	Reserve.			Fund.	Surplus.
	Sums Assured.	Existing Bonuses.	Total.		
	£	£	£	£	£
5	109,027	...	109,027	129,983	20,956
10	206,430	20,882	227,312	270,890	43,578
15	302,288	66,023	368,311	414,438	46,127
20	394,912	114,893	509,805	559,549	49,744
25	487,400	169,273	656,673	707,599	50,926

TABLE 22. 4 *per cent, with surrenders.*

End of Year.	Bonus.		Rate of Bonus.			Annual Rate of Bonus.	
	Cash.	Revy	Cash.	Revy.	Total Revy.	Cash.	Revy.
	£	£	£	£	£	£	£
5	20,956	35,605	2·721	4·622	4·622	·544	·924
10	43,578	65,360	6·431	9·646	14·268	1·286	1·929
15	46,127	60,847	7·557	9·968	24·236	1·511	1·994
20	49,744	57,388	9·096	10·493	34·729	1·819	2·099
25	50,926	50,926	10·449	10·449	45·178	2·090	2·090

TABLE 25.—REVERSIONARY BONUS—MORTALITY PROFIT.

DURATION.	QUINQUENNIUM.					
	1st	2nd	3rd	4th	5th	6th
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
Without Surrenders—						
30 Years, .	0 2 5	—5	0 0 2	0 1 2	0 1 2	0 0 3
25 ,, .	0 7 4	0 3 4	0 2 4	0 1 11	0 0 5	...
20 ,, .	0 11 11	0 5 8	0 3 2	0 1 0	...	...
With Surrenders—						
25 Years, .	0 8 5	0 3 7	0 2 7	0 1 11	0 0 6	...

TABLE 26.—REVERSIONARY BONUS—LOADING PROFIT.

DURATION.	QUINQUENNIUM.					
	1st	2nd	3rd	4th	5th	6th
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
Without Surrenders—						
30 Years, .	0 7 5	1 2 3	0 19 7	0 17 3	0 15 1	0 13 2
25 ,, .	0 3 0	1 0 0	0 17 8	0 15 7	0 13 8	...
20 ,, .	—1 10	0 18 6	0 16 5	0 14 6	...	...
With Surrenders—						
25 Years, .	0 0 8	1 1 2	0 18 4	0 16 2	0 14 0	...

TABLE 27.—REVERSIONARY BONUS—INTEREST PROFIT.

DURATION.	QUINQUENNIUM.					
	1st	2nd	3rd	4th	5th	6th
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
Without Surrenders—$3\frac{3}{4}$ %—						
30 Years, .	0 2 0	0 6 0	0 9 8	0 13 2	0 16 6	0 19 9
25 ,, .	0 2 3	0 6 9	0 10 10	0 14 10	0 18 7	...
20 ,, .	0 2 7	0 7 9	0 12 8	0 17 6	...	...
Without Surrenders—4 %—						
30 Years, .	0 2 9	0 8 1	0 13 1	0 18 0	1 2 8	1 7 4
25 ,, .	0 3 0	0 9 0	0 14 9	1 0 3	1 5 7	...
20 ,, .	0 3 5	0 10 5	0 17 3	1 3 10	...	...
With Surrenders—						
25 Years, $3\frac{3}{4}$ %,	0 2 6	0 7 4	0 11 9	0 16 1	0 19 11	...
25 Years, 4 %,	0 3 4	0 9 10	0 16 0	1 1 11	1 7 6	...

The profit from mortality is small, and may be said to be a decreasing quantity. It is very small when the duration is long; indeed, there is an actual loss in the second quinquennium for the 30 year endowment assurance. This is due to the rate by the O^M table used in the valuation being slightly less, at some young ages, than 80 per cent of the rate by the corresponding select table. The increase in the profit from mortality as the original duration decreases, and the age at entry increases, is due to the rate of mortality by the O^M table gradually becoming larger than 80 per cent of the rate by the corresponding select table.

The profit from loading is considerable, and forms an important item in the bonus; but it also, after the first quinquennium, is a decreasing quantity. The effect of a high rate of initial expenditure is shown very clearly in the first quinquennium, where the profit diminishes very rapidly with the duration, until in the 20 year endowment assurance there is an actual loss.

The profit from interest is small at first, but it rapidly increases, until in the later years it becomes the most important of all the sources of profit. It is the one that counteracts the decreasing tendency of the other two, and it is to it, therefore, that we must look for the maintenance of the rate of bonus in this class of assurance. The investment of the funds of an office has always been an important duty, demanding much skill and judgment, but it looks as if it were destined to become more important in the future, and to require even more attention than it has hitherto received.

APPENDIX A.

Age at entry, 20. *Duration*, 30 *years*. *Interest*, $3\frac{3}{4}$ *per cent.*

End of Year.	Reserve.			Fund.	Surplus.
	Sums Assured.	Existing Bonuses.	Total.		
	£	£	£	£	£
5	108,099	...	108,099	123,064	14,965
10	226,117	16,541	242,658	280,212	37,554
15	354,181	59,606	413,787	457,598	43,811
20	493,811	113,823	607,634	659,428	51,794
25	646,150	181,954	828,104	887,126	59,022
30	811,200	263,817	1,075,017	1,140,246	65,229

End of Year.	Bonus.		Rate of Bonus.			Annual Rate of Bonus.	
	Cash.	Revy.	Cash.	Revy.	Total Revy.	Cash.	Revy.
	£	£	£	£	£	£	£
5	14,965	29,196	1·525	2·974	2·974	·305	·595
10	37,554	64,544	3·929	6·753	9·727	·786	1·351
15	43,811	66,204	4·731	7·149	16·876	·946	1·430
20	51,794	68,594	5·799	7·680	24·556	1·160	1·536
25	59,022	68,169	6·897	7·966	32·522	1·379	1·593
30	65,229	65,229	8·041	8·041	40·563	1·608	1·608

Age at entry, 20. *Duration*, 30 *years*. *Interest*, 3¾ *per cent*.

	QUINQUENNIUM.					
	1st	2nd	3rd	4th	5th	6th
	£	£	£	£	£	£
Quinquennial Profit—						
Mortality, .	3,069	—576	220	1,940	2,096	525
Interest, . .	2,538	8,364	14,857	22,242	30,612	39,992
Loading, .	9,358	30,917	30,036	29,057	27,965	26,693
	14,965	38,705	45,113	53,239	60,673	67,210
Less Interim Bonus, . .	...	1,151	1,302	1,445	1,651	1,981
Net Profit, .	14,965	37,554	43,811	51,794	59,022	65,229
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
Cash Bonus— Annual Rate.						
Mortality, .	0 1 3	—3	0 0 1	0 0 10	0 1 0	0 0 3
Interest, . .	0 1 0	0 3 6	0 6 5	0 10 0	0 14 4	0 19 9
Loading, . .	0 3 10	0 13 0	0 13 0	0 13 0	0 13 0	0 13 2
	0 6 1	0 16 3	0 19 6	1 3 10	1 8 4	1 13 2
Less Interim Bonus, . .	...	0 0 6	0 0 7	0 0 8	0 0 9	0 1 0
	0 6 1	0 15 9	0 18 11	1 3 2	1 7 7	1 12 2
Reversionary Bonus— Annual Rate.						
Mortality, .	0 2 5	—5	0 0 2	0 1 2	0 1 2	0 0 3
Interest, . .	0 2 0	0 6 0	0 9 8	0 13 2	0 16 6	0 19 9
Loading, . .	0 7 5	1 2 3	0 19 7	0 17 3	0 15 1	0 13 2
	0 11 10	1 7 10	1 9 5	1 11 7	1 12 9	1 13 2
Less Interim Bonus, . .	...	0 0 10	0 0 10	0 0 11	0 0 11	0 1 0
	0 11 10	1 7 0	1 8 7	1 10 8	1 11 10	1 12 2

Age at entry, 20. *Duration*, 30 *years*. *Interest*, 4 *per cent*.

End of Year.	Reserve.			Fund.	Surplus.
	Sums Assured.	Existing Bonuses.	Total.		
	£	£	£	£	£
5	108,099	...	108,099	123,920	15,821
10	226,117	17,487	243,604	284,037	40,433
15	354,181	63,821	418,002	467,073	49,071
20	493,811	124,251	618,062	677,953	59,891
25	646,150	202,307	848,457	918,939	70,482
30	811,200	298,644	1,109,844	1,190,443	80,599

End of Year.	Bonus.		Rate of Bonus.			Annual Rate of Bonus.	
	Cash.	Revy.	Cash.	Revy.	Total Revy.	Cash.	Revy.
	£	£	£	£	£	£	£
5	15,821	30,866	1·612	3·144	3·144	·322	·629
10	40,433	69,492	4·230	7·271	10·415	·846	1·454
15	49,071	74,153	5·299	8·008	18·423	1·060	1·602
20	59,891	79,317	6·705	8·880	27·303	1·341	1·776
25	70,482	81,405	8·236	9·512	36·815	1·647	1·902
30	80,599	80,599	9·936	9·936	46·751	1·987	1·987

Age at entry, 20. *Duration*, 30 *years*. *Interest*, 4 *per cent*.

	QUINQUENNIUM.					
	1st	2nd	3rd	4th	5th	6th
	£	£	£	£	£	£
Quinquennial Profit—						
Mortality, .	3,069	—574	218	1,954	2,121	527
Interest, . .	3,394	11,241	20,119	30,325	42,047	55,360
Loading, . .	9,358	30,917	30,036	29,057	27,965	26,693
	15,821	41,584	50,373	61,336	72,133	82,580
Less Interim Bonus, . .	...	1,151	1,302	1,445	1,651	1,981
Net Profit, .	15,821	40,433	49,071	59,891	70,482	80,599
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
Cash Bonus— Annual Rate.						
Mortality, .	0 1 3	—3	0 0 1	0 0 10	0 1 0	0 0 3
Interest, . .	0 1 4	0 4 8	0 8 8	0 13 7	0 19 8	1 7 4
Loading, . .	0 3 10	0 13 0	0 13 0	0 13 0	0 13 0	0 13 2
	0 6 5	0 17 5	1 1 9	1 7 5	1 13 8	2 0 9
Less Interim Bonus, . .	...	0 0 6	0 0 7	0 0 8	0 0 9	0 1 0
	0 6 5	0 16 11	1 1 2	1 6 9	1 12 11	1 19 9
Reversionary Bonus— Annual Rate.						
Mortality, .	0 2 5	—5	0 0 2	0 1 2	0 1 2	0 0 3
Interest, . .	0 2 9	0 8 1	0 13 1	0 18 0	1 2 8	1 7 4
Loading, . .	0 7 5	1 2 3	0 19 7	0 17 3	0 15 1	0 13 2
	0 12 7	1 9 11	1 12 10	1 16 5	1 18 11	2 0 9
Less Interim Bonus, . .	...	0 0 10	0 0 10	0 0 11	0 0 11	0 1 0
	0 12 7	1 9 1	1 12 0	1 15 6	1 18 0	1 19 9

Age at entry, 40. *Duration* 20 *years.* *Interest*, $3\frac{3}{4}$ *per cent.*

End of Year.	Reserve.			Fund.	Surplus.
	Sums Assured.	Existing Bonuses.	Total.		
	£	£	£	£	£
5	183,836	...	183,836	204,571	20,735
10	378,654	22,328	400,982	455,110	54,128
15	579,390	81,163	660,553	718,129	57,576
20	782,300	145,140	927,440	988,361	60,921

End of Year.	Bonus.		Rate of Bonus.			Annual Rate of Bonus.	
	Cash.	Revy.	Cash.	Revy.	Total Revy.	Cash.	Revy.
	£	£	£	£	£	£	£
5	20,735	30,774	2·136	3·170	3·170	·427	·634
10	54,128	70,985	5·859	7·684	10·854	1·172	1·537
15	57,576	66,307	6·686	7·699	18·553	1·337	1·540
20	60,921	60,921	7·787	7·787	26·340	1·557	1.557

	Quinquennium.			
	1st	2nd	3rd	4th
Quinquennial Profit—	£	£	£	£
Mortality, . . .	19,577	9,960	5,877	1,876
Interest, . . .	4,221	13,723	23,732	34,153
Loading, . . .	— 3,063	32,582	30,759	28,417
	20,735	56,265	60,368	64,446
Less Interim Bonus, .	...	2,137	2,792	3,525
Net Profit, . . .	20,735	54,128	57,576	60,921
Cash Bonus— Annual Rate.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
Mortality, . . .	0 8 1	0 4 4	0 2 9	0 1 0
Interest, . . .	0 1 9	0 5 11	0 11 0	0 17 6
Loading, . . .	— 1 3	0 14 1	0 14 3	0 14 6
	0 8 7	1 4 4	1 8 0	1 13 0
Less Interim Bonus, .	...	0 0 11	0 1 3	0 1 10
	0 8 7	1 3 5	1 6 9	1 11 2
Reversionary Bonus— Annual Rate.				
Mortality, . . .	0 11 11	0 5 8	0 3 2	0 1 0
Interest, ...	0 2 7	0 7 9	0 12 8	0 17 6
Loading, . . .	— 1 10	0 18 6	0 16 5	0 14 6
	0 12 8	1 11 11	1 12 3	1 13 0
Less Interim Bonus, .	...	0 1 2	0 1 6	0 1 10
	0 12 8	1 10 9	1 10 9	1 11 2

Age at entry, 40. *Duration*, 20 *years*. *Interest*, 4 *per cent*.

End of Year.	Reserve.			Fund.	Surplus.
	Sums Assured.	Existing Bonuses.	Total.		
	£	£	£	£	£
5	183,836	...	183,836	205,994	22,158
10	378,654	23,860	402,514	461,383	58,869
15	579,390	87,823	667,213	733,210	65,997
20	782,300	160,916	943,216	1,016,567	73,351

End of Year.	Bonus.		Rate of Bonus.			Annual Rate of Bonus.	
	Cash.	Revy.	Cash.	Revy.	Total Revy.	Cash.	Revy.
	£	£	£	£	£	£	£
5	22,158	32,886	2·282	3·387	3·387	·456	·677
10	58,869	77,202	6·372	8·357	11·744	1.274	1·671
15	65,997	76,005	7·663	8·826	20·570	1·533	1·765
20	73,351	73,351	9·376	9·376	29·946	1·875	1·875

	Quinquennium.											
	1st			2nd			3rd			4th		
Quinquennial Profit—	£			£			£			£		
Mortality,	19,577			9,970			5,892			1,885		
Interest,	5,644			18,454			32,138			46,574		
Loading,	—3,063			32,582			30,759			28,417		
	22,158			61,006			68,789			76,876		
Less Interim Bonus,	...			2,137			2,792			3,525		
Net Profit,	22,158			58,869			65,997			73,351		
Cash Bonus— Annual Rate.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
Mortality,	0	8	1	0	4	4	0	2	9	0	1	0
Interest,	0	2	4	0	8	0	0	14	11	1	3	10
Loading,	—1	3		0	14	1	0	14	3	0	14	6
	0	9	2	1	6	5	1	11	11	1	19	4
Less Interim Bonus,	...			0	0	11	0	1	3	0	1	10
	0	9	2	1	5	6	1	10	8	1	17	6
Reversionary Bonus— Annual Rate.												
Mortality,	0	11	11	0	5	8	0	3	2	0	1	0
Interest,	0	3	5	0	10	5	0	17	3	1	3	10
Loading,	—1	10		0	18	6	0	16	5	0	14	6
	0	13	6	1	14	7	1	16	10	1	19	4
Less Interim Bonus,	...			0	1	2	0	1	6	0	1	10
	0	13	6	1	13	5	1	15	4	1	17	6

APPENDIX B.

A table of distribution of withdrawals is given in the Endowment Assurance Experience for quinquennial groups of ages for the first ten years after entry, for 0 to 2, 2 to 6, 6 to 8, and 8 to 12 months in each year. Those withdrawing in 0 to 2 months were assumed by me to do so at the end of the previous year, as the great majority of them would no doubt lapse at the expiry of the days of grace, and the premium due at the end of that year would not be paid. These added to the total of the 2 to 6, 6 to 8, and 8 to 12 months in that year, gave the total withdrawals in the year. They were all assumed to withdraw at the end of the year, thus giving the rate of withdrawal or non-renewal. An example will make this clear. The figures, except those for withdrawals, are taken from pages 58-66 of the original statistics :—

AGES AT ENTRY, 28-32.

First Year.

Number of Entrants,		34,955
Add Entered,		...
Total,		34,955
Deduct—Existing,	3,641	
Died,	88	
Terminations,	...	3,729
Exposed to risk of withdrawal,		31,226
Number of withdrawals in first year,		1,008
Add withdrawals in first two months of second year,		1,782
Assumed number of withdrawals at end of first year,		2,790
Rate of withdrawal at end of first year,	2,790 ÷ 31,226 = ·08935	
Number left at end of first year,	31,226 − 2,790 = 28,436	

Second Year.

Number at beginning of year,		28,436
Add Entered,		179
Total,		28,615
Deduct—Existing,	3,127	
Died,	83	
Terminations,	...	3,210
Exposed to risk of withdrawal,		25,405

Number of withdrawals in second year, . . .	2,232
Deduct withdrawals in first two months, . . .	1,782
	450
Add withdrawals in first two months of third year, .	738
Assumed number of withdrawals at end of second year,	1,188
Rate of withdrawal at end of second year,	1,188 ÷ 25,405 = ·04676
and so on.	

The Aggregate Table, excluding the first ten years, is given on page 138 of the Endowment Assurance Experience, where the data have been arranged for the purpose of ascertaining the rate of mortality. The withdrawals there given were assumed to have taken place at the exact age, and they accordingly require to be added to the exposed to risk of death in order to get the corresponding exposed to risk of withdrawal. The number of withdrawals, divided by the sum thus obtained, gave the rate of withdrawal. An example will make this clear :—

AGE 40.

Exposed to risk of death, ...	10,275
Add withdrawals, ...	140
Total, . .	10,415
Rate of non-renewal, ...	140 ÷ 10,415 = ·01344
and so on.	

It is not possible in this case to make any correction for the distribution of withdrawals throughout the year, but this is immaterial, as the rates after all are only approximate.

DISCUSSION.

THE PRESIDENT (Mr. N. B. GUNN).—It is gratifying at the commencement of the session to have had such an important paper as this by Mr. Chatham. The subject of the paper must appeal to us all, and I shall be glad if there is a full discussion. We have all studied to a certain extent the topic with which he deals. I trust, therefore, that as many as possible will mention their views.

Mr. GORDON DOUGLAS.—I think we as a Faculty are to be congratulated on having such an excellent paper submitted to us by Mr. Chatham at this time. Mr. Chatham's papers are habitually practical in character, clear in expression, and convincing in argument, and the present is no exception. It deals with an important point in our everyday business life, and one that is becoming more and more important as years go on, seeing that in many offices endowment assurances are pushing aside whole-life policies almost altogether. Formerly it was difficult to say what effect this great change in class of business would have on the rate of bonus, but Mr. Chatham's paper to a great extent meets and overcomes that difficulty. We must all have been impressed by the amount of labour involved in the preparation of the voluminous tables submitted, and it might truly be said that only a very busy man like Mr. Chatham could have found time to go into the whole matter so fully and exhaustively. As profit from interest enters so largely into the building up of all bonuses, no investigation into the bonuses under endowment assurances can be complete unless it takes into account the difference between the rate of interest assumed in the valuation and that realised on the funds. As endowment assurances carry so much larger reserve values than ordinary policies, the surplus rate of interest tells the more heavily in their case. As regards the assumptions made by Mr. Chatham, little exception can, in my opinion, be taken to them. The only one to which I would refer is that as to the probable rates of mortality in this class. I am inclined to think that 80 per cent of the $O^{[M]}$ is just on the low side. My view is that, in future, endowment assurances will not show quite so favourable rates of mortality as those brought out in the recent joint investigation. There is, I think, a distinct tendency, justifiable in certain cases, but not so in others, to encourage doubtful lives to take endowment assurances by accepting them on easier extra terms than would apply to ordinary policies. Formerly, too, the selection of an endowment assurance lay much more with the individual himself, but agents are finding out that, by appealing to the selfish side of human nature, they can often get a man to effect an endowment assurance who otherwise would not assure at all. The selection of the particular form of policy is, therefore, not so much based as it used to be upon the proposer's confidence in his own life prospects. However, Mr. Chatham shows that after the first quinquennium the profit from mortality does not enter largely into the ground-work of bonuses, and it is therefore of comparatively little consequence whether a somewhat higher rate of mortality should have been assumed. One point clearly brought out in the paper is the strain involved in the case of offices doing a relatively large new business, if the same rate of bonus is given to policies participating for the first time as to those of longer standing. The weight of expenses is heaviest, and the profit from mortality is lightest, in the first quinquennium. To give the same rate of bonus to new as to old policies means, therefore, that the office must look to profit from lapses, etc., to equalise the rate; and not only so, but the older policyholders must, to a certain extent, help to make up the bonuses to

those who have newly joined. The great effect of a large rate of realised interest is very clearly brought out in the paper, where it is seen that, when surrenders are allowed for, a difference of ¼ per cent means differences in bonus of 10d., 2s. 7d., 4s. 3d., 5s. 10d., and 7s. 7d. per cent per annum in the several quinquenniums respectively. Another way of looking at the matter is that overhead every shilling by which the realised rate of interest is raised increases the endowment assurance bonus by 9d. or 10d. per £100 per annum. One's thoughts are therefore not unnaturally led in the direction of considering what would be the different effect of a high as against a low rate of income-tax. A difference of 6d. per £1 in the rate of tax involves a difference of about 1s. 6d. a year in the rate of bonus per £100 under such policies as those now being dealt with. Not the least interesting of the tables in the paper are those relating to withdrawals and surrender-values. As regards the former, I was surprised to find that in 15 out of the 24 ages or periods in the table, the rate of withdrawal is higher in the case of endowment assurances than under whole-life policies. This, I think, is largely due, as Mr. Chatham suggests, to the very favourable terms on which endowment assurances can be discontinued when a considerable portion of the original period has expired. It will be seen also that the combined effect of deaths and withdrawals is that only about one-half of the original number of policies become claims by survivance of the full term. As regards surrender-values, I was not aware that any offices actually based their surrender-values on the values at 4½ per cent of the proportionate paid-up policies, although I have often employed the method myself as a check. The drawback to it is that the proportionate paid-up policy itself is somewhat arbitrary. Reverting to the substance of the paper, I think the one great lesson to be gathered from the results brought out may be summed up thus: "Be naturally careful as regards expenses and the selection of lives, but make a point of looking well to the realised rate of interest, and the bonuses will look after themselves." The greatest strain in the case of bonuses under endowment assurances is in the last few years, when the cash and reversionary values closely approximate. It is just in these years that the surplus rate of interest tells most heavily, and helps to relieve the pressure. In thanking Mr. Chatham for this, the latest of his valuable contributions to the Transactions of the Faculty, I would suggest that one thing still remains to be done. That is, to have a paper prepared on similar lines applicable to whole of life policies, with a view to the comparison of the "model" bonuses under the two classes, so that it may be seen to what extent the rates of allocation should differ when the funds applicable to the different kinds of policies are not kept separate. If at some future time Mr. Chatham sees his way to bestow his labours in this direction, he will lay us under a still deeper debt of gratitude to him, substantial as our debt already is.

Mr. H. W. Brown.—I should like to voice our admiration of Mr. Chatham's tireless energy, and our gratitude that the latest effort of his indefatigable pen should have been presented to the Faculty. The paper is well calculated to hearten us after the rather gloomy vaticinations in the two papers first referred to by Mr. Chatham. At first sight the deficiency in the bonus of the first quinquennium is somewhat alarming, but the mischief is perhaps more apparent than real. There may be some theoretical truth in the contention that there is a tendency to give too large bonuses to recent entrants, but it must be recognised that the heavy initial expenditure is incurred for the good of the office as a going concern, and that for bonus purposes a certain proportion of such expenditure may equitably be thrown over the whole business. An O^M 3 per cent valuation can perhaps scarcely be defended until an office acquires a settled position. For an office giving

a uniform reversionary bonus such a valuation is only justifiable inasmuch as in the case of a well-established office under normal conditions it affords a wonderfully close approximation to a more precise estimate of the liabilities obtained by using the true rate of interest, making provision for future bonuses, and taking credit for the office premiums receivable, less a reasonable deduction for expenses. When an office has only been a few years in existence an O^M 3 per cent valuation puts an unnecessarily severe strain on its resources. If, then, an O^M 3 per cent valuation can only be vindicated on the broad average of the whole business, would it not be preferable to have tables showing how the bonuses would result under actual working conditions in a well-established office with new entrants each year? Of course, this suggestion involves the laborious construction of a model office. The following, however, is a rough attempt in this direction. Using Mr. Chatham's figures in Table 3, which necessitates the assumption that the whole of the new business of each quinquennium is effected at the commencement of that period, and assuming further that the amount of the new business is uniform, the following table shows the results for an office twenty and twenty-five years in existence:—

Age of Office.	Liability.	Fund.	Surplus.	Annual Rate of Reversionary Bonus.
	£	£	£	£ s. d.
20 years, . .	1,721,161	1,894,558	173,397	1 6 4%
25 years, . .	2,728,222	2,964,620	236,398	1 7 6%

Of course, in a proper model office the figures would be duly weighted. In a paper read before the Actuarial Society of Edinburgh in April 1899, an attempt was made to show that a margin of $\frac{1}{2}$ to $\frac{3}{4}$ of one per cent above the valuation rate of interest will maintain a constant reversionary bonus of 30s. per cent per annum under endowment assurance policies, and that conclusion seems to be confirmed by the results of the present paper, although it was reached through an entirely different avenue. I was curious to learn the effect of Mr. Chatham's mode of procedure when applied to whole-life policies, and accordingly, adopting the same hypotheses as Mr. Chatham, but taking the mortality experienced to be that of the $O^{[M]}$ table, and using £2, 9s. 6d. as the average office whole-life premium for age 30, tables corresponding to Nos. 1, 2, 3, 4, and 5 were constructed with the following results:—

WHOLE LIFE POLICIES.

Age at entry, 30. *Interest*, $3\frac{3}{4}$ *per cent.*

End of Year.	Bonus.		Rate of Bonus.			Annual Rate of Bonus.	
	Cash.	Revy.	Cash.	Revy.	Total Revy.	Cash.	Revy.
	£	£	£	£	£	£	£
5	17,540	41,267	1·802	4·240	4·240	·360	·848
10	28,801	61,423	3·086	6·581	10·821	·617	1·316
15	29,375	56,892	3·320	6·431	17·252	·664	1·286
20	32,015	56,448	3·869	6·821	24·073	·774	1·364
25	33,530	54,033	4·424	7·129	31·202	·885	1·426

	1st	2nd	3rd	4th	5th
Profit during Quinquennium—	£	£	£	£	£
Mortality, . .	5,783	— 1,187	— 2,680	— 1,812	— 1,374
Interest, . . .	1,756	5,454	9,084	12,659	15,992
Loading, . . .	10,001	26,320	25,124	23,704	22,009
	17,540	30,587	31,528	34,551	36,627
Less Interim Bonus, .	...	1,786	2,153	2,536	3,097
Net Profit, . . .	17,540	28,801	29,375	32,015	33,530
Annual Cash Bonus—	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
Mortality, . .	0 2 5	— 6	— 1 3	— 11	— 9
Interest, . . .	0 0 9	0 2 4	0 4 1	0 6 2	0 8 5
Loading, . . .	0 4 1	0 11 3	0 11 5	0 11 6	0 11 8
	0 7 3	0 13 1	0 14 3	0 16 9	0 19 4
Less Interim Bonus, .	...	0 0 9	0 1 0	0 1 3	0 1 7
	0 7 3	0 12 4	0 13 3	0 15 6	0 17 9
Annual Reversionary Bonus—					
Mortality, . . .	0 5 7	— 1 1	— 2 4	— 1 7	— 1 2
Interest, . . .	0 1 9	0 5 0	0 8 0	0 10 10	0 13 7
Loading, . . .	0 9 8	1 4 1	1 2 0	1 0 3	0 18 9
	0 17 0	1 8 0	1 7 8	1 9 6	1 11 2
Less Interim Bonus, .	...	0 1 8	0 1 11	0 2 2	0 2 7
	0 17 0	1 6 4	1 5 9	1 7 4	1 8 7

For a model office on the lines referred to above the whole-life results are:—

Age of Office.	Liability.	Fund.	Surplus.	Annual Rate of Reversionary Bonus.
	£	£	£	£ s. d.
20 years, . .	772,549	880,280	107,731	1 4 3%
25 years, . .	1,174,803	1,316,064	141,261	1 5 2%

Comparison of these tables with those in the paper seems to point to the welcome conclusion, that under present conditions no falling off in bonus rates need be apprehended by reason of the supplanting of whole-life by endowment assurance business. Apparently Mr. Chatham has not thought it worth while, in calculating his reserves, to make any adjustment in respect of the claims being paid two months after death—no doubt in view of the trifling nature of such a correction where endowment assurances are concerned. The liability under whole-life policies would be increased by one per cent, but this simple adjustment is not applicable to endowment assurances, because the endowment portion requires no

correction. Strictly speaking, the corrected reversion and net premium should be employed in the valuation, but the calculation of these for all the terms and ages entails a considerable amount of labour, and in general it will be found that an increase of ·15 of one per cent of the reserve value of the endowment assurance contracts exclusive of bonuses, and an increase of ·2 of one per cent of the reserve value of the bonus additions, is an ample allowance. Of course, in analysing the surplus the net premiums corrected for the immediate payment of claims should, properly speaking, be employed.

Mr. Archibald Hewat.—Mr. Chatham always gives us a good practical paper on sound principles carefully worked out. Those of us who have looked into this matter must be glad to find our observations again confirmed. As regards mortality profit it is, of course, natural to expect that it would diminish along with the assurance part of the benefit as the policy approaches maturity. Another consideration in the lessening of the profit from mortality is brought out clearly by the introduction of the surrender element. The general opinion is that the best class of lives surrender and the worst remain. Further, the O^M table now in use gives a nearer approximation to the actual experience than the tables formerly available. The interest profit, on the other hand, naturally increases as the result of the increasing amount reserved. It is natural again to expect a not inconsiderable amount of profit from the loading, seeing it is charged on the endowment, or investment part, as well as on the assurance part of the premium. The commission on the modern £1 per £100 scale, the medical fees, the policy stamps, and other incidental expenses are of the same actual amount for endowment assurances as for whole-life policies, but are set against a much larger premium. I have often wondered why an agent does not prefer to seek whole-life business rather than endowment assurance business, seeing that for an equal premium the whole-life assurance, and consequently his commission, is greater; but possibly he finds endowment assurance more attractive generally. In 1899 I had occasion to look into this matter, and to-day I referred to my figures. In 1870 the endowment assurances, I found, represented 6 per cent of the total business of the offices, while in 1899 the proportion was 36 per cent, and now I should expect it to be not less than 50 per cent. There is no doubt that the whole-life class is the real life assurance business, and should be as steadily cultivated as in bygone days. The withdrawals are no doubt greater in endowment assurances than in other classes. The assured can estimate pretty nearly what the policy will ultimately be worth, especially in the later years, and can discount it, as he frequently does, when the date of maturity approaches. As to the rate of interest, there is no doubt that it is becoming more and more difficult to keep the ever-growing funds closely invested at really profitable rates, though rates have improved during recent years, and, by vigilance combined with financial skill, may be maintained for some years to come.

Mr. Stenhouse.—When Mr. Hardy submitted the paper to which Mr. Chatham refers, pointing out that the surplus on endowment assurances decreased, and when Mr. Gunn's figures seemed to confirm this view, there was necessarily a feeling of unrest as to how that class would ultimately work out. But the present paper is very reassuring, and demonstrates that more potent factors are at work in this class of our business than mortality. A few facts, taken from the practical experience of a large office, which I am permitted to give, may be interesting. It commenced endowment assurance business in 1851, in which year one policy of the kind was issued. In the ten years following there were only 49 such

policies issued. In the following ten years there were 410, the next ten showed an increase to 1804, while the last four years gave 962 such policies, or 240 each year. As in these four years the average number of policies of all classes issued was 2182, the endowment assurances were about one in nine. The total number of policies of this kind issued last year was considerably more than one-half of the total, and I think that will bear out what Mr. Douglas said as to the change in the class of business, and the probable inferior quality of the lives insured as compared with past experience, through the lives in different classes becoming more assimilated. Owing to a change of rates in 1884 the business up to that year only was included and traced up to the end of 1899. The total number of policies dealt with was 3225, and the total sum assured £1,090,255. The premiums were accumulated at interest, the payments made for surrenders, claims, etc., were deducted, due allowance being made for expenses, and a valuation was made of the policies in force at the end of the investigation. The particulars were as follows:—

	Number.	Per Cent.	Amount.	Per Cent.
			£	
Surrendered, . .	888	27·54	310,152	28·45
Died, . . .	370	11·47	120,844	11·07
Matured, . .	601	18·63	225,035	20·65
Existing, . .	1366	42·36	434,224	39·83
	3225	100	1,090,255	100

It will be observed that the percentages of lives and of sums assured are not very unlike. The surrenders appear rather heavy; but I think the explanation given by Mr. Chatham accounts for a good many. Formerly all endowment assurances that were paid before maturity were called surrenders, but those within a year or so of maturity are now treated as claims paid under discount. How far this would have affected the percentages I cannot tell. A little over 50 per cent of the endowment assurances were for periods of from 21 to 30 years; nearly 20 per cent were for periods under 21 years, and about 30 per cent were for periods over 30 years. Very few indeed were for very long or very short periods. Then another point was, that as the term of the endowment assurance increased the proportion of surrenders increased. This I fancy is what we must expect, for a short-period endowment assurance is more like a deposit than anything else, and in addition to that the value increases very quickly, so that if the policyholder has any difficulty in meeting his premium he can get a loan, or, if he desires, when near maturity he may discount. And this suggests another thing. Seeing that there is little profit from interest and little chance of profit from surrenders, while expenses are considerable, premiums for short terms ought to be fairly high. Then it was found that in half-yearly and quarterly cases the proportion of surrenders was larger than in yearly cases. Of the very few quarterly cases the longest was only kept up for 5 years. This would seem to show that the more frequently premiums fall due the greater is the chance of discontinuance. Further, the great majority of the surrenders took place when a premium was due. The policies in force at the close of the investigation were valued by the H^M table (then the basis of valuation) with 3 per cent interest by three different methods, and

it was very satisfactory to find that the valuation by each of these three methods came so near.

By Lidstone's method, . .	£397,200
By Woolhouse's method, . . .	396,000
By Office method, ...	397,400

The last method was by grouping according to the age at maturity. Taking any of these the experience was found to have been very favourable. In regard to valuations in this class, I am rather doubtful as to the ordinary method of applying a section of a table applicable to another class of assurances; but hitherto there has been no alternative. Of course the experience to which I have referred is not yet complete, but the results show that the class has paid its way and proved not unprofitable to the general fund. With Mr. Chatham's conclusions before me, and keeping in view the above experience, I must say I now look forward with greater confidence. Referring for a moment to Mr. Douglas's remarks in regard to the different methods of bonus distribution, I think in some of the discussions at the Institute both Mr. G. F. Hardy and Mr. King have stated it as their opinion that, taking an average premium, the method that worked out very well and fairly on this particular class of business was the compound reversionary bonus system. Given an average premium, it seems an equitable method for both this class and the whole-life class.

Dr. A. E. Sprague.—I desire to point out a risk which is suggested by the change made eleven years ago in the death duties, namely from a fixed rate to a graduated scale, increasing with the amount of the estate. It is quite conceivable that a similar change may be made in the income-tax, in which case the life assurance companies will be hit very hard, as they probably have bigger incomes than most other companies or individuals. It is bad enough at the present time, when a gross rate of say 4 per cent realised on the funds is reduced by the tax to about $3\frac{3}{4}$ per cent net; but if the tax is graduated so as to fall more heavily upon large incomes, the result will be very serious, as the maintenance of present rates of bonus depends so largely on the net rate of interest being kept up to the present level.

Mr. J. J. M'Lauchlan.—The general question of the change which has come about in life business by the immense expansion of this class of assurance is interesting and deserves attention. The question of the special mortality to which this class is subject has been already investigated. Then there is the question of the shorter duration of these policies as compared with ordinary policies. They go off the books sooner, and consequently a larger new business is required to replace waste than under the old system. Again, the larger premiums received on these policies open the way to increased expenditure. Ten per cent on these endowment assurance premiums each year is a good deal more than ten per cent on the whole-life premiums we used to get. And now this other question of the bonuses on endowment assurances has been discussed by Mr. Chatham very fully, and in his usual clear and admirable manner. I am glad that he has given such a complete series of figures. This is a great advantage for all who desire to study the paper.

Mr. A. D. L. Turnbull.—With regard to the very important question, as to which is the most equitable manner of dividing profits, I have, after suitably modifying the premiums, compared the bonuses brought out by two other methods with Mr. Chatham's results. The first method is where

bonuses are declared in cash, and afterwards turned into reversion, an addition being made in respect of bonuses previously allotted and still existing. I find these results agree very closely with Mr. Chatham's figures for the 25- and 30-year groups, where he has taken interest at 4 per cent. As this does not allow for income-tax, the rate is really say 4¼ per cent, which is somewhat high, and besides, nothing is allowed for shareholders' profits. The second method considered was the compound reversionary system, which, I agree with Mr. Stenhouse, is probably the fairest method. I find that where interest is taken at 3¾ per cent the bonuses agree with those brought out by Mr. Chatham for the 25- and 30-year groups. With regard to the shortest period—20 years—the bonuses under both methods are considerably larger than those which Mr. Chatham's figures showed to be equitable. As regards short-period endowment assurances, it is a question for the consideration of all offices, whether the premiums should be raised, or the granting of such policies discontinued altogether.

Mr. Walter A. Smith.—In reference to the results of a comparison of the bonuses on endowment assurances by different systems, I may mention that a considerable number of years ago I went into the matter with my friend, Mr. George F. Hardy, in order to ascertain the practical results of the different methods of allotting the bonuses on this class of assurances. I looked up my notes and I find, for instance, that after about 20 years the amounts of bonuses resulting, from three methods, to endowment assurance policies approximate very closely. The three methods I refer to are—1st, the Simple Bonus; 2nd, the Compound Bonus; and 3rd, the amount of bonus got by the "contribution" method of analysing profits as shown in Mr. Chatham's paper. The results for a £1000 policy payable at age 60 or death (age at entry 30) were :—

After	5 years.	15 years.	25 years.
	£	£	£
Simple Bonus, ...	100	300	500
Compound Bonus, ...	68	257	480
Contribution Method, ...	60	252	511

The simple bonus assumed is of course a uniform reversionary bonus of £2 per cent per annum. The compound bonus is assumed at 34s. per cent, excluding the first year; and the bonus from the "contribution" plan is the result of the same general basis of profit as the latter. While the contribution plan is probably the fairest method of allocation of profit to the individual policyholder, it will be observed that the compound bonus plan closely approximates to it and, as has been already remarked, is quite a fair method for this class of assurances. It has also the advantage of simplicity, which the other has not.

Mr. Armstrong.—I would suggest that Mr. Chatham might have included among his tables the statement of a fund worked out on the lines indicated in the remarks quoted at the commencement of the paper, *i.e.* a fund valued on the basis of its own experience. Excluding interim bonus, the results corresponding to Tables 3 and 4 would be substantially as follows :—

EXPERIENCE AND VALUATIONS 80 PER CENT O[M] AND 3¾ PER CENT.

End of Year.	Fund.	Reserve.	Surplus.	5 Years Bonus.	Total Bonus.
	£	£	£		
5	156,600	132,300	24,300	4·8	4·8
10	354,900	303,400	51,500	9·1	13·9
15	573,100	522,800	50,300	7·9	21·8
20	813,600	766,400	47,200	6·6	28·4
25	1,073,000	1,028,500	44,500	5·5	33·9

Mr. Gunn's remark as to the relative bonuses was thus confirmed. But the table was instructive as showing that, whatever the incidence of the bonuses might be, the total bonus was practically the same,—that Mr. Chatham's 3 per cent valuations by an aggregate table simply equalised or rather graduated the successive bonuses. In fact, the results in his Table 4, as a whole, might have been immediately deduced from the conditions he had postulated. Given certain experience rates of mortality and interest the true net premium for the original assurance was definitely fixed. In this case it worked out at 2·792 per cent. Deducting this and ·423, the necessary annual provision for expenses, from the office premium of £4, 0s. 7d. left a margin of ·814, which was the equivalent, under the conditions laid down, of an annual reversionary bonus of 26s. 6d. or a total addition for the 25 years of about 33 per cent. In this way the bonuses were traced to their true source, viz. margin of premium received over and above what was required for the original assurance under the conditions experienced. Mr. Chatham's earlier valuations merely created a reserve out of which the later additions were increased.

Dr. BUCHANAN.—Mr. Chatham's results appear to confirm those of other papers on the same subject, such as those by Mr. Sunderland and Mr. Lidstone; but in each of these papers the valuation rate is 3 per cent, and it seems a pity that he has not carried his interesting analysis further, so as to show the effect of a reduction of the rate. Mr. Chatham has emphasised the importance of a sufficient interest margin, and if the rate of interest fell, a reduction in the valuation rate would be necessary. We know generally that the effect of such a change is to diminish profit in the earlier years, and in the later years to increase it; but looking to the fact that so large a proportion of existing endowment assurance business is comparatively new business, it would have been instructive to have had the actual figures. Table 12 shows that the rate of non-renewal is fairly constant after a time; but it seems to me that the explanation given can scarcely be the true one. The number of withdrawals during the few years preceding maturity would probably be very small, for a man is not likely to drop his policy when he can look forward to the near approach of the time when he will receive payment of the claim in full. Of course claims are often paid under discount, but in the supplementary volume of the *New Experience*, Mr. Ackland tells us that out of about 6200 recorded terminations, 800 occurred at odd dates, showing them to be largely maturities paid in advance, but treated as maturities in the records on the cards. The effect would not be to increase the rate of withdrawal, as Mr. Chatham suggests, and the explanation must be sought elsewhere. Another difficulty suggests itself. We know that in whole-life cases the rate of withdrawal depends on age at entry as well as on duration, but in endowment assurance there

is the additional element of endowment term. In the select tables for each age at entry policies of all endowment terms have been thrown together; but the investment element enters more largely in some than in others, and the rates of withdrawal may be very different at the same age in a 10-year endowment assurance and in one taken out for a 40-year period. Of course Mr. Chatham in dealing with average terms may be justified in taking average rates of non-renewal; but the figures can scarcely be taken as applicable to policies of all terms. The difficulty seems a very real one, and I should have liked to have had the benefit of Mr. Chatham's ideas on the subject.

The PRESIDENT.—It only remains for me to thank Mr. Chatham in your name for his very admirable paper. We are now anxious to hear what he has to say in view of the discussion, which has been one of the most interesting I have had the pleasure of listening to in this room. I would only desire to add that it is a great satisfaction to find that one's fears regarding endowment assurance bonuses were uncalled for. We can go forward and cultivate endowment assurance business with much greater pleasure and vigour than after hearing Mr. Hardy's remarks, and having, as I thought, had them confirmed by the investigation which I myself made. It is satisfactory to find that endowment assurance profits do not decrease with the duration of the policy.

Mr. CHATHAM, in reply, said—I am very much indebted to you indeed for the manner in which you have received my contribution, and I quite agree that the discussion has been most interesting. A number of points have been mentioned, and I will endeavour to deal with some of them. Mr. Douglas referred to 80 per cent of the $O^{[M]}$ mortality being too low in his opinion; but I was desirous to take a rate sufficiently low to bring out clearly that it was not in any way due to the mortality that the surplus would be reduced. The rate I have used agrees in the later years almost exactly with the $E^{M(5)}$ table given by Mr. Ackland; that is to say, after five years have passed away in the endowment assurance experience, the $O^{[M]}$ rates reduced to 80 per cent agree almost exactly with his ultimate rate. Then there is the other circumstance that in assurance offices the actual rate of mortality is less, sometimes considerably less, than the expected; for instance, you see offices stating that all along their mortality has been about 70 to 80 per cent of the expected. With regard to surrender-values, Mr. Douglas is quite right in saying that the proportionate paid-up policy is not exactly correct. It is at some ages too much, and at others too low; but of course it is a widely recognised rule, and as the assured has the right to demand the paid-up policy, so it seems to me one must be guided by that in quoting the surrender-value. I may mention that, in the office with which I am connected, surrender-values are obtained by discounting at 4½ per cent the proportionate paid-up policies, not only for endowment assurance, but also for limited payment policies. Mr. Brown referred to the 3 per cent valuation being too severe for young offices. Well, I think it says all the more for those offices which are able to stand it. I am not quite sure I followed his remark about the bonus coming out overhead, by his method, 27s. 6d., because the figures that he quoted for the whole-life class seem rather to uphold what I gave for the endowment assurance class; at least the difference was not very great. I agree with Mr. Hewat and Mr. M'Lauchlan in saying the expenses are now charged on a much larger premium; it is an interesting point, and sufficient attention does not appear to have been given to it yet. Quite a number of speakers have referred to endowment assurances being discounted on favourable terms before maturity, and the figures of Mr. Stenhouse were very interesting. That

is my own experience, I may say, and probably Dr. Buchanan did not observe what Mr. Stenhouse had said, because it has been proved that in practice the fact of a high rate of withdrawal was due to discounted claims. With regard to Mr. Stenhouse's treatment of them now as discounted claims, we had both come to the same conclusion. I also treat them as discounted claims, and I think that is the regular practice in cases of this kind.

Mr. A. D. L. TURNBULL.—Is it not an infringement of the Act?

Mr. CHATHAM.—I don't think so. I think it is perfectly legitimate.

Dr. SPRAGUE.—You would treat a case as a discounted claim if it is within a year of maturity, and all premiums had been paid, I suppose?

Mr. CHATHAM.—Yes. It is not a real surrender.

Mr. CHATHAM, continuing, said — Short-term endowment assurance premiums should be high. There is not sufficient time to make interest profit in these cases, and some offices discourage them. Mr. Turnbull referred to some interesting points. It was gratifying to hear that the results of the other methods referred to agreed so closely for certain durations of endowment assurances with those brought out in my paper, and it was interesting to have Mr. Smith's figures. With regard to Mr. Armstrong's remarks, I thought I had stated very clearly in my paper the conditions under which I worked out the results. If the profit he referred to is not profit from interest, I don't know what it is. With regard to Dr. Buchanan's remark that I might have gone further and given analyses at lower rates of interest—some of the members have said, and I think rightly, that the tables are very numerous already. I did not see how I could dispense with any, having regard to clearness, but I think I have gone far enough when I show the conditions under which endowment assurances contribute their fair proportion of the bonuses.

Vol. III. Part II.

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THE FACULTY OF ACTUARIES IN SCOTLAND.

JUBILEE, 1906.

THE Faculty of Actuaries in Scotland celebrated the Jubilee of its foundation by a Dinner on Friday, 19th January 1906, in the North British Station Hotel, Edinburgh, at which one hundred gentlemen were present. Mr. Niel B. Gunn, Manager of the Scottish Widows' Fund, President of the Faculty, occupied the Chair. He was supported on the right by The Right Hon. Sir J. H. A. Macdonald, Lord Justice-Clerk of Scotland; Mr. Henry Cockburn, President of the Institute of Actuaries; Mr. William Campbell, K.C., Dean of the Faculty of Advocates; and Mr. C. D. Higham, Chairman of the Life Offices Association; and on the left by The Hon. Lord Ardwall; Mr. R. P. Hardy; Sir George Anderson, Treasurer of the Bank of Scotland; and the Very Rev. Dr. Gray. The Croupiers were Messrs. George M. Low, Archibald Hewat, J. J. M'Lauchlan, and W. G. Walton.

The following gentlemen were present as Guests of the Faculty:—

The Right Hon. Sir J. H. A. Macdonald, Lord Justice-Clerk of Scotland; The Hon. Lord Ardwall; Mr. William S.

Haldane, W.S., Crown Agent for Scotland; Mr. William J. Dundas, W.S., late Crown Agent.

Messrs. Henry Cockburn, President of the Institute of Actuaries; George King and T. G. Ackland, Vice-Presidents of the Institute of Actuaries; Charles D. Higham, Chairman of the Life Offices Association; David Paulin, President of the Insurance Society of Edinburgh; J. Buyers Black, President of the Insurance and Actuarial Society of Glasgow; R. P. Hardy and Spencer C. Thomson, Honorary Fellows of the Faculty of Actuaries.

Mr. William Campbell, K.C., Dean of the Faculty of Advocates; Mr. George M. Paul, Deputy Keeper of the Signet; Dr. John Playfair, President of the Royal College of Physicians of Edinburgh; Dr. Charles W. MacGillivray, President of the Royal College of Surgeons of Edinburgh; Mr. William Grant, Master of the Merchant Company of Edinburgh; Mr. John Campbell, President of the Society of Solicitors before the Supreme Courts; Mr. Frederick W. Carter, President, and Mr. Richard Brown, Secretary of the Society of Accountants in Edinburgh; Mr. Thomas Jackson, President of the Institute of Accountants and Actuaries in Glasgow; Mr. W. T. Armour, Chairman of the Edinburgh Stock Exchange.

The Very Rev. Dr. W. H. Gray; Dr. George Chrystal, Professor of Mathematics in the University of Edinburgh; Mr. J. S. Nicholson, Professor of Commercial and Political Economy and Mercantile Law in the University of Edinburgh; Mr. John Rankine, K.C., LL.D., Professor of Scots Law in the University of Edinburgh; Mr. R. Addison Smith, Registrar of Friendly Societies for Scotland.

Messrs. Henry Brown, Manager of the Century Insurance Co., Ltd.; Robert Chapman, General Manager of the Caledonian Insurance Co.; J. A. Cook, General Manager of the Scottish Union and National Insurance Co.; Leonard W. Dickson, Manager of the Standard Assurance Co.; Martin L. Martin, Manager of the Scottish Accident Life and Fidelity Insurance Co., Ltd.; Adam K. Rodger, Manager of the Scottish Temperance Life Assurance Co., Ltd.

Sir George Anderson, Treasurer of the Bank of Scotland; Messrs. Alexander Bogie, General Manager of the Commercial Bank of Scotland, Ltd.; T. Hector Smith, General Manager of the National Bank of Scotland, Ltd.; Robert Blyth, General Manager of the Union Bank of Scotland, Ltd.; John Henderson, General Manager of the Clydesdale Bank, Ltd.

The following noblemen and gentlemen, who had received invitations, wrote expressing their regret that they were unable to be present:—

The Right Hon. Sir Robert Cranston, Lord Provost of Edinburgh; The Right Hon. The Earl of Rosebery, Lord-Lieutenant of Mid-Lothian; Mr. Charles C. Maconochie, K.C., Sheriff of the Lothians; The Most Hon. the Marquis of Linlithgow, late Secretary for Scotland; The Right Hon. John Sinclair, M.P., Secretary for Scotland; The Right Hon. Lord Dunedin, Lord Justice-General; The Right Hon. Thomas Shaw, Lord Advocate; The Right Hon. Richard B. Haldane, M.P., Lord Rector of the University of Edinburgh; The Right Hon. Charles Scott Dickson, K.C., late Lord Advocate; Sir William Turner, K.C.B., Principal of the University of Edinburgh; Sir Stair Agnew, K.C.B., Registrar General for Scotland; Sir James Guthrie, President of the Royal Scottish Academy; The Very Rev. Dr. J. Cameron Lees, Dean of the Chapel Royal; Mr. Alexander Ure, K.C., M.P., Solicitor General for Scotland; Mr. James Avon Clyde, K.C., late Solicitor General for Scotland.

Mr. Ernest Woods, a Vice-President of the Institute of Actuaries; Messrs. George Todd and S. G. Warner, Hon. Secretaries of the Institute of Actuaries; Mr. P. R. D. Maclagan, Hon. Secretary of the Associated Scottish Life Offices; Mr. S. J. Pipkin, President of the Federation of Insurance Institutes; The Presidents of the Actuarial Society of America, the German Society for the Science of Insurance, and the Holland Society of Actuaries; and the Secretary of the Permanent Committee of International Actuarial Congresses; Dr. T. B. Sprague, F.I.A.; Dr. Frederick Fuller and Mr. D. Clunie Gregor, Honorary Fellows of the Faculty; Mr. James Howden, C.A., an Original Member of the Faculty.

Mr. James Milne, Jun., President of the Society of Accountants in Aberdeen.

Mr. Thomas Hunter, W.S., Town Clerk of Edinburgh; Dr. James C. Dunlop, Superintendent of the Statistical Department at the General Register Office.

Mr. H. E. Marriott, Manager of the Scottish Metropolitan Life Assurance Company.

Mr. D. R. W. Huie, Cashier of the Royal Bank of Scotland; Mr. Hamilton A. Hotson, Manager of the British Linen Company Bank; Mr. James Hutcheon, Manager of the North of Scotland Bank; Mr. Thomas Cochrane, Manager of the Town and County Bank; Mr. Charles M. Brown, Manager of the Caledonian Banking Company.

The following Fellows of the Faculty were present:—

Messrs. William Adamson; William L. Adamson; John R. Armstrong (Hon. Editor); Hugh Blair, C.A.; James C. Boyd; Robert R. Brodie, A.I.A.; Hugh W. Brown, F.I.A.; Dr. James Buchanan, F.I.A.; Messrs. Neil Campbell; James Chatham, F.I.A.; Stewart F. M. Cumming; A. Graham Donald, A.I.A.; Gordon Douglas, F.I.A.; Campbell K. Duff, C.A.; John Edgar (Secretary); Francis K. Fenton; James Fenton; Alexander Fraser, Jun., F.I.A.; William Gibson; John B. Gillison, F.I.A.; Niel B. Gunn, F.I.A. (President); Archibald Hewat, F.I.A.; Thomas Kyd, F.I.A.; John Lamb, C.A.; George Lisle, C.A.; George M. Low, F.I.A.; William R. Macdonald, A.I.A.; James J. M'Lauchlan (Hon. Secretary); Vyvyan Marr, F.I.A.; Robert Murrie; John Nicoll, A.I.A.; John Notman; Lewis P. Orr, A I.A.; David F. Park, C.A., A.I.A.; Henry J. Pearce, Jun., A.I.A.; David Pearson, C.A.; Alexander Prentice; James Pringle, C.A., A.I.A.; Walter A. Reid, C.A.; William J. M. Rennie; William Robertson, A.I.A.; Robert M. M. Roddick; Frederick A. Ross, C.A.; William A. Sim, F.I.A.; William J. Slimon; Dr. J. Turnbull Smith, C.A.; Messrs. R. Gordon Smith, F.I.A.; Walter A. Smith; Dr. A. E. Sprague, F.I.A. (Hon. Librarian); Messrs. Graham Steell; George C. Stenhouse; R. Hill Stewart; James Stirling; Robert Stirling, F.I.A.; David J. Surenne; John W. Thomson, A.I.A.; A. D. Lindsay Turnbull, A.I.A., C.A.; Thomas Wallace, F.I.A.; William G. Walton, A.I.A. (Hon. Treasurer); John M. Warden.

The following Members of the Faculty wrote expressing their regret that they were unable to be present:—

Messrs. J. J. P. Anderson; David M. Carment; John W. Cousin; Alexander Duncan; Austyn J. C. Fyfe; Edward J. Harrison, C.A.; William Hutton, F.I.A.; C. W. S. Jamieson, A.I.A.; John N. Lewis, F.I.A.; William R. M'Ilvenna; Robert Mackay; W. Smith Nicol; William L. Ramsay; J. A. Robertson-Durham, C.A.; James Sorley, F.I.A., C.A.; Herbert C. Thiselton, F.I.A.; Andrew H. Turnbull, F.I.A. (Vice-President); Thomas Wallace, Jun.; James Graham Watson, C.A.

On the Menu Card were displayed portraits of the following original Officials of the Faculty—Mr. Wm. Thomas Thomson, Chairman of the first Meeting, and Messrs. Robert Balfour and J. M. M'Candlish, first Hon. Secretaries.

The toast list was as follows:—

The King, ...	THE CHAIRMAN
The Queen, The Prince and Princess of Wales, and other Members of the Royal Family, . . .	THE CHAIRMAN
The College of Justice, . . .	DR. TURNBULL SMITH
Reply, ...	THE LORD JUSTICE-CLERK
The Faculty of Actuaries in Scotland, . . .	LORD ARDWALL
Reply, ...	THE CHAIRMAN
The Institute of Actuaries and other Societies connected with Actuarial Science or the business of Life Assurance, ...	Mr. LOW *Vice-President of the Faculty of Actuaries*
Reply, ...	Mr. COCKBURN *President of the Institute of Actuaries*
The Guests, ...	Mr. HEWAT
Reply, ...	Mr. CARTER *President of the Society of Accountants*
The Chairman, ...	Mr. R. P. HARDY

The loyal toasts having been honoured,

DR. TURNBULL SMITH, in proposing the toast of the College of Justice, said:—Mr. President, my Lords and Gentlemen, I have the honour to submit to you the toast of the College of Justice. To a Scotsman there are few pages in our history of more interest than those which tell us of the development of the system under which law and justice are administered. Most of you, perhaps, know that before 1532 civil causes in Scotland were judged by Standing Committees of the three Orders of the Scottish Parliament,—the Clergy, the Peers, and the Commissioners of Burghs, who made an annual circuit in each of the quarters of the country. You can easily imagine that such a system was by no means an ideal one. The expense fell upon those who composed the committees, and the office was naturally not much sought after. When those who did accept it went upon such an errand they made the business go through as quickly as possible. In 1532, however, James V. instituted the Court of Session. At that time the Court consisted of the Lord Chancellor, the Lord President, and fourteen ordinary Judges or Senators of the College of Justice, as well as an indefinite number of Peers who only seem to have come upon the scene when there was something important connected with the Crown to be dealt with in the Court. This state of matters prevailed for some considerable time. It is very peculiar, looking back upon it now, to think that what we should call the first sederunt of the Court of Session took place in the Tolbooth Prison, the old Heart of Mid-Lothian. At this first sederunt on the 27th May 1532, James V. was present in person.

Of the fourteen Judges I have mentioned, seven were laymen appointed by the King, while the other seven were ecclesiastics appointed by the Church. One thing the Church at that time did not care about was that all the expenses of this Court were saddled upon its revenues. It is not at all to be wondered at that it remonstrated in regard to this, but ineffectually. The taxation, or whatever you like to call it, was persisted in. After 1532 various modifications were from time to time introduced in the Court until in 1830, almost within the memory of some of us here, the Court of Session, as we now know it, came to its present position with its thirteen Judges—the Inner House with its two Divisions, and the five Judges in the Outer House. All this is more or less known to us, but we have some friends from south of the Border with us this evening, and I thought it would be not uninteresting to mention these points for their benefit. I need scarcely say that although my memory is a very good one, it does not go back to, and I have no personal recollection of, the days of three or four hundred years ago; but the sources from which I have culled my information are open to you all should you feel inclined to pursue the matter further. What can I say with regard to the College of Justice, the subject of my toast? What more can I say than that we, here in Scotland, are proud of our Scottish Court? The Solicitors who form a portion of the College of Justice are almost all of them men of the utmost integrity, men upon whom we lean, men with whom we are associated in many ways. The Members of the Bar are, in general, men of the highest ability and character. The Senators of the College of Justice demand and receive the absolute confidence of their fellow-countrymen. I remember hearing a very eminent President of the Court of Session advise all sensible men to have nothing whatever to do with law, or rather with lawsuits, saying that there was only one thing worse, and that was going to arbitration. Now it struck me at the time that that was all very well for a gentleman who held office in the Law Courts, but I could not help feeling that it was not at all likely his view would be homologated by the Members of the Bar and by the Solicitors. If they did they would be rather more than human. However, I am sure of this, that if we do such an unwise thing as to go to law, we shall have a straight run for our money. I had hoped to have the great pleasure and satisfaction of coupling this toast with the name of the Lord President, who unfortunately is unable to be present. It is with no less satisfaction that I couple the toast with the name of the Lord Justice-Clerk. He is always welcome at all our social gatherings, and stands in the highest estimation of the whole of us.

THE LORD JUSTICE-CLERK, in reply, said:—Mr. Chairman, croupiers, and gentlemen, I join with my friend Dr. Turnbull Smith in expressing my regret that my chief is not able to be here to-night, the more so that the cause is that he is not very well, but I trust that he will be all right in a day or two. I may say this that I must make a confession. I know a great deal more about the history of the College of Justice in Scotland now than I did ten minutes ago. I am glad to feel, in learning those points of history, that I have not learned anything to its discredit. The Courts of Justice in Scotland may not be as efficient as the Courts in other places. As to that I say nothing. We do not expect to be flattered by any assurance that you have perfect confidence in our decisions. I am quite sure that criminals who come before me so often have no confidence in my decisions whatever. I am equally certain many litigants who appear in the Civil Court are equally dissatisfied. I remember a story of a litigant on one occasion, who had had his case decided against him, going down the High Street in a state of towering indignation. A friend who passed by heard him say, "The College o' Justice! It's ma opeenion they were a' the waur o' drink." (Laughter.) I am not quite sure whether the impeachment was true or not. (Renewed laughter.) I can only say for myself as I have an opportunity of seeing what is the present luncheon of the Members of the College of Justice who sit on the Bench, that there is no risk of any decision being given in such a state as that. Well, gentlemen, there is one thing about our Courts and the administration of justice in the Civil Court in this country, which I will say without any doubt or difficulty. I had a case before me recently, and I found on looking up the references that it had been in the Court of Session a great many years ago, and that after it left the Court of Session it took four years to decide it in the House of Lords. I can assure you of this, that if any of you are unfortunate enough, as I trust you will not be, to have any litigation in the Court of Session, you will be sure of being put out of misery before that. The ordinary duration of a case going through the Outer House and then through the Inner House, in which I have the honour to sit, will certainly not average more than about five or six months. I think if there is nothing else to be said for the credit of the administration of justice in Scotland than that you can put a case through the Supreme Court in six months, that is a great deal to say in our favour. Perhaps you think we give too hasty decisions; that it would be better if we took a couple of years, as we used to do, to bring a case to a close. I am not of that opinion. If you think we are wrong, as we sometimes may be—we do not set ourselves up as infallible—you have your remedy by going elsewhere. But it is

in favour of the litigation in this country that it does not involve long and serious delays. Wherever there is long delay there is always vast expense. You get your cases more fairly and cheaply through in our Courts than anywhere else I know. There is the Criminal Court, which I suppose none of you hopes to make an acquaintance with. From my very long experience in connection with that Court I may be entitled to express my belief that in no country in the world is the criminal system of conducting the law more satisfactory, more fair to the prisoner, and more certain to bring about true justice both to the Crown and to the prisoner, than it is in Scotland. We used to have a system of indictments which I do not think any juryman could possibly understand. It took about four pages of print to express the fact that John M'Callum had stolen a purse from a lady in Princes Street. I may tell you a curious and interesting fact—that in consequence of the Act of Parliament, which I was by great luck able to get through the Legislature, no less a sum than £1550 was saved to the State in the very first year after the Bill was passed, in the printing of indictments alone. You may fancy what an expensive and roundabout mode the old way was Now the unfortunate criminal has very little chance indeed of escaping on a plea to the relevancy. The indictment leaves no loopholes. There is also an excellent arrangement, which may be very convenient if any of you gentlemen ever get into trouble, which will save a great deal of delay in your case. Any criminal has only to give notice that he is going to plead guilty, and he is brought up and sentenced, saving him two or three months' extreme anxiety, and two or three months added to his sentence. Gentlemen, I thank you for the kindness with which you have received this toast. It is a good sign, when the College of Justice consists of all those engaged in the administration of the law—Judges, Counsel, Writers to the Signet, Solicitors, —that there is nothing to reprove, nothing I am glad to say of corruption, and you may be assured of this, that everything that is done by your Bench is done in the earnest desire to do justice as between man and man, and to hold up the morale of the country before all the world.

LORD ARDWALL gave the toast of the Faculty of Actuaries in Scotland. He said:—Mr. President, croupiers, and gentlemen, I assure you I deem it a very great honour to have been asked to propose this toast—the Faculty of Actuaries in Scotland. It is in some ways an easy thing to propose the toast of one's host, especially when that host is a body so deserving of public respect and esteem as the Faculty of Actuaries. But when one comes to expound in any way, however short, who Actuaries are, what

work they do, and the history of the Faculty, I have found it not so very easy. To begin with the name itself of Actuary; as far as I have been able to discover, and recollecting a little of my classical lore, the *Actuarii* were the clerks who recorded the *Acta* of the Senate or of the Courts of Justice in ancient Rome. It then came to mean business men generally, where, as in most European countries, Latin became the language of the learned professions. How the name has come in this country to be limited to those who turn their attention to monetary questions, which involve the consideration of the rate of interest on money and the probabilities affecting the duration of human lives, I really have been unable to discover. And although I have consulted your President to-night, I am sorry to say he has not been able to afford me much enlightenment. But however it came about, the word is now indissolubly connected with gentlemen who turn their attention to those subjects, and I think, briefly stated, the function of the Actuary may be described as the application of the doctrine of probabilities to the affairs of life, and, of course, it is hardly necessary to say, in this company, that the scientific profession of the Actuary has taken its rise almost entirely in connection with the business of life assurance, annuities, and reversionary interests. I understand that this particular Faculty of Actuaries is linked historically, first, with an Association of Life Assurance Managers, which still exists, and second, with the Institute of Actuaries of London, which originally embraced both Scottish and English Actuaries. The idea of forming an association of those connected with the management of life assurance companies was, I understand, originally suggested by the very eminent Actuary Mr. William Thomas Thomson, whose portrait you will find on the fly-leaf of the menu, and whom most of us recollect, even if we were very young at the time, as the distinguished Manager of the Standard Assurance Office. It was found in the course of a few years, I suppose for very good reasons, that it was better that the Scottish Actuaries should form themselves into a Faculty by themselves. This body accordingly was formed in 1856, and we are now in the happy position of seeing it arrive at the Jubilee Anniversary of its founding. In that time it has done a great deal of good work and made very rapid progress. The Faculty has had the advantage of bringing together the various members of the insurance and cognate professions, of stimulating the study of actuarial science among young men who intend to follow the business of insurance and cognate pursuits, and enabling them, after passing examinations which, to judge from some papers I have before me, seem sufficiently difficult, to go over all the world with a diploma given by the Faculty of Actuaries, which is a testimony to their education and their ability to

perform their duties as Actuaries in connection with life assurance in any part of the world. That it has been successful in so doing is certain, because this Faculty has members in almost every part of the globe, who have distinguished themselves in the various colonies and dependencies to which they have gone, and who have spread the fame of this Faculty, I may say, all over the world. The progress of the Faculty has been something very remarkable. In 1870 there were only 62 members; in 1880 the number had risen to 79; in 1890 it had reached 109; and now I understand there are no fewer than 184 members. Such figures are the best testimony to the Faculty's usefulness and efficiency. It has come to be recognised as a body which devotes its attention to a speciality and a very important speciality in modern life, which sends out to the service of the public young men well equipped for that most interesting pursuit, the pursuit of actuarial science. There are a great many questions which are submitted to and determined by Actuaries. I have seen specimens of their work, more or less, as a Director of an Insurance Company, and all I can say is that while the matters were very much beyond my comprehension, and the foundation of their calculations very far beyond me, yet I was convinced of the soundness of their principles and the sufficiency of the methods by which they arrived at their results. There are some very curious and out-of-the-way inquiries which may be gone into in judging the probability of human life. A very eminent member of my own profession wrote a great poem—I can call it nothing else—on the Annuitant. I think that is the most valuable actuarial treatise that a member of the Bar has ever written upon annuities or cognate questions, and I would suggest that some gentlemen here to-night should devote their attention to this question: "Given an ordinary human life—a male, or say a female of a certain age—what is the probability of that life being indefinitely prolonged by the person taking out an annuity, or having an annuity bequeathed by some one who is keeping the fiar out of the enjoyment of what he considers his lawful property, for the time?" According to Mr. Outram, the taking out of an annuity is a prophylactic against shooting, drowning, accidents or any other evil that can attack human life, and I think that is a subject which might in their leisure moments, perhaps on their holidays, amuse some members of this learned Faculty. I am sorry to say that is the only contribution, as I have pointed out, that lawyers in my experience have made to this science at all. I wish I knew more about it, and perhaps I would have said more, but probably my deficiencies will be made up by the President, who is to respond to this toast. He was an Edinburgh man originally, and I am sure all his old friends are very glad to

see him back again. On this occasion I very heartily propose to you the toast of long life and increasing usefulness to this admirable Society, the Faculty of Actuaries in Scotland, and I couple with it the name of your President, Mr. Gunn.

THE CHAIRMAN, in responding, said:—Croupiers, my Lords and gentlemen, before replying to this toast proposed in so genial and sympathetic a way by Lord Ardwall, I would like to thank the guests who have done us the honour of coming here to-night to help us to celebrate our Jubilee. The only grudge I have against Lord Ardwall is that he has anticipated much of what I was going to say. With the capacity of the advocate, he has assimilated thoroughly the subject he had to deal with, and has really left very little for me to add in reply.

Although Lord Ardwall has explained generally the distinctive duties of an Actuary, there might be some reason in almost any other company for supplementing his remarks, and for entering more fully into a description of his functions; but in Edinburgh, which has Assurance as one of its staple industries, along with banking, brewing, education, and printing—the Actuary is a familiar figure. In my wanderings in other parts of Scotland I have been asked to explain what an Actuary is, and I am afraid, like Lord Ardwall, I have been unable to explain how the term has been derived, but I may say also that there are other titles which, familiar in our ears as household words, are not understood outside the bounds of Mid-Lothian. Some people even do not understand the meaning of the letters "W.S." or "S.S.C.," and I think that if they do not know what those are, we may forgive them for being ignorant of the qualifications of an Actuary!

The Faculty of Actuaries, as you have heard, was founded in 1856. The first meeting was held on 4th January of that year, so that we are within a very few days of the Jubilee of the foundation of the Faculty. I will not trouble you with a lengthy history, but I may mention a few facts and figures that may be interesting. The original members numbered thirty-eight, of whom Mr. Howden and Mr. Clunie Gregor survive. I am sorry that neither of them is here to-night. One of our Vice-Presidents, Mr. A. H. Turnbull, was one of the original students of the Faculty, although he did not become a member till some years later. Mr. Meikle, who not very long since was with us, was one of the first Examiners. The original memorandum on which the Faculty was founded was signed by many well-known names in the assurance and accounting world, and among others by James and Archibald Borthwick, John Mackenzie, William Spens, Charles Pearson, James Watson, William Thomas Thomson, John M'Candlish, Dr. Smith, William Finlay, James Howden, Thomas Goldie Dickson,

Samuel Raleigh, A. W. Robertson, and Donald Lindsay. (I am glad to say that we have among the members of the Faculty a considerable number of descendants of these founders of the Faculty.) It is to these that we, as you know, owe the fabric of life assurance, which has proved such a blessing to the country, teaching it lessons of thrift and self-denial, the value of which cannot be exaggerated. The Faculty owes its existence to their determination that the good work should not be spoilt by any possibility of the practice of life assurance being based on unsound theory, and owing to their far-seeing judgment Scotland now possesses the magnificent institutions of which we are all so proud, and with which I myself and so many others here have the honour of being connected.

As might be expected, the Faculty owed at first a considerable portion of its strength to the help of members of the Society of Accountants. There were then very few professional men who had been trained in actuarial science proper, and the profession of accountant naturally approached the actuarial more closely than any other. I am glad to say that we have still among our number many accountants who strengthen our body and form a pleasant link between us and the Society of Accountants in Scotland.

You will, I hope, pardon me if I take up a small portion of your time with a word or two as to the value of the work performed by the Faculty. It was important enough in 1868 to warrant the granting of a Royal Charter of Incorporation. In the preamble to that document it is mentioned that the Assurance Institutions established in Scotland possessed funds amounting to upwards of £20,000,000. In the period of less than forty years which has elapsed since then, the responsibility of the Actuary has grown very largely, for the £20,000,000 in 1868 have increased to £83,000,000 in 1904. In one way or another that large sum is controlled by members of the Faculty, and if we add the funds of Offices in England and in British dependencies such as Canada and Australia, which are under the charge of Fellows of the Faculty, the amount is increased to £127,000,000. This leads me to say that the Faculty has, like so many other Scottish institutions, spread its branches over the world, and you will find its members almost everywhere. The principal Office in Canada, the principal Office in Australia, and the principal Office in South Africa, each of them has a Fellow of the Faculty for its Actuary, and in New York you will find several of our number. It was very interesting to receive the following cablegram to-night from New York,

"We greet our Alma Mater. Congratulations on past and best wishes for even increased prosperity in future from Cathles, Gibb, Hunter, Hutcheson, Moir, Webster, and Young."

These gentlemen are all Fellows of the Faculty of Actuaries, two of them holding leading positions in the American giant concerns, and I trust you will sanction our sending them a cablegram in reply embodying our thanks and reciprocating their kind wishes. This leads me to the next point. We have heard a good deal lately about the management of the leading American Offices. While perhaps a good deal of the abuse bestowed on the leading officials of these institutions has been manufactured for the purpose of providing sensational headlines for the New York newspapers, yet there is no doubt we should not like to see the financial methods of some of these Offices repeated in this country. There are probably many reasons why it is possible such a condition of things can exist in America and not on this side of the Atlantic, but while the main safeguard we have here is the more thorough supervision exercised by our Boards of Directors, yet—if I may be allowed to say so—there is another of almost equal importance, namely, the difference in the duty and power of the Actuary in America and the Actuary in Great Britain, and in the amount of responsibility entrusted to each. I say merely in the duty and power, for as far as the individual Actuaries are concerned, America will stand comparison with any other country. I had the pleasure, along with a good many others who are here tonight, of paying a visit to the American Actuaries about two years ago, and I am sure my companions on that occasion will join in my testimony to the sterling ability and uprightness of our hosts. They are, however, according to the custom of the United States, divorced from the management of the Offices with which they are connected, and almost invariably subordinated to those who have charge of the financial arrangements, and the agency departments; and they are thus unable to control the commercial policy of the Offices. In Great Britain, on the other hand, as you are aware, the Actuary usually controls the Office, and if he does not control, he has an important voice in the management. One cannot help imagining—and I think we are almost warranted in doing so—that the training which an Actuary receives has its effect on the morals as well as on the intellect. We may be sure that if the three great American Companies had been ruled by Actuaries instead of financiers, we would not have heard of what are called the "Assurance Scandals." While the American method may have tended to help the development of life assurance business, it does not ensure that the business shall be conducted on the straightest of lines, nor do I think it conduces to the profit of the individual policy-holders. We have learned, and have probably still to learn, a good deal from the American Offices, but I hope we shall be able to pick out what is really for our advantage, and leave the rest. Probably the most

important lesson we should learn from them does not follow so much from their example, but is rather suggested as a method of meeting the difficulties they have experienced. I refer to the importance of having our Actuaries thoroughly equipped for all the duties connected with the management of an Assurance Office, in fact that they should possess the triple qualification of mathematical acquirements, of knowledge and discretion in the investment of funds, and of skill in the development of business. Both the Faculty and the Institute have in the past gradually widened the scope of the examinations for their diploma, but we must not be satisfied to stand still. We must constantly keep in view the varied requirements of life assurance in all its branches, so that our Fellows may be fitted for all the work which may fall to their lot in the management of assurance offices. I have perhaps dwelt too long on the life assurance side as far as the Faculty of Actuaries is concerned. You know, however, that the object of the Faculty is the promotion of Actuarial science which is mainly the adaptation of science to the problems of human life. It finds its principal outlet in the development of life assurance, and I have dealt perhaps too exclusively with that phase of the subject, but I hope my colleague, the President of the Institute, may devote himself more fully to the scientific side of the question. I have again to thank the guests for coming here, and hope that the younger members among us, who may live to celebrate the centenary of the Faculty, will leave it as flourishing as it is at present. Gentlemen, I have to thank you for coupling my name with the toast, and for the kind manner in which it has been received.

Mr. G. M. Low then proposed the toast of the Institute of Actuaries and other Societies connected with Actuarial Science or the business of Life Assurance. He said:—Mr. President and gentlemen, while we are engaged in looking back upon our past history as a Faculty and congratulating ourselves that after half a century's existence we can feel that we have in some satisfactory measure fulfilled the objects and purposes for which we were established, it is very natural and proper that we should at the same time turn our thoughts to our relations with other bodies having objects of a more or less similar nature to our own, and think of these relations as they now exist—as relations of the most harmonious and satisfactory character—and that we should extend to the representatives of those other bodies who have honoured us with their presence here to-night a cordial welcome, and express our grateful thanks for their presence and for their countenance of our Jubilee Celebration. When we speak of other bodies having objects similar to our own, we almost naturally think first of

the Societies of Accountants in this country. As you have reminded us, sir, we were originally very much in the same boat with the accountants. Many of our original members were accountants; many accountants still think it worth their while to go through the arduous course of study we prescribe for our students, and we count them among our most valued members. Indeed, as we have also been reminded, it was only by degrees, and by a somewhat slow process, that the functions of the Actuary came to be differentiated in fact and in popular estimation from the functions of the Accountant. It is quite within living memory that even such a highly enlightened body as the College of Justice has sought the opinion and the assistance of the Accountant upon questions which really fell within the province of the Actuary; but now the functions of the Accountant and of the Actuary are very well recognised and have become sufficiently distinct, certainly in the minds of the Judges, and I think also in the minds of the public. But our relations with the Societies of Accountants continue to be of the most pleasant and cordial nature, perhaps all the more that this differentiation has taken place. We are happy to welcome among us the representatives of some of those bodies—the President and Secretary of the Edinburgh Society, the President of the Society in Glasgow, and but for preventing circumstances we should have had the President of the Aberdeen Society also. We welcome them heartily and thank them for their presence. But while we rejoice in our friendly relations with another profession we do not forget, and you, sir, have reminded us, that our very existence as a separate profession depends on the business of life assurance, with which most of us are more or less intimately connected. That leads me to think of the associations of insurance officials formed for the purpose of conserving and furthering the objects of the great institutions which carry on that business. First of all we naturally think of our Managers' Association in Scotland, which we are proud to look upon as the parent of all similar Associations. As the first Association of its kind, it is one that has done in the past, and we hope will continue to do in the future, a great deal of useful work. Well, sir, it was for many years a cause of regret to the Managers' Association in Scotland that they had not in London a body like their own with which they could communicate on the subjects they had to deal with. It was necessary for them to have individual communication with their professional brethren in London, and although this was always very pleasant, it did not yield the fruitful results which would have arisen from communication with a united body. To our great satisfaction such an Association was at length formed—The Life Offices Association of London—and it is very gratifying to

think that that body has been able to accomplish a great deal of useful work which we in Scotland could not have attempted. We have the great satisfaction of having with us this evening Mr. Higham, the recently elected Chairman of that Association, a warm friend of many of us, and a gentleman whose name stands very high indeed in the estimation of Actuaries and of insurance men. Long may that Association flourish, and may it and the Managers' Association continue to be in the very friendly and satisfactory relations in which they now are. If I may be allowed to wander for one moment a little out of the line of thought I have been endeavouring to pursue, I should like to say this, that I do hope Mr. Higham and his Association will always give their Scottish brethren the opportunity of conferring with them when any question of general importance arises. We saw the other day a report of a meeting in London at which another distinguished member of the Association discoursed on the subject of life assurance, and discoursed with the great ability we always expect from that gentleman. But in the course of what he said he appears to have thrown out a suggestion which some of us here would not, I fancy, be prepared to concur in; and that is why I say that I hope we shall have the privilege of communicating and interchanging our views on all occasions of importance with the Life Offices Association. If I have read correctly the report of that address, it was suggested that a Royal Commission or a Parliamentary Committee should be appointed to inquire into the working of the Life Assurance Companies Act with a view to its amendment. Now, I suppose, anybody who knows anything about that Act is prepared to suggest amendments for its improvement, but these in the main would only amount to dotting the "i's" and stroking the "t's" with regard to certain particulars which are not of vital importance; and I do think the speaker who immediately followed (Lord Rothschild) was on the right line when he said in effect, "Let the Life Insurance Companies confer among themselves and agree upon what is desirable in the way of amendment; and then let them go to the Government and ask what they want." There are other institutions connected with insurance which we wish to remember. There are the Insurance Institutes established in various important centres throughout the United Kingdom. Mr. Pipkin, President of the Federation of those Institutes, is unfortunately not able to be with us, but he sent a most cordial message, and we heartily reciprocate his kind feeling. We have, however, representatives of two of the local Associations—Mr. Black, the President of the Glasgow Society, and our friend Mr. Paulin, who is President of the Edinburgh Insurance Society. But when we come to think of yet another distinguished body we feel ourselves

on the ground of closer kinship, more intimate relationship, and if possible more cordial fellowship—I mean the Institute of Actuaries. Lord Ardwall has said something of the origin of our own body and of the ideas out of which arose the Institute of Actuaries. When the Faculty was formed in 1856 the Institute was comparatively in its infancy. It was then eight years old, and it is now fifty-eight years old. No Actuary, be he a member of the Faculty of Actuaries or be he an Actuary in any part of the world, can think of Actuarial science without recognising that beyond measure the Institute of Actuaries and the members of the Institute have formed and promoted the science in which we are interested. We as Scottish Actuaries—or rather I should say our predecessors, the Scottish Actuaries—found it necessary, for reasons which I am sure our friends of the Institute now fully recognise and appreciate, to form an Association of our own, with what results and what success we to-night have been reminded. But the Institute went on its own course, growing and extending and developing in a way which the Faculty with its more limited membership and resources has not been able to rival. With the Institute of Actuaries we are happily in the most cordial and intimate relationship, and it is a great satisfaction to us that they have on this occasion sent such a generous representation of their body. Particularly it is a satisfaction to us as a Faculty that on this our Jubilee we are able to receive among us, as President of the Institute and as representing them, one of our own sons, one who received his training here in the Standard Life Assurance Office, probably in those days the most prolific training-school of Actuaries and of Insurance Managers in the United Kingdom. Mr. Cockburn went south after having qualified himself by study and examination, and having received his diploma as an Actuary from the Faculty. He went as one of the great army of Scotsmen who for centuries have been gradually accomplishing the peaceful conquest of our southern neighbour, and he has attained a proud and eminent position at the head of his profession. We have also with us two of the Vice-Presidents of the Institute, two men whose names are known the world over as distinguished Actuaries, Mr. King and Mr. Ackland. We have also another member of the Institute, whom we all regard with peculiar favour and even affection, Mr. Ralph Price Hardy. I hardly know whether we welcome Mr. Ackland and Mr. Hardy more as distinguished members of the Institute of Actuaries or as honorary members of our own Faculty. We congratulate ourselves on having had the honour to elect them to that position, the first occasion, I think, in our history on which the Honorary Fellowship was conferred on professional brethren south of the Tweed Members of the Faculty know why that was done—that those

gentlemen were in the forefront of the labours connected with that very large and important undertaking in which the Institute and the Faculty were recently engaged—the mortality investigation and resulting tables of mortality of the experience of assured lives, the greatest undertaking of its kind hitherto accomplished. I fear I have detained you too long, and I now only ask you to join with me in the toast of "The Institute of Actuaries and other Societies connected with Actuarial Science or the business of Life Assurance," and I couple with it the name of Mr. Cockburn.

MR. COCKBURN, in responding, said :—Mr. Chairman, Mr. Low, and gentlemen, I find very great pleasure in coming here to-night as your guest, and as representing the Institute of Actuaries on this interesting occasion, for it is the wish of all of our members, I know, to offer to the Faculty of Actuaries our sincere congratulations upon the good work they have now for fifty years carried on, and not only that, but to express to them some of that additional respect which is due to advancing age, and to wish them well in all future work and endeavour. As Mr. Low has so ably expressed it, the Institute of Actuaries and the Faculty have long been on a most friendly footing and in close connection. We have many members in common; we are generally privileged to have one or two of our northern friends on our Council Board in London, and we have undertaken important work together. I trust we may have the opportunity of carrying out together in an equally agreeable and smooth manner some future work, be it great or small. To-night we are looking back a good deal, and therefore I should like to say that during the last fifty years the progress of life assurance has in my opinion been largely due to two principal causes; firstly, the foundation of the Faculty of Actuaries and the Institute of Actuaries, which have provided a safe scientific basis for life assurance; and, secondly, to the passing of the Life Assurance Act of 1870, the legislative provisions of which have proved beneficial to the cause that we all have at heart. In the progress of life assurance during the last fifty years you all know to what an honourable place Offices directed from Edinburgh and from Glasgow have attained. Life assurance, no doubt, is our principal care, but I think as time goes on, and as, I think, you yourself, Mr. President, hinted, the functions of the Actuary will range over a wider field. The time has come when by our training or experience, or by opportunity, I think we may reach out into a larger field of usefulness than has been possible in the past. It is very desirable that we should do so if for no other reason than that our members are increasing in a very marked manner, and we have not got quite enough work for all of them to do. But apart from that, I think there is much

work before us in a wider sphere. We are concerned, of course, with all forms of thrift, to see that that should flow through proper channels, but beyond that there is in our modern life a great deal of statistical inquiry and financial inquiry, in which the Actuary may take, I think, a worthy part. If a Committee of the Faculty of Actuaries and the Institute of Actuaries had been asked, for instance, to look into the question of Tariff Reform—well, gentlemen, it is perhaps rather late in the day to confide in you my opinion as to what the result might have been. But whatever our work may be—and it is largely life assurance, no doubt—I am glad to think that not only in this country, but in all our colonies, in the United States, on the Continent of Europe, even in far Japan, there are Actuaries, many of them our own sons, who are prepared and who are striving to carry on the beneficent work which it is our privilege to fulfil. I am very glad, sir, that you have included in this toast the Kindred Associations, for that takes one's thoughts to brethren in Greater Britain across the seas, and other countries I have mentioned, all of whom, as we continually have evidence, express and feel the greatest loyalty and affection for their Alma Mater, be it the Institute or the Faculty. In our own country we have the Provincial Insurance Institutes, as we call them. I believe the oldest of these Institutes hails from Glasgow. The Insurance and Actuarial Society of Glasgow, if I mistake not, was one of the earliest Societies of that kind. In much later days we have Institutes in the leading cities of the United Kingdom. They are doing very good work; they are educating and encouraging their young men to pass examinations, and take a greater interest in and raise the level of their work, and in that I am sure they have our hearty sympathy. In conclusion, may I say that it is a particular pleasure to me to be here to-night, because I am myself an old member of the Faculty, as Mr. Low has told you, and I do feel, shall I say, a great pride in coming back amongst you in the proud position of President of the Institute, and receiving the kind welcome you have given me. I know that between the North and the South there is a close bond of feeling which is a very pleasant fact in our daily life, and I shall go back to my friends in the South and tell them of the kind way in which you have received all of us here—not only myself but also the friends in London whom you associated in your most hospitable invitation, and two or three of whom could not come, to their very great regret. I thank you most sincerely on behalf of all for the great honour you have paid to us.

Mr. Hewat, in proposing "The Guests," said that he looked upon many of them as representatives of the Actuary's professional

ancestry. Some years before the founding of the Faculty the professions began to specialise, more particularly the accountant, the legal, the insurance, and the actuarial. We actuaries are indebted more or less to them all. It is to the clerical profession that we are indebted for one of the earliest and, in its day, one of the best of our mortality tables—the Northampton, the work of the Rev. Dr. Price. The cautious actuary, in strengthening reserves against future liabilities, no doubt receives encouragement from the clergy when hearing them discourse upon the "seven years of plenty and the seven years of famine." In this city, 163 years ago, an eminent Doctor of Divinity conceived the idea of establishing a Fund for the Widows and Orphans of the Ministers of the Church and of the Professors in the Universities of Scotland. There being no actuaries in those days, as we now know them, this minister did his best by seeking the nearest help he could find in half an actuary—the Professor of Mathematics in the University of Edinburgh—who happened to be also a brother in the ministry. But even a Professor of Mathematics has his limitations, so he sought the other half of the actuary, and found him in the person of the Accountant to the City of Edinburgh, who was also Accountant to the Board of Excise for Scotland. These two set about making the necessary calculations, and reported to the enterprising Doctor of Divinity, who called in the Lord Provost of this city, of the day, to complete the quartette. As that Fund has had a successful and useful career of upwards of 160 years, and is in a more prosperous condition to-day than ever—thanks to the guidance of Fellows of this Faculty during the past half-century—we may safely say that the foundations were well and truly laid by these four gentlemen, though in the actuarial dark ages. We all know how close the medical profession comes to the actuarial when we deal with the probabilities of human life. As is well known, the *Carlisle* table of mortality was the joint production of a physician and an actuary—an excellent table, still used occasionally, and one on which we look with veneration and respect.

The legal profession, too, has been helpful to us, as we to it. That profession has much to do with intricate matters—entails and such like—which require the skilled assistance of the actuary. He would be a bold and foolish lawyer who would himself attempt to wander through the actuarial mazes of such problems. The College of Justice has already been referred to. We actuaries are always glad to appear, for a consideration, in the Court of Session to assist the learned Judges in arriving at wise and reliable decisions. We cannot get on without the bankers, nor can they get on without the actuary. In banking business, questions and interests arise which can only be safely dealt with

after they have passed through the actuarial mill. Their Widows' Funds, too, require the oversight and investigation of the actuary—indeed, their members "wont be happy till they get it."

We are also much indebted to the mercantile community. Trade and commerce help the actuary, as he in turn strengthens in some measure the bases on which these rest. I am glad to see my old friend and schoolfellow, Mr. Grant, Master of the Merchant Company, among our guests this evening. That ancient and honourable Corporation knows when it requires the services of an actuary, and knows also to come to the Faculty for such.

We have also among our welcome guests several prominent Insurance Officials who have carved out useful niches for themselves in the Insurance business—in its many and varied forms—untrammelled by the mysteries of the actuary's craft. But I need not enlarge. We are glad to see all our guests this evening, and we thank them for the honour they have done us by their presence here in this the time of our Jubilee.

In calling upon you to respond, as I know you will do most heartily, to this toast, I desire to associate it with our nearest relative—nearest in affinity and nearest in age—the Edinburgh Society of Accountants, as represented here by Mr. Carter, their President.

MR. CARTER, in reply, said that this had indeed been a night of instruction, of welcome instruction, upon subjects which many of them thought they already knew something about until they now had discovered how little they really knew. Lord Ardwall had given them a very clear and intelligible definition of the word "Actuary," but he would ask Lord Ardwall to take it from him that his Lordship had not quite arrived at the proper definition. There was a better and a later one, one more understandable to the common mind, and one which, by its very utterance, was much more clear and not so complicated by technicalities. The "definition" was given in that very building and in this wise. A certain Edinburgh Society was holding a banquet there about fifteen months ago, and the man who should have announced the guests and called the toasts was, unfortunately, owing to illness, unable to attend. They got a substitute, however, at the last moment, and when the latter gentleman came, it was plain that whatever he knew about his own business he knew very little about official titles. He began reading the list, and the guests filed slowly into the room. In due time he read, "The President of the Faculty—of the Faculty," and then he came to a dead stop. Then, looking round in desperation, he called out, "The President of the Faculty of *Accurates!*" (Laughter.) The guests all pressed forward to see who the distinguished President

of this unique Faculty was, but nobody moved. Presently a gentleman, accepting the situation, stepped into the room with considerable courage, and, at the same time, with becoming modesty. That gentleman was his (Mr. Carter's) good friend, their worthy Chairman, the President of the Faculty of Actuaries. So they would see that their President, by his presence of mind and prompt action, secured to them, in all time coming, the term "Accurate," a definition even more appropriate than his Lordship's. Mr. Carter then asked Mr. Hewat to accept, on behalf of the Society of Accountants, his grateful thanks for the very kind allusions he had made to the Society. He trusted the two bodies would always continue to work hand in hand, and to assist one another. In conclusion, he thanked the Faculty for the welcome they had given all their guests, and wished them, in the name of the latter, increased prosperity and continued good luck.

MR. HIGHAM, at the Chairman's request, interpolated a toast conveying the thanks of the company to the Croupiers. He was, he said, a Southron who until then had not had the slightest idea what a croupier was. If the croupiers meant the Dinner Committee, he thought they would all agree that their heartiest thanks were due to them. Nothing struck an Englishman more than the good fellowship which seemed to exist among Scottish gentlemen. He trusted the Faculty would long continue in prosperity not alone for their scientific attainments and their skill, but in the hope that they would from time to time invite poor Southrons to their hospitable board.

MR. M'LAUCHLAN, in responding, said it had always been a very great pleasure to him to do anything he could for the profession, and particularly for the Faculty. He could assure them that the Committee to whom the arrangements for that night were entrusted, and their Secretary, Mr. Edgar, had given their best attention and thought to these, and he was sure if the company had enjoyed themselves, and if they were able to feel that their meeting that evening had been an adequate commemoration of the Jubilee of the Faculty, the Committee would feel amply repaid for their trouble.

MR. R. P. HARDY then gave the last toast, that of "the Chairman." In doing so he asked them to place on record the feelings of respect and personal regard for that gentleman, which they unanimously entertained, and by which the London brethren who were there that night were deeply impressed. It was unnecessary that he should enlarge upon the dignity and the grace with which Mr. Gunn had presided over that representative meeting of that

great profession of which, within that realm, he was the official head. It was in the view of them all eminently fitting that on such an occasion the profession should be represented in the Chair by the responsible chief officer of one of the Scottish Life Assurance Companies whose names stood as admitted types of British solidity, as bright examples of personal integrity in the unselfish administration of the high trusts committed to them. It was such men, men of character quite apart from professional skill, who had lifted the Faculty to the high position it now deservedly held, and who would ever maintain it in its sphere of honour and transcendent usefulness as the guardian and feeder of those cisterns of learning which it had so abundantly filled and placed at the free disposal of all. Mr. Hardy concluded by wishing the President long life and much happiness, a wish in which he was sure they all joined.

THE CHAIRMAN said he felt very much gratified by the way in which Mr. Hardy had proposed the toast. He could only thank them all for the kind way in which his efforts had been received. It was a great gratification to him that, in consequence of the accident of being President of the Faculty at that time, he had been privileged to fill such an important position, and he had to thank the Faculty for placing him there. Before he sat down he would like to thank his colleagues on the Dinner Committee for the labours they had gone through in carrying out the preparations, and especially he would re-echo the words of Mr. Higham with reference to Mr. M'Lauchlan for the unwearied trouble Mr. M'Lauchlan had taken and for all he had done for them.

Music was played during dinner; Mr. Robert Burnett sang in the intervals between the toasts; and Mr. Paulin, by request, gave a recitation of "The Annuity," Mr. George Outram's well-known production, referred to by Lord Ardwall.

The following cable messages, in addition to that mentioned in the President's speech, were received in the course of the evening:—

From New York.—"Council Actuarial Society in session extends heartiest congratulations.—Hunter, Secretary."

From Magdeburg.—"Zum Jubelfeste Ihres 50-jährigen Bestehens senden wir Ihnen unsere herzlichsten Glückwünsche in dankbarer Würdigung der grossen Dienste welche die Fakultät in dieser langen Zeit als bahnbrechende Führerin unserer Wissenschaft geleistet hat. Möchten gleiche Erfolge und gleiche Ehren der Fakultät auch in aller Zukunft beschieden sein. Möchten stets

die herzlichsten Beziehungen zwischen der gefeierten Fak und unserem Vereine obwalten. Möchte es uns vergönnt recht zahlreiche Ihrer hochverehrten Mitglieder zum fü Kongress für Versicherungswissenschaft in unserer Reichsh stadt begrüssen zu können.—Deutscher Verein für Versicheru wissenschaft: Dr. Hahn, Vorsitzender."

"On the occasion of your Jubilee Festival we send you heartiest good wishes in grateful appreciation of the great serv which the Faculty has rendered during this long period pioneer in our science. May continued success and furt honours attend the Faculty in the future. May the same frien relations between the Faculty and our Society be always maintai and may we have the privilege of welcoming very many of yo esteemed members at the Fifth Actuarial Congress in our Capi —From the German Society for the Science of Insurance: D Hahn, President."

From Rotterdam.—"Cordial congratulations, Holland Society Actuaries."

Suitable replies to these kind messages were sent by the Ho Secretary in the name of the Faculty.

Vol. III. Part III.

TRANSACTIONS

OF THE

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CONSTITUTED 1856. INCORPORATED 1868.

No. 27.

Notes on Summation and Interpolation.

By Austyn J. C. Fyfe, F.F.A.

Abstract of Discussion thereon.

Actuarial Note.

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Notes on Summation and Interpolation. *By* AUSTYN J. C. FYFE, F.F.A., *of the Northern Assurance Company, Aberdeen.*

[Read before the Faculty, 12 March 1906.]

ONE element underlying the whole theory of Life Contingencies is that of a SERIES. Thus in the Life Table we have a collection of the more usual and elementary functions, every column forming a series, and it will at once suggest itself that as soon as the actual calculation of any problem is faced, we must sooner or later arrive at a Series, and the solution of the problem will finally depend on our ability to deal with this Series. The same is obviously true of other branches of our Science, such as those derived from statistics of Sickness, of Marriage and of Issue, and, of course, in the purely algebraical theory of Compound Interest.

Regarded in this light, the principle of a Series may be said to be the "bedrock" of the process, so far as the actual calculation is concerned—and it is not surprising, therefore, that Summation and Interpolation have received their full share of attention from contributors to actuarial literature. Indeed, Summation has been so much treated and discussed, directly or incidentally, that a paper on the subject at this stage may seem presumptuous, or, at best, somewhat out of date.

It will be agreed, however, that the formulas which have been receiving this attention are those generally known as "Quadrature" formulas, and the method is more fittingly termed "Integration" than Summation. This distinction between two different classes of formula should be borne carefully in mind. SUMMATION, from its nature, purports a more exact result than Integration, being in fact more an algebraical operation, while INTEGRATION is primarily a geometrical problem. Summation refers to a series, Integration more to a curve.

Taking Lubbock's formula as representative of the first of these classes, and Mr. Hardy's formulas as representative of the second, there can be no doubt that, having regard to the object in view (which is generally the evaluation of some complex benefit), the formulas of the latter class are far superior, developed as they have been to the utmost conciseness, and consequent facility of application, and it is needless to repeat their many advantages here. For this very reason, however, the natural tendency has been to discard the former class entirely, and there is a danger, for students especially, from this repeated reference to the more advanced formulas, hastily to throw aside the older formulas as cumbersome and obsolete. On this plea, therefore, it is hoped that the following remarks on Lubbock's formula may not be devoid of interest, whatever their practical value may be.

My purpose is to show that the formula which we know by the name of Lubbock's, and which is due to the late Sir J. W. Lubbock, Bart., is the most general form of summation of a series of equidistant values, at whatever interval, in terms of successive finite differences, and in the symbolical form which we shall deduce, is of still wider application. In this paper I have made a somewhat extensive use of the method of Separation of Symbols, or Calculus of Operations as it has been called. As this method of proof does not seem to be too well known, it may be best, for the sake of completeness, to deduce the formula *ab initio*, following De Morgan (*Diff. Calc.*, p. 315) and Dr. Sprague (*J. I. A.*, xviii., 305).

Let $u_0, u_1, u_2 \ldots u_{n-1}$ be terms of a series, and let the series be differenced in the usual way, Δu_0, $\Delta^2 u_0$, etc., representing the successive orders of differences, the interval of differencing being 1, and let the series to be summed be denoted by $u_0, u_{\frac{1}{m}}, u_{\frac{2}{m}} \ldots$, etc., $\frac{1}{m}$th of an interval apart, and let it be required to find the sum of all terms from u_0 to $u_{n-\frac{1}{m}}$, both inclusive, mn terms in all.

Then we have for the required sum

$$u_0+u_{\frac{1}{m}}+u_{\frac{2}{m}}+\&c.+u_{n-\frac{1}{m}}$$

$$=u_0+(1+\Delta)^{\frac{1}{m}}u_0+(1+\Delta)^{\frac{2}{m}}u_0+\&c.+(1+\Delta)^{n-\frac{1}{m}}u_0.$$

$$=\left[1+(1+\Delta)^{\frac{1}{m}}+(1+\Delta)^{\frac{2}{m}}+\&c.+(1+\Delta)^{n-\frac{1}{m}}\right]u_0$$

$$=\frac{(1+\Delta)^n-1}{(1+\Delta)^{\frac{1}{m}}-1}u_0$$

$$=\frac{1}{(1+\Delta)^{\frac{1}{m}}-1}(u_n-u_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The fraction may now be expanded in ascending powers of Δ, either by actual division, or by recourse to the co-efficients in the expansion of $\frac{x}{(1+x)^t-1}$, originally given by De Morgan, and incorporated by Mr. King in Part II. of the *Text Book*. This gives for the required sum :—

$$u_0+u_{\frac{1}{m}}+\&c.+u_{n-\frac{1}{m}}$$

$$=\left[m\cdot\frac{1}{\Delta}+\frac{m-1}{2}-\frac{m^2-1}{12m}\Delta+\frac{m^2-1}{24m}\Delta^2\right.$$

$$\left.-\frac{(m^2-1)(19m^2-1)}{720m^3}\Delta^3+\&c.\right](u_n-u_0) \quad . \quad . \quad (2)$$

To put a value on the first term, put $m=1$ in the equation and we have

$$u_0+u_1+u_2+\&c.+u_{n-1}=\frac{1}{\Delta}(u_n-u_0),$$

and substituting this in (2), we obtain

$$u_0+u_{\frac{1}{m}}+u_{\frac{2}{m}}+\&c.+u_{n-\frac{1}{m}}$$

$$=m(u_0+u_1+u_2+\&c.+u_{n-1})+\frac{m-1}{2}(u_n-u_0)-\frac{m^2-1}{12m}(\Delta u_n-\Delta u_0)$$

$$+\frac{m^2-1}{24m}(\Delta^2u_n-\Delta^2u_0)-\frac{(m^2-1)(19m^2-1)}{720m^3}(\Delta^3u_n-\Delta^3u_0)+\&c. \quad (3)$$

This is the formula in its usual and familiar shape. But before proceeding further I think it will be well to make a slight modification. The series to be summed has been referred to an original series $u_0, u_1, u_2 \ldots u_{n-1}$, at "unit" intervals apart, and in the aggregate containing n units of value, while the series to be summed contains mn units of value. To bring the required sum, therefore, into line with the original series as it were, and also for a better comparison with other and analogous formulas (as will

afterwards appear), we must multiply every term by $\frac{1}{m}$, and so obtain from equation (3)

$$\frac{1}{m}\left[u_0+u_{\frac{1}{m}}+u_{\frac{2}{m}}+\&c.+u_{n-\frac{1}{m}}\right]$$
$$=(u_0+u_1+u_2+\&c.+u_{n-1})+\frac{m-1}{2m}(u_n-u_0)$$
$$-\frac{m^2-1}{12m^2}(\Delta u_n-\Delta u_0)$$
$$+\frac{m^2-1}{24m^3}(\Delta^2 u_n-\Delta^2 u_0)$$
$$-\frac{(m^2-1)(19m^2-1)}{720m^4}(\Delta^3 u_n-\Delta^3 u_0)$$
$$+\&c. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

It is to the symbolical form of this formula, however, that I wish to call attention. Let us write

$\Sigma_{\overline{n}|}$ to denote $(u_0+u_1+u_2+\&c.+u_{n-1})$

and $\Sigma^{(m)}_{\overline{n}|}$ to denote $\frac{1}{m}\left(u_0+u_{\frac{1}{m}}+u_{\frac{2}{m}}+\&c.+u_{n-\frac{1}{m}}\right)$

then from equations (1) and (3) we write at once

$$\Sigma^{(m)}_{\overline{n}|}=\frac{1}{m}\cdot\frac{1}{(1+\Delta)^{\frac{1}{m}}-1}(u_n-u_0) \quad . \quad . \quad . \quad (5)$$

or, since $\quad u_n-u_0=\{(1+\Delta)^n-1\}u_0$

$$\Sigma^{(m)}_{\overline{n}|}=\frac{(1+\Delta)^n-1}{m[(1+\Delta)^{\frac{1}{m}}-1]}u_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

But, when $m=1$ this becomes

$$\Sigma_{\overline{n}|}=\frac{(1+\Delta)^n-1}{\Delta}\cdot u_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Whence we have, from these last two equations

$$\Sigma^{(m)}_{\overline{n}|}=\frac{\Delta}{m[(1+\Delta)^{\frac{1}{m}}-1]}\Sigma_{\overline{n}|} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

A glance at this equation suggests at once the analogy of the formulas for amounts and present values of annuities-certain, namely

$$s^{(m)}_{\overline{n}|}=\frac{i}{m[(1+i)^{\frac{1}{m}}-1]}s_{\overline{n}|}, \text{ or } \frac{i}{j_{(m)}}s_{\overline{n}|}$$

$$a^{(m)}_{\overline{n}|}=\frac{i}{m[(1+i)^{\frac{1}{m}}-1]}a_{\overline{n}|}, \text{ or } \frac{i}{j_{(m)}}a_{\overline{n}|}$$

and I think this analogy is worth a little examination.

We know that in these two formulas the fraction on the right is a *factor*, the application of which will convert the ordinary annuity into one payable m times a year, always under the operation of the same effective rate of interest. But we may otherwise describe this factor as one which will give the sum of a series of "$m$thly" values (mn in number but of total value n) in terms of a series of "unit" values.

So now in an exactly analogous way we may regard the function

$$\frac{\Delta}{m[(1+\Delta)^{\frac{1}{m}}-1]},$$

as an *operator*, which if applied to the ordinary series of "unit" values $\Sigma_{\overline{n}|}$ will give instead the sum of a series of $m$thly values (mn in number, but of total value n); or otherwise:—the effect of this operator is

(1) to interpose $(m-1)$ terms between each of the terms of the original series, thus subdividing each interval into m parts

(2) to multiply the whole series so obtained by $\frac{1}{m}$

all in place of the original sum $\Sigma^{(1)}_{\overline{n}|}$.

Let us carry the analogy a little further. In the case of the annuity, the result (whether amount or present value) will be determined by the value of the factor $i/j_{(m)}$, which will in turn vary according to the value of m. Now as m varies from 1 to ∞, $j_{(m)}$ varies from i to $\log_e(1+i)$,* and the annuity will vary from $a_{\overline{n}|}$ to $\bar{a}_{\overline{n}|}$.

So now again, taking the symbolical formula

$$\Sigma^{(m)}_{\overline{n}|}=\frac{\Delta}{m[(1+\Delta)^{\frac{1}{m}}-1]}\Sigma_{\overline{n}|}$$

we may say that as m varies from 1 to ∞ the form of the denominator will vary from Δ to $\log_e(1+\Delta)$, and the resulting sum $\Sigma^{(m)}_{\overline{n}|}$ will vary from $\Sigma_{\overline{n}|}$ to

$$Lt_{m=\infty}\left[u_0+u_{\frac{1}{m}}+u_{\frac{2}{m}}+\&c.+u_{n-\frac{1}{m}}\right]\frac{1}{m}$$

that is from $\Sigma^{(1)}_{\overline{n}|}$, to $\int_0^n u_x d_x$.

That this is true may be shown by taking as examples the two extreme cases.

* The limit, when m is infinite, of $m\,[(1+x)^{\frac{1}{m}}-1]$ is $\log_e(1+x)$. See also *Institute of Actuaries' Text Book.* Part I. (New Edition.) Ch. I. Art. 10.

(1) If $m=1$ we have as before

$$\Sigma^{(1)}_{\overline{n}|}=\frac{(1+\Delta)^n-1}{(1+\Delta)-1}u_0=\frac{(1+\Delta)^n-1}{\Delta}u_0$$

or expanding the right hand side

$$\Sigma_{\overline{n}|}=nu_0+\frac{n(n-1)}{\underline{|2}}\Delta u_0+\frac{n(n-1)(n-2)}{\underline{|3}}\Delta^2 u_0+\ .\ .\ .\ (9)$$

which is a familiar elementary formula.

[See *Text Book II.*, p. 464.]

This may also be written

$$\Sigma_{\overline{n}|}=\frac{u_n-u_0}{\Delta}=\frac{u_n}{\Delta}-\frac{u_0}{\Delta}=\Sigma u_n-\Sigma u_0\ .\ .\ .\ .\ (10)$$

which is recognised at once as a fundamental formula, and which also furnishes a definition of the symbol $\Sigma=\Delta^{-1}$.

(2) If $m=\infty$ we have

$$Lt_{m=\infty}\Sigma^{(m)}_{\overline{n}|} \text{ or}$$

$$\int_0^n u_x dx=Lt_{m=\infty}\left[\frac{u_n-u_0}{m[(1+\Delta)^{\frac{1}{m}}-1]}\right].$$

$$=\frac{u_n-u_0}{\log_e(1+\Delta)}\ .\ .\ .\ .\ .\ .\ .\ .\ (11)$$

$$=\frac{1}{\Delta-\frac{\Delta^2}{2}+\frac{\Delta^3}{3}-\frac{\Delta^4}{4}+\ .\ .\ .}(u_n-u_0)$$

$$=\left[\frac{1}{\Delta}+\frac{1}{2}-\frac{\Delta}{12}+\frac{\Delta^2}{24}-\frac{19}{720}\Delta^3+\frac{9}{480}\Delta^4-\ .\ .\ .\right](u_n-u_0)\ (12)$$

$$=\Sigma_{\overline{n}|}+\frac{1}{2}(u_n-u_0)-\frac{1}{12}\left(\Delta-\frac{\Delta^2}{2}+\frac{\Delta^3}{3}-\frac{\Delta^4}{4}+\ .\ .\ .\right)(u_n-u_0)$$

$$+\frac{1}{720}\left(\Delta^3-\frac{3}{2}\Delta^4+\ .\ .\ .\right)(u_n-u_0)-\&c.$$

Now $\left(\Delta-\frac{\Delta^2}{2}+\frac{\Delta^3}{3}-\frac{\Delta^4}{4}+\ .\ .\ .\right)u_x=\frac{du_x}{dx}$ and $\left(\Delta^3-\frac{3}{2}\Delta^4+\ .\ .\ .\right)u_x=\frac{d^3u_x}{dx^3}$ approximately, see Text Bk. II., pp. 453-4

so that the last equation may be written

$$\int_0^n u_x dx=\Sigma_{\overline{n}|}+\frac{1}{2}(u_n-u_0)-\frac{1}{12}\left(\frac{du_n}{dx}-\frac{du_0}{dx}\right)+\frac{1}{720}\left(\frac{d^3u_n}{dx^3}-\frac{d^3u_0}{dx^3}\right)-\&c.$$

$$.\ .\ .\ (13)$$

which is one of Woolhouse's formulas.

I propose to use the symbol $\delta_{(m)}$ to represent $m[(1+\Delta)^{\frac{1}{m}}-1]$, in the same way that $j_{(m)}$ stands for $m[(1+i)^{\frac{1}{m}}-1]$. This matter of the analogy between $\delta_{(m)}$ and $j_{(m)}$ I shall return to later on, but meanwhile we may regard $\delta_{(m)}$ as the "$m$thly difference," or

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Now $a_0=1$ and $b_{-1}=1$

$$a_1=t \qquad b_0=+\frac{m-1}{2m}$$

$$a_2=\frac{t(t-1)}{2} \qquad b_1=-\frac{m^2-1}{12m^2}$$

$$a_3=\frac{t(t-1)(t-2)}{6} \qquad b_2=+\frac{m^2-1}{24m^2}$$

$$a_4=\frac{t(t-1)(t-2)(t-3)}{24} \qquad b_3=-\frac{(m^2-1)(19m^2-1)}{720m^4}.$$

Whence, substituting these values in the expansion, multiplying out, and remembering that $\Delta^{-1}=\Sigma$, $\Delta^0=1$, etc., we have, after reduction:—

$$_t\Sigma_{\overline{n}|}^{(m)}=\Sigma_{\overline{n}|}+\left(\frac{m-1}{2m}+t\right)(u_n-u_0)-\left\{\frac{m^2-1}{12m^2}+\frac{t}{2}\left(\frac{1}{m}-t\right)\right\}(\Delta u_n-\Delta u_0)$$
$$+\left\{\frac{m^2-1}{24m^2}+\frac{t}{4}\left(\frac{1}{m}-t\right)+\frac{t}{12}\left(\frac{1}{m}-t\right)\left(\frac{1}{m}-2t\right)\right\}(\Delta^2u_n-\Delta^2u_0) \quad (17)$$

taken as far as second differences.

So long ago as the year 1869, in *J. I. A.*, xv, 95, Mr. Woolhouse gave a formula for the summation of the same series but taken to infinity, and in terms of *continuous* values and successive differential coefficients, in place of ordinary values and finite differences. His formula (with the notation modified and the upper limit of summation being n in place of ∞) is as follows:—

$$_t|\Sigma_{\overline{n}|}^{(m)}=\int_0^n u_x dx-\left(\frac{1}{2m}-t\right)(u_n-u_0)+\frac{1-6mt+6m^2t^2}{12m^2}\left(\frac{du_n}{dx}-\frac{du_0}{dx}\right)$$
$$+\frac{t}{12}\left(\frac{1}{m}-t\right)\left(\frac{1}{m}-2t\right)\left(\frac{d^2u_n}{dx^2}-\frac{d^2u_0}{dx^2}\right)-\&c. \quad . \quad (18)$$

It is easy to show that these two formulas are interchangeable. We have only to put in the last formula for $\int_0^n u_x dx$ its equivalent as found in formula (13) and

for $\frac{d}{dx}$ write $\Delta-\frac{1}{2}\Delta^2+\frac{1}{3}\Delta^3$

" $\frac{d^2}{dx^2}$ " $\Delta^2-\Delta^3$

" $\frac{d^3}{dx^3}$ " Δ^3

and our formula No. 17 will be reproduced.

It seems almost necessary to state that the Finite Difference formula (17) was arrived at quite independently, in the manner shown; but it is obvious that it might have been got from Woolhouse's by simply making these substitutions. Reference

to Woolhouse's paper, however, will show that his method of proof is entirely different from the present.

APPLICATIONS.

It now remains to show the application of these formulas to the actual calculation of benefits, and for the sake of illustration it will be sufficient to take the case of the single-life annuity as representative. Here it may be remarked that in the examples used in this connection a somewhat undue prominence has been given to the *whole-life* benefit, with the result that when temporary annuities are taken up, the formulas seem to acquire a formidable appearance, and this more especially when any irregularity presents itself, as for instance when payments are made fractionally, or are deferred. From the greatly increased number of Endowment Assurances, however, and the consequent importance of their valuation, the temporary annuity has become a function hardly less important than the whole-life annuity, and a thorough familiarity with expressions for the valuation factors is an evident necessity. In the following examples, therefore, I have worked mainly on temporary annuities.

Starting with our formula (17) we have in this case

$$
\begin{aligned}
{}_t|\Sigma^{(m)}_{\overline{n}|} &= {}_t|a^{(m)}_{x\overline{n}|} \times D_x = {}_t|N^{(m)}_{x\overline{n}|} \\
\Sigma_{\overline{n}|} &= a_{x\overline{n}|} \times D_x = N_{x\overline{n}|} \\
u_0 - u_n &= D_x - D_{x+n} \\
\Delta(u_0 - u_n) &= \Delta D_x - \Delta D_{x+n}, \\
&\quad \&c. \qquad\quad \&c.
\end{aligned}
$$

Whence

$$
\begin{aligned}
{}_t|N^{(m)}_{x\overline{n}|} = N_{x\overline{n}|} &- \left(\frac{m-1}{2m} + t\right)(D_x - D_{x+n}) \\
&+ \left[\frac{m^2-1}{12m^2} + \frac{t}{2}\left(\frac{1}{m} - t\right)\right](\Delta D_x - \Delta D_{x+n}) - \&c.
\end{aligned}
$$

or, dividing by D_x

$$
\begin{aligned}
{}_t|a^{(m)}_{x:\overline{n}|} &= a_{x\overline{n}|} - \left(\frac{m-1}{2m} + t\right)(1 - {}_nE_x) \\
&+ \left[\frac{m^2-1}{12m^2} + \frac{t}{2}\left(\frac{1}{m} - t\right)\right]\frac{\Delta D_x - \Delta D_{x+n}}{D_x} - \&c. \quad . \quad . \quad (19) \\
&= a_{x\overline{n}|} + \left(1 - t - \frac{m-1}{2m}\right)(1 - {}_nE_x) + \&c. \\
&= a_{x\overline{n}|} + \left(\frac{m-1}{2m} + \frac{1}{m} - t\right)(1 - {}_nE_x) + \&c. \quad . \quad . \quad . \quad (20)
\end{aligned}
$$

If in this last equation we put $m=1$ and take only the first two terms, we have

$$\begin{aligned} {}_{t|}\mathrm{a}_{x\overline{n}|} &= a_{x\overline{n}|}+(1-t)(1-{}_n\mathrm{E}_x) \\ &= a_{x\overline{n}|}+(1-t)(\mathrm{a}_{x\overline{n}|}-a_{x\overline{n}|}) \\ &= ta_{x\overline{n}|}+(1-t)\mathrm{a}_{x\overline{n}|} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21) \end{aligned}$$

This is a convenient approximation and one commonly used. It will be seen from (19) that the error arising from its use may be measured by

$$\frac{1}{\mathrm{D}_x}\cdot\left\{\frac{t}{2}(1-t)[\Delta\,\mathrm{D}_x-\Delta\,\mathrm{D}_{x+n}]\right\}$$

subsequent terms being insignificant.

Another annuity frequently required in practice is the *continuous temporary annuity* $\bar{a}_{x\overline{n}|}$.

Here we simply put $m=\infty$ and $t=0$ in the formula and

$$\bar{a}_{x\overline{n}|}=a_{x\overline{n}|}+\tfrac{1}{2}(1-{}_n\mathrm{E}_x)+\tfrac{1}{12}\left(\frac{\Delta\,\mathrm{D}_x-\Delta\,\mathrm{D}_{x+n}}{\mathrm{D}_x}\right) \quad . \quad . \quad . \quad . \quad (22)$$

So that the third term being generally very small we may safely write

$$\bar{a}_{x\overline{n}|}=\tfrac{1}{2}(a_{x\overline{n}|}+\mathrm{a}_{x\overline{n}|}) \text{ or } \tfrac{1}{2}(1+a_{x:\overline{n-1}|}+a_{x\overline{n}|}). \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

An alternative formula to the last is obtained by Mr. Woolhouse's thus

$$\bar{a}_{x\overline{n}|}=a_{x\overline{n}|}+\tfrac{1}{2}(1-{}_n\mathrm{E}_x)-\tfrac{1}{12}\left[\mu_x+\delta-\frac{\mathrm{D}_{x+n}(\mu_{x+n}+\delta)}{\mathrm{D}_x}\right]-\&c. \quad (24)$$

The annuity represented by ${}_{t|}\mathrm{a}^{(m)}_{x:\overline{n}|}$ is the most general expression for an annuity on a single life, and by assigning particular values to t, m, and n, almost every case may be met. It is needless to detail examples here; the scope of the formula is evident. An expression is given in the Text Book, ch. ix. equation 27, for ${}_{\frac{1}{t}|}\mathrm{a}_x$; but it is now seen that this is merely a particular case of our formula (20) when $m=1$, $n=\infty$, and $\frac{1}{t}$ is written for t.

A rather curious case is that where m is fractional, or less than 1. As an example of this we may find the value of a bonus of £1 °/₀ per annum declared quinquennially, on a life aged 60. Let the next declaration fall to be made 3 three years hence. Then we have only to take formula (17) and put $m=\frac{1}{5}$ $t=3$, and $n=\infty$, and using the Text Book Graduation of the H^{M} Table at $4\frac{1}{2}$ per cent. interest we have, for the value of the quinquennial bonus

$$\frac{5}{\mathrm{D}_{60}}\cdot[\mathrm{M}_{63}+\mathrm{M}_{68}+\mathrm{M}_{73}+\ .\ .\ .]$$

$$=\frac{1}{D_{60}}\{\Sigma M_{60}-[-2+3]M_{60}+[-2+\tfrac{3}{2}(5-3)]\Delta M_{60}\}$$
$$=\frac{1}{D_{60}}[R_{60}-M_{60}+C_{60}]$$
$$=\frac{1}{D_{60}}[27683-2370+119{\cdot}7]$$
$$=25913\div D_{60}$$
$$=6{\cdot}0058.$$

Actual summation of the series on the left gives $25915\div D_{60}$.

INTERPOLATION.

A table of the amount of £1 at the end of 1, 2, 3 . . . n years, that is a table of $(1+i)^n$ at rate of interest i, is formed by continuous multiplication by $(1+i)$, i being the rate of increment of each value over the preceding. And if it were desired to divide each of these intervals into m parts, thus giving a table of amounts at the end of each mth part of a year, then always assuming an effective rate of interest i, our factor for continuous multiplication would be $\left(1+\frac{j_{(m)}}{m}\right)$, where $j_{(m)}=m[(1+i)^{\frac{1}{m}}-1]$, the rate of increment of each successive value being $j_{(m)}/m$, corresponding to a yearly rate $j_{(m)}$.

Now if we have a series of terms u_0 u_1 . . . u_n the problem of subdividing each interval into m parts, interpolating $m-1$ terms between each consecutive pair, is precisely analogous to this. We have seen that regarded an an *operation* on the *sum* of a series, the function $\Delta/\delta_{(m)}$, or $\dfrac{\Delta}{m[(1+\Delta)^{\frac{1}{m}}-1]}$ will give us the *sum* of a series subdivided into $m$thly intervals, or the sum of the original series together with that of the interpolated terms. If, then, a process of interpolation is included, why not make direct use of the same function of operation, viz. $\delta_{(m)}$, and construct the series by continuous application of this operator, in the same way as would be done in a table of $(1+i)^{\frac{1}{m}}$?

The method generally followed to get the series subdivided, is to find the "subdivided differences," by forming a series of equations expressing u_x in terms of u_0 and its differences Δu_0, $\Delta^2 u_0$ etc., then by differencing out the series of equations so obtained, the subdivided differences are ready to hand. My point is, however, that these required "subdivided differences" are nothing more nor less than the successive powers of $\left(\frac{\delta_{(m)}}{m}\right)u_0$. For by

actual expansion it may be found (stopping at fourth differences) that

$$\left.\begin{aligned}
\left(\frac{\delta_{(m)}}{m}\right)u_0 &= [(1+\Delta)^{\frac{1}{m}}-1]u_0 \\
&= \left[\frac{\Delta}{m}-\frac{m-1}{2}\cdot\frac{\Delta^2}{m^2}+\frac{2m^2-3m+1}{6}\cdot\frac{\Delta^3}{m^3} \right. \\
&\qquad \left. -\frac{6m^3-11m^2+6m-1}{24}\cdot\frac{\Delta^4}{m^4}\right]u_0 \\
\left(\frac{\delta_{(m)}}{m}\right)^2 u_0 &= [(1+\Delta)^{\frac{1}{m}}-1]^2 u_0 \\
&= \left[\frac{\Delta^2}{m^2}-(m-1)\frac{\Delta^3}{m^3}+\frac{11m^2-18m+7}{12}\cdot\frac{\Delta^4}{m^4}\right]u_0 \\
\left(\frac{\delta_{(m)}}{m}\right)^3 u_0 &= [(1+\Delta)^{\frac{1}{m}}-1]^3 u_0 \\
&= \left[\frac{\Delta^3}{m^3}-\frac{3m-3}{2}\cdot\frac{\Delta^4}{m^4}\right]u_0 \\
\left(\frac{\delta_{(m)}}{m}\right)^4 u_0 &= [(1+\Delta)^{\frac{1}{m}}-1]^4 u_0 \\
&= \frac{\Delta^4}{m^4}u_0.
\end{aligned}\right\} \quad (25)$$

So far there is nothing new. The method of obtaining the subdivided differences though more direct, simply bringing us back to the usual method—fully described in the Text Book, Part II., pp. 441-443.

But this process of expanding $\left(\frac{\delta_{(m)}}{m}\right)$ has suggested a slight modification, which gives the same results, from somewhat simpler expressions.

We notice that

$$\begin{aligned}
\delta u_0 &= u_{\frac{1}{m}}-u_0{}^{*} && =[(1+\Delta)^{\frac{1}{m}}-1]u_0 \\
\delta^2 u_0 &= u_{\frac{2}{m}}-2u_{\frac{1}{m}}+u_0 && =[(1+\Delta)^{\frac{2}{m}}-2(1+\Delta)^{\frac{1}{m}}+1]u_0 \\
\delta^3 u_0 &= u_{\frac{3}{m}}-3u_{\frac{2}{m}}+3u_{\frac{1}{m}}-u_0 && =[(1+\Delta)^{\frac{3}{m}}-3(1+\Delta)^{\frac{2}{m}} \\
& && \quad +3(1+\Delta)^{\frac{1}{m}}-1]u_0 \\
\delta^4 u_0 &= u_{\frac{4}{m}}-4u_{\frac{3}{m}}+6u_{\frac{2}{m}}-4u_{\frac{1}{m}}+u_0 && =[(1+\Delta)^{\frac{4}{m}}-4(1+\Delta)^{\frac{3}{m}} \\
& && \quad +6(1+\Delta)^{\frac{2}{m}}-4(1+\Delta)^{\frac{1}{m}}+1]u_0
\end{aligned}$$

But instead of using $\delta^2 u_0$ we may use $\delta^2 u_{-\frac{1}{m}}$

" " $\delta^3 u_0$ " $\delta^3 u_{-\frac{1}{m}}$

" " $\delta^4 u_0$ " $\delta^4 u_{-\frac{2}{m}}$

* In future I write δ for $\frac{\delta_{(m)}}{m}$.

and so obtain

$$\delta^2 u_{-\frac{1}{m}} = u_{\frac{1}{m}} - 2u_0 + u_{-\frac{1}{m}} \qquad = [(1+\Delta)^{\frac{1}{m}} - 2 + (1+\Delta)^{-\frac{1}{m}}]u_0$$

$$\delta^3 u_{-\frac{1}{m}} = u_{\frac{2}{m}} - 3u_{\frac{1}{m}} + 3u_0 - u_{-\frac{1}{m}} \qquad = [(1+\Delta)^{\frac{2}{m}} - 3(1+\Delta)^{\frac{1}{m}} + 3 - (1+\Delta)^{-\frac{1}{m}}]u_0$$

$$\delta^4 u_{-\frac{2}{m}} = u_{\frac{2}{m}} - 4u_{\frac{1}{m}} + 6u_0 - 4u_{-\frac{1}{m}} + u_{-\frac{2}{m}} \qquad = [(1+\Delta)^{\frac{2}{m}} - 4(1+\Delta)^{\frac{1}{m}} + 6 - 4(1+\Delta)^{-\frac{1}{m}} + (1+\Delta)^{-\frac{2}{m}}]u_0$$

from which it may be seen that the powers of Δ tend to range more symmetrically round the central term u_0, and the expansions will involve somewhat simpler co-efficients.

Thus we have, expanding, after reduction :—

$$\left.\begin{aligned} \delta^2 u_{-\frac{1}{m}} &= \left[\frac{\Delta^2}{m^2} - \frac{\Delta^3}{m^3} + \frac{1+11m^2}{12}\cdot\frac{\Delta^4}{m^4}\right]u_0 \\ \delta^3 u_{-\frac{1}{m}} &= \left[\frac{\Delta^3}{m^3} - \frac{1+3m}{12}\cdot\frac{\Delta^4}{m^4}\right]u_0 \\ \delta^4 u_{-\frac{2}{m}} &= \frac{\Delta^4}{m^4}u_0 \end{aligned}\right\} \quad . \quad (26)$$

It will be allowed that these coefficients are simpler than those of the usual differences $\delta^2 u_0$, $\delta^3 u_0$, and $\delta^4 u_0$ (see p. 76) I have only carried the work out as far as 4th differences, but it will be found that even to 5th differences the formulas are simpler.

As examples, to subdivide the intervals into 5 parts (*i.e.* $m=5$)

$$\left.\begin{aligned} \delta u_0 &= \cdot 2\Delta u_0 - \cdot 08\Delta^2 u_0 + \cdot 048\Delta^3 u_0 - \cdot 0336\Delta^4 u_0 \\ \delta^2 u_{-\frac{1}{m}} &= \cdot 04\Delta^2 u_0 - \cdot 04\Delta^3 u_0 + \cdot 0368\Delta^4 u_0 \\ \delta^3 u_{-\frac{1}{m}} &= \cdot 008\Delta^3 u_0 - \cdot 0106\Delta^4 u_0 \\ \delta^4 u_{-\frac{2}{m}} &= \cdot 0016\Delta^4 u_0 \end{aligned}\right\} \quad . \quad (27)$$

and to subdivide the intervals into 10 parts (*i.e.* $m=10$)

$$\left.\begin{aligned} \delta u_0 &= \cdot 1\Delta u_0 - \cdot 045\Delta^2 u_0 + \cdot 0285\Delta^3 u_0 - \cdot 0206625\Delta^4 u_0 \\ \delta^2 u_{-\frac{1}{m}} &= \cdot 01\Delta^2 u_0 - \cdot 01\Delta^3 u_0 + \cdot 009175\Delta^4 u_0 \\ \delta^3 u_{-\frac{1}{m}} &= \cdot 001\Delta^3 u_0 - \cdot 002583\Delta^4 u_0 \\ \delta^4 u_{-\frac{2}{m}} &= \cdot 0001\Delta^4 u_0 \end{aligned}\right\} \quad (28)$$

In applying this method, of course, we must bear in mind that these differences are not the "leading differences" of the series, with which we are familiar, but are the differences of

u_0, $u_{-\frac{1}{m}}$, $u_{-\frac{1}{m}}$, and $u_{-\frac{2}{m}}$ respectively.

A numerical example will make this clear; and for the sake of comparison we may take the same example as in the Text Book II, p. 446.

Having given

$$\left.\begin{array}{l}\log p_{20}=\bar{1}\cdot9969724\\ \log p_{30}=\bar{1}\cdot9964260\\ \log p_{40}=\bar{1}\cdot9959051\\ \log p_{50}=\bar{1}\cdot9943048\end{array}\right\}$$ to complete the series by interpolating all the intervening values from $\log p_{20}$ to $\log p_{50}$.

Here we have

$$\begin{aligned} u_0 &= \bar{1}\cdot9969724\\ \Delta u_0 &= -\ \cdot0005464\\ \Delta^2 u_0 &= +\ \cdot0000255\\ \Delta^3 u_0 &= -\ \cdot0011049. \end{aligned}$$

Whence, applying the above formulas and treating δ^3 as constant

$$\delta u_0 = -\cdot0000872770$$

$$\delta^2 u_{-\frac{1}{m}} = +\cdot0000113040$$

$$\delta^3 u_{-\frac{2}{m}} = -\cdot0000011049$$

and the following shows the work, the figures under the heavy line representing the interpolated values:—

δ^3	δ^2	δ	u		
−11049	+113040				
	101991	−872770	+9969724000	$=\log p_{20}$	u_0
	90942	770779	9968851230	$=\log p_{21}$	$u_{1/m}$
	79893	679837	9968080451	$=\log p_{22}$	$u_{2/m}$
	68844	599944	9967400614	$=\log p_{23}$	$u_{3/m}$
	57795	531100	9966800670	$=\log p_{24}$	$u_{4/m}$
	46746	473305	9966269570	$=\log p_{25}$	$u_{5/m}$
	35697	426559	9965796265	$=\log p_{26}$	$u_{6/m}$
	24648	390862	9965369706	$=\log p_{27}$	$u_{7/m}$
	13599	366214	9964978844	$=\log p_{28}$	$u_{8/m}$
		352615	9964612630	$=\log p_{29}$	$u_{9/m}$
			9964260015	$=\log p_{30}$	u_1

In closing, I feel that some apology is needed for bringing for-

ward a paper so fragmentary, and containing at the best so little of any practical value. The truth is, however, that the paper was originally intended merely as a Note for the *Transactions*. It had its origin in an attempt to find a general formula for the annuity ${}_{t|}a^{(m)}_{xn}$, or rather the sum of the series underlying it; and it was thought that the symbolical method of proof, by expansion of the formula No. 16, might be of some interest. I am convinced that by the use of the symbolical method some further developments may still be made on the standard processes of dealing with Series.

DISCUSSION.

THE paper having been read, in Mr. Fyfe's absence, by the Secretary, Mr. Edgar,—

Mr. VYVYAN MARR said—The paper is a welcome contribution to the Transactions as showing in a concrete shape the general expression from which so many formulas in the Text Book are derived. Students especially will benefit from Mr. Fyfe's work, and they would perhaps have been even better pleased had he dealt more fully with the method—Separation of Symbols—he has employed. My experience has been that the Text Book, while giving most complete solutions for the various benefits, does not put the work in such a way that the average student actually grasps the main idea underlying them; but a study of this paper, after a preliminary reading of the Text Book, should give a good grounding of the actual mathematical work involved in the theory of Life Contingencies. I fear I have little to say on the subject of the paper, except to mention that some time ago I investigated the error involved in using $a_x + 1 - t$ as the value of ${}_{t|}a_x$ and when Mr. Fyfe thought of undertaking a paper for us I was glad he adopted a suggestion which I suppose was the origin of this paper. An interesting point in connection with the error involved in the approximation is that it varies according to t and attains a maximum and then diminishes. Taking Mr. Fyfe's expression—given after formula (21)—and differentiating it with respect to t, it will be found to attain a maximum value when $t = \cdot 5$—that is to say, under no circumstances does the error, in the case of a yearly annuity, exceed

$$\frac{1}{8}\frac{\Delta D_x - \Delta D_{x+n}}{D_x}.$$

Mr. C. C. NICHOLL.—Mr. Fyfe's paper is undoubtedly an interesting example of the method of Separation of Symbols, and forms a useful supplement to Chapters XXII and XXIII of the Text Book. The Calculus of Operations is not, as a rule, gone into very extensively by young students, and, as the paper is of special interest to them, it would perhaps have been well if Mr. Fyfe had given something more by way of explanation of the method, as it is only just touched upon in the Text Book, and people who are not familiar with it may be excused for regarding it with a certain amount of suspicion. The principle on which it is based, so far as the Calculus of Finite Differences is concerned, may be stated as follows:—

Any direct function of Δ_x and constant quantities may be transformed as if Δ_x were itself a quantity, and the result, on being interpreted according to the definitions of the symbols, will be found to be correct. Constant terms and factors therefore retain their character throughout, but the index of the symbol of operation, which is treated in the course of the transformations as indicating the power to which the fictitious quantity Δ_x is to be raised, is taken in the final result as showing the extent to which the operation is to be performed. But it should be carefully noted that, although for the purpose of our operations we may treat Δ_x as following the Distributive and the Index laws, it can only be taken as following the Commutative law so far as constants are concerned. Thus, for instance, $\Delta f(n)\phi(n)$ is not, except in special cases, the same as $f(n)\Delta\phi(n)$.

Turning to another portion of the paper we may note that the error involved in finding the value of a deferred annuity by interpolating to first differences is very small. In the general case of the $m$thly annuity payable for n years and deferred t years $\left(\text{supposing } t<\frac{1}{m}\right)$, the maximum absolute error occurs when $t=\frac{1}{2m}$, when it amounts to

$$-\frac{2m^2+1}{24m^2}\,\frac{\Delta D_x-\Delta D_{x+n}}{D_x},$$ neglecting second and higher difference of D_x.

The following short note shows the *percentage* error, as distinct from the absolute error, arising from the use of the usual approximation to the value of the deferred yearly annuity, and is based on the H^M 3 per cent. *whole of life* annuity table.

Age.	Value of t for which percentage error is a maximum	Maximum error per cent.
20	·506	·0153
30	·506	·0221
40	·507	·0272
50	·509	·0382
60	·512	·0668
70	·517	·154
80	·529	·475
90	·556	1·50

The error is a positive one, and therefore leads us to over-estimate our assets in a valuation, but would hardly be worth while taking into account even at the oldest ages, unless we were dealing with a case where m was fractional. In such a case the error might acquire considerable dimensions at an advanced age. In the case of the deferred *temporary* annuity, an investigation of the expression for the percentage error shows that it will, generally speaking, be less than in the case of the corresponding whole life annuity.

Dr. BUCHANAN.—Mr. Fyfe's paper is of a very technical nature, but students will no doubt find old and familiar formulæ presented in ways which had not occurred to them before. He has distinguished between two groups of summation formulæ, of which Lubbock's and Hardy's may be taken as the types; and has referred to the former as belonging to the older class. So far as their application to life contingencies is concerned that is no doubt correct; but the other group are really much older than Lubbock's formula. The method of expressing the area of a curve in terms of equidistant ordinates is due to Newton and Cotes, and some of the oldest of this class of formulæ date back to the beginning of the eighteenth century. Woolhouse's formula too, or as it should more correctly be called the Euler-Maclaurin summation formula, also belongs to the eighteenth century; and it has been shown that practically all the best known quadrature formulæ involving equidistant terms can be derived from it, so that it is both older and of greater generality than Lubbock's. Its relation to Lubbock's formula had long been known; but it is instructive to have the student's attention drawn to it in this way. An integral is the limit of a summation, and the method employed in the paper of deriving the one formula from the other is an example of the Newtonian method of integration—of taking the sum of a finite number of terms and then passing to the limit by making the number infinite. It may be pointed out, however. that the transformation could be more easily effected by starting with formula (4), making m infinite, and then expressing the differences in terms of differential coefficients. Formula (8) too could be derived at once from (1) without any of the intermediate steps by the self-evident substitution of $\Delta\Sigma_{\overline{n}|}$ for $u_n - u_0$. There is a danger in these symbolical methods that operators should come to be regarded as algebraical symbols of quantity. The chief interest of the paper centres in the application of Lubbock's formula to the calculation of deferred benefits; and it is useful to have a general expression for a deferred temporary annuity payable m times a year. Mr. Fyfe's principal formula (17) is, for purposes of calculation, open to the same objection as Lubbock's, to which Dr. Sprague drew attention many years ago, viz.: that it involves differences of u_n, which requires a knowledge of terms following u_n. The point is not of great practical importance; but it is possible to express formula (17) in terms of differences which depend only on $u_0, u_1, \ldots u_n$, and the transformation might be suggested as an exercise for students. In the example worked out at the end of the paper the results agree with those of the Text Book, because the interpolation is in each case carried to third differences, which are supposed to be constant. That is as close an approximation as any one would wish; but there does not seem to be any shortening of the work. The mode of arriving at the $m$thly differences is shorter than that given in the Text Book; but the method has probably suggested itself before to many of those present. The symbolical method of treating difference questions is a very powerful one, and has recently been extensively used to develop the theory of central differences as distinct from the theory of advancing differences: but the meaning of the Operators should never be lost sight of, for without clear ideas of their nature the manipulation of symbols of operation is apt to lead to distrust of the results obtained.

Dr. A. E. SPRAGUE.—Mr. Fyfe has no need to apologise for his paper. It puts the matters dealt with in a fresh light and advances our knowledge of the subject. I do not, however, agree with him when he says that Summation purports a more exact result than Integration. In both cases the theory is mathematically accurate, and in practice numerical results can be obtained to as close a degree of approximation as may be

desired. Mr. Fyfe speaks of formula (8) as analogous to the two formulas immediately following, but the connection is closer than mere analogy. The amount of an annuity-certain can be expressed as the sum of a series whose general term u_x is of the form $(1+i)^x$. Hence

$$\Delta u_x = (1+i)^{x+1} - (1+i)^x = i(1+i)^x = iu_x \text{ or symbolically } \Delta = i.$$

Therefore if formula (8) be applied to find the amount of such an annuity, the result will be got simply by substituting i for Δ, that is to say, the second formula is a particular case of the first. The same holds for the present value of the annuity, since this is equal to the amount of the annuity multiplied by v^n, which is independent of x, and is therefore constant so far as Δ is concerned. Personally I have found the paper most interesting. In particular, the deduction of Lubbock's formula by Separation of Symbols has always seemed to me preferable to the demonstration in the Text Book, which is to my mind more complicated and difficult to follow.

THE PRESIDENT expressed his sense of the value of the paper as an assistance to students, and as suggesting interesting methods of dealing with the development of actuarial formulæ. On his motion a hearty vote of thanks was accorded to Mr. Fyfe for his paper.

Mr. FYFE'S reply.—I should like to thank the various speakers for their remarks on my paper, which I have read with much interest. Mr. Nicholl and Dr. Buchanan have referred to the caution necessary in manipulating Symbols of Operation. The scope of the paper did not involve, to any great extent, the direct use of Operators as such, beyond the more usual symbols belonging to the Calculus of Finite Differences; otherwise it would have been necessary, as Mr. Nicholl says, to have given some explanation. The general question is as intricate as the method is powerful, especially when considering its limitations. But Mr. Nicholl's remarks on the subject are very clear. With regard to the transformation of Lubbock's formula into Woolhouse's, it is true that, as Dr. Buchanan says, it could be more easily effected in the way he describes. But the object I had in view was to lay more stress on the generality of the Symbolical method, by transforming the symbols themselves, as in formula (11), rather than the derived expansion. Indeed it is by this process alone that so many transformations may be made upon series, such, for instance, as the expression to which he refers as preferable to formula (17), which may be obtained by transforming expression (16).

Thereafter the Secretary read a short paper submitted by Mr. GEO. C. M'LAREN, F.F.A., of the South African Mutual Life Assurance Society, Cape Town, on the results of A GROUP VALUATION OF JOINT LIFE POLICIES, in which the policies were grouped for valuation according to their duration, the average ages for each group being based upon the average premium payable. A vote of thanks was accorded to Mr. M'Laren for his paper.

ACTUARIAL NOTE.

On a General Expression embracing Lubbock's and Woolhouse's Summation Formulas. *By* JOHN GOVAN, F.F.A.

It has been shown that Woolhouse's formula may be derived from Lubbock's and *vice versa*, but it would seem to have escaped notice that both are cases of a more general equation, viz. :—

$$\begin{aligned} h(u_0+u_h+u_{2h}+\&c.+u_{x-h}) \\ =n(u_0+u_n+u_{2n}+\&c.+u_{x-n})-\frac{n-h}{2}(u_0-u_x) \\ +\frac{n^2-h^2}{12n}(\Delta u_0-\Delta u_x) \\ -\frac{n^2-h^2}{24n}(\Delta^3 u_0-\Delta^3 u_x) \\ +\&c. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1) \end{aligned}$$

where Δ stands for the operation of differencing for interval n. Mr. Fyfe's ingenious paper affords a convenient means of verification. Give to h the value $\frac{1}{m}$ and to n the value 1, and we have his equation (4) or Lubbock's formula. Give to h the value 1 and diminish n indefinitely, in which case Δ becomes $n\frac{d}{dx}$, then transpose, and we have his equation (13), which is Woolhouse's formula. It may be well to show the latter operation step by step. Thus, $n(u_0+u_n+\&c.+u_{x-n})$ becomes the integral $\int_0^x u_x dx$. The second term presents no difficulty. Write the third term in the the form $-\frac{h^2}{12}\left(\frac{\Delta}{n}u_0-\frac{\Delta}{n}u_x\right)$ and it is seen to be $-\frac{h^2}{12}\left(\frac{du_0}{dx}-\frac{du_x}{dx}\right)$.

Write the fourth term $\frac{nh^2}{24}\left(\frac{\Delta^3}{n^3}u_0-\frac{\Delta^3}{n^3}u_x\right)$ and substitute $\frac{d^3}{dx^3}$ for $\frac{\Delta^3}{n^3}$ when the term is seen to vanish; and so on.

Again, writing $2n$ for h and 0 for n we obtain the first of the formulas given in the Text Book under the name of Mr. G. F. Hardy (No. 30). To follow the artifices by which Mr. Hardy arrives at the particular form there exhibited, we have simply to shift the origin (say by writing $x-n$ for the variable x), deduct nu_{-n} from both sides of the equation, and then express the functions of $-n$ in terms of the functions of 0.

This compendious rule is only one of a large number w
either cases of, or immediately derivable from, the fund
equation

$$u_x = u_{\frac{x}{n}.n} = (1+\Delta)^{\frac{x}{n}} u_0 \quad . \quad . \quad . \quad . \quad . \quad .$$

where n is the interval involved in the operation of diffe
The proof is familiar. Similarly, where h is the interva
stands for the operation, we have

$$u_x = (1+\delta)^{\frac{x}{h}} u_0$$

From these two equations, eliminating x, we get

$$\delta = \left\{(1+\Delta)^{\frac{h}{n}} - 1\right\}$$

and generally, $$\delta^r = \left\{(1+\Delta)^{\frac{h}{n}} - 1\right\}^r \quad . \quad . \quad . \quad . \quad . \quad .$$

When r is integral and positive, and h and n appreciable qua
this last is the formula which enables us to supply val
interval h in a series given at intervals of n. If r is a p
integer, h infinitesimal and n appreciable, division by h^r gi
the rth differential coefficient—say $\left\{\frac{1}{n}\log_e(1+\Delta)\right\}^r$ in ter
the differences for interval n, whence it is easy to get it in
of values of the function.

If now r be given the value -1 we have

$$\delta^{-1} u_x = \frac{1}{(1+\Delta)^{\frac{h}{n}} - 1} u_x \quad . \quad . \quad . \quad . \quad . \quad .$$

Here $\delta^{-1}u_x$, as is shown in Mr. Fyfe's paper, signifies Σ
interval h, and, expanding the right hand expression (see
Book II., pp. 432-3), we at once obtain the equation given
as embracing the Summation formulas.

Again, reverting to the fundamental equation (2) and
ing n infinitesimal, we have $u_x = \left(1 + n\frac{d}{dx}\right)^{\frac{x}{n}} u_0 = e^{x\frac{d}{dx}} u_0$ whi
Maclaurin's modification of Taylor's Theorem in symbolic for

To show how it is that the alternate coefficients in Woolh
expansion vanish (commencing with the fourth), take the expr
for it corresponding to the right-hand side of equation (4)
$(e^{h\frac{d}{dx}} - 1)^{-1} u_x$, say $(e^a - 1)^{-1} u_x$. Let $f(a)$ stand for $(e^a - 1)^{-1}$
will be found that $f(a) + f(-a) = -1$; so that if we expand
in powers of a, we must get a series such that when the alte
signs are changed and the resulting series added to the ori
all the coefficients in the sum from the first power upwards, va
This can only be if the coefficients of the even powers of
the expansion of $f(a)$ are all zero.

Vol. III.

TRANSACTIONS

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No. 28.

Notes on Recent Legal Decisions relating to Life Assurance. By John L. Wark, M.A., LL.B., Advocate.

Abstract of Discussion thereon.

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TRANSACTIONS

OF THE

FACULTY OF ACTUARIES.

VOL. III.—PART IV.

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Notes on Recent Legal Decisions relating to Life Assurance.

By John L. Wark, M.A., LL.B., Advocate.

Abstract of Discussion thereon.

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FACULTY OF ACTUARIES.

Notes on Recent Legal Decisions relating to Life Assurance.

By JOHN L. WARK, M.A., LL.B., Advocate.

[Read before the Faculty, 4th June 1906.]

IN 1893 Mr. William Harvey delivered to the Actuarial Society of Edinburgh a paper on "Misrepresentation and Concealment as affecting policies of Insurance,"[1] and in the following year a series of "Lectures on Insurance Law."[2] I have no desire to trench upon the ground covered in these lectures, but what I now propose is to supplement them by putting before you, briefly, the results of the more important decisions relating to life assurance which have occurred in the English and Scottish Courts since these lectures were delivered. In dealing with the decisions I propose to follow the order of treatment adopted by Mr. Harvey, and I shall endeavour, also following him, to notice the American cases, and to indicate how far these are likely to be useful as authorities in our law.[3] The subject of Trust and Settlement Policies has been separately dealt with by Mr. Constable in his paper on "The Married Women's Policies of Assurance (Scotland) Act, 1880";[4] but there are a number of decisions since that paper was delivered which also I think it right to bring under your notice.

I. MISREPRESENTATION AND CONCEALMENT.

(*a*) *Warranty and Representation.*—The recent cases in this connection have mostly turned on the question of whether statements made by an assured in answer to questions contained in the application for the policy amounted to warranties, or were merely representations as to the truth of the facts contained in them. The difference between a warranty and a representation

[1] *Transactions of the Actuarial Society of Edinburgh*, vol. iii. p. 117.
[2] *Ibid.* p. 267.
[3] American decisions are indicated by the abbreviated titles of the Reports, viz., "U.S." for Otto's Supreme Court Reports, and "Am. St. R." for the American State Reports.
[4] *Ibid.* p. 341.

has been very well put in an American case: "A representation is clearly distinguishable from a warranty. The former is part of the proceedings which propose a contract. The latter is part of the completed contract. The falsity of the former may render the contract voidable for fraud, but a non-compliance with the latter is an express breach of the contract."[1] Fraud includes at common law such misrepresentation, although innocent, as has materially induced the contract. But the language of modern policies almost invariably excludes the right to challenge a mere representation—not amounting to a warranty—except on the ground of fraud.

In determining whether a statement in the proposal or the policy is a warranty or merely a representation, the whole terms of the documents must be considered, and if it appears from the whole that the statement was not intended as a warranty it will not be so construed.[2] The mere use of the word "warrant" will not of itself decide the question. The court will "look at the business of the matter"[3] to ascertain the real intention of parties. On the other hand the use of that word is an element which the court will consider in determining the intention of parties.[4] In the case of *Hambrough* the proposal contained the following declaration: "I agree that all the foregoing statements and answers, as well as those that I make to the Company's examiner in continuation of this application, are by me warranted to be true and are offered to the Company as a consideration of the contract, which I hereby agree to accept as issued by the Company in conformity with this application." A provisional policy was issued in consideration of the application, which was thereby made part of the contract. The clause above narrated was construed by the Court of Appeal as a warranty that the statements in the proposal were accurate, and a condition that if any of them were untrue in fact the policy should not be enforceable.

(*b*) *Extent of Warranty.*—It was not necessary in that case to decide whether the warranty extended to statements made in answer to the medical examiner of the Insurance Company as well as to those in the proposal. This question came up sharply for decision in the subsequent case of *Delahaye* v. *The British Empire Mutual Life Assurance Company.*[5] The policy in that case recited that the assured had delivered to the Company a proposal and declaration, and agreed that these should be the basis

[1] *Wheaton* v. *The North British Insurance Company* (1888), 9 Am. St. R. 216.
[2] *National Bank* v. *Union Insurance Company* (1891), 22 Am. St. R. 324; *Phœnix Life Insurance Company* v. *Raddin* (1887), 120 U.S. 183.
[3] *Barnard* v. *Faber*, L.R. (1893), 1 Q.B. 340, p. Mr. Justice A. L. Smith.
[4] *Hambrough* v. *The Mutual Life Insurance Company of New York* (1895), 72 Law Times Reports, 140, p. Lord Justice Lopes.
[5] (1897), 13 Times Law Reports, 245.

of the contract between him and the Company. In the proposal the assured answered in the negative the question whether he had ever suffered from any serious illness or disease tending to shorten life, and made the following declaration: "I do hereby declare that I am at present in good health and that I am not, to the best of my knowledge, the subject of any disease tending to make assurance more than usually hazardous; and I agree that this declaration shall be the basis of the contract between me and the Company, and that if any untrue statement be made therein or in the answers to the questions put by the Company's medical examiner in reference to this proposal, all sums paid to the Company on account of this assurance shall be forfeited, and any policy granted hereon shall be null and void." On December 13th, 1895, the applicant was examined by the Company's doctor. The doctor put to him the question, "Have you ever suffered from indigestion or jaundice?" to which he answered "No." He died in February 1896. At the trial evidence was given on behalf of the defendants that the assured had had an attack of jaundice in February 1893. The jury found that the answers in the proposal and to the doctor were not untrue to the knowledge of the assured. In an application for a new trial, the Court of Appeal held that any warranty given by the assured did not extend beyond the answers given by him in the proposal, that his answers to the doctor were not intentionally false, and that the Company was liable upon the policy. Lord Esher (M.R.) said, "The questions contained in the proposal were before the applicant when he signed the document, but he could not then know what questions would be asked by the medical man. There was a great difference between the two cases, and they must construe the document so as to give effect to that difference. Though there might be a warranty with regard to those matters which were before the applicant when he signed the document, there was no warranty at all with regard to the other matters, except a warranty only that he would tell the truth to the best of his knowledge."

The American Courts have displayed a similar reluctance in the absence of the most explicit and unequivocal stipulations to infer that the insured took and the insurer issued a life-policy on the distinct understanding that it should be void if any statements made in the proposal or in the course of the medical examination should be false, whether the assured was conscious of the falsity or not.[1] At the same time there is nothing to prevent such an understanding being made part of the contract, even as regards matters of which the insured could have no knowledge, *e.g.* latent

[1] *March* v. *The Metropolitan Life Insurance Company* (1898), 65 Am. St. R. 887; *Globe Mutual Life Insurance Company* v. *Wagner* (1900) 80 Am. St. R. 169.

diseases,[1] or, as in *Wagner*, the death of a relative abroad whom the assured believed to be and had represented to be alive; and it is well to note that where such is the case the condition will be strictly enforced even as regards statements on matters which would appear to be matter of opinion or belief, unless such statements are qualified by the use of words showing that they are only intended to be statements of opinion or belief.

(*c*) *Contradictory and ambiguous provisions in proposal and in policy.*—The familiar difficulty caused by contradictory provisions in the policy and in the declaration appended to the proposal arose again recently in a Scotch case dealing with an accident policy. The declaration was in these terms: "I do hereby agree that the above particulars are true, and I agree that this proposal and declaration shall be the basis of the contract between myself and the company." The policy provided that any fraudulent misdescription in the particulars furnished by the insured should render the policy void. The insured had in the proposal made a false statement with regard to claims against him in respect of accidents prior to the date of the proposal, but the proof established that this statement was not made fraudulently. The Second Division held that the proposal and policy must be construed together; that the declaration in the proposal was a warranty that the statements in it were true in point of fact, and that this warranty was not limited by the clause in the policy. They accordingly held the policy void.[2] This decision was given by a majority consisting of Lord Young and Lord Trayner in a court of three judges. Lord Moncreiff, who was the third judge, strongly dissented, and in an opinion, which humbly appears to me to be sound, found that although taken by itself, the declaration in the proposal amounted to a warranty of the statements contained therein, the provision in the policy that it should be rendered void by any fraudulent misdescription in the particulars furnished by the insured applied to the particulars furnished in the proposal and was not limited to particulars mentioned in the policy or its conditions. His Lordship agreed that the policy and proposal must be read together, but the result of doing so he found to be that the insurance company had excluded challenge of the policy on any ground except fraud. This opinion would seem to be the sound one if *Fowkes* v. *The Manchester and London Life Assurance Association*[3] is to be regarded as well decided. *Fowkes'* case was even less favourable to the insured, for there the clauses were reversed, the provision against fraudulent misstatement being contained in the proposal, and the policy embody-

[1] *Thomson* v. *Weems* (1884), L.R., 9 A.C. 671, p. Ld. Watson.

[2] *Reid and Company* v. *The Employers Accident Assurance Association* (1899), 1 Fraser 1031.

[3] (1863), 3 B. & S. 917.

ing the wider provision that if any statement in the declaration was untrue the policy should be void.

(*d*) *Indisputable policies.*—The term "indisputable policy" is used by Mr. Harvey to describe a policy, by the terms of which the insurer abandons his right to dispute the policy except on the ground of fraud —as opposed to a "conditional policy" in which the accuracy in point of fact of the statements made in the proposal is made a condition precedent to the right of the insured to recover on the policy. But the term "indisputable" or "incontestable" has also been used to describe a form of policy recently adopted by some American companies, containing a provision that either after delivery and acceptance, or within a specified period from its date, the policy shall be incontestable except for non-payment of premiums or misstatement of age. A policy of this nature cannot, after the terms of this provision have been complied with, be avoided even on the ground of fraudulent misstatements by the assured regarding his health,[1] or on the ground of suicide, even where the policy contains in another clause an exception against suicide.[2] It has further been held that a stipulation in a life policy that it shall be incontestable for fraudulent statements in the application or in the course of the medical examination is not void as against public policy.[3] I do not think that Scottish Insurance Companies will be eager to introduce a form of policy which entails such heavy risks as some American companies are apparently willing to undertake. An unnecessary straining at a gnat after swallowing a camel is found in one recent American case in which it was held that notwithstanding an incontestable clause of the nature above referred to, the company might successfully deny liability to an assignee of the policy on the ground that he was not such in good faith, but that his assignation was an evasion of the rule against wager policies.[4]

(*e*) *The doctrines of Waiver and of Agency as affecting the right to avoid a policy for misrepresentation or concealment.*—Two cases are referred to by Mr. Harvey as establishing the proposition that failure on the part of the assured to communicate facts of which the agent of the company making out and receiving the proposal for insurance is or ought to be aware, will not avoid the insurance. In the first of these, *Follette* v. *The United States Mutual Accident Association*,[5] the assured was afflicted with deafness. The agent who received the proposal was well

[1] *Patterson* v. *The Natural Premium Mutual Life Insurance Company* (1898), 69 Am. St. R. 399.

[2] *Mareck* v. *The Mutual Reserve Fund Life Association* (1895), 54 Am. St. R. 613; *Goodwin* v. *The Provident Savings Life Assurance Association* (1896), 59 Am. St. R. 411; *Sun Life Insurance Company* v. *Taylor* (1900), 94 Am. St. R. 383.

[3] *Insurance Company* v. *Fox* (1901), 82 Am. St. R. 885.

[4] *Clement* v. *Insurance Company* (1898), 70 Am. St. R. 650.

[5] (1890), 22 Am. St. R. 878.

acquainted with the assured and aware of his infirmity, and it was held the policy could not be avoided on the ground that the proposal contained a statement that the assured had no mental or bodily infirmity. In the other case, *Bawden* v. *The London, Edinburgh and Glasgow Assurance Company*,[1] the assured was blind of one eye, a circumstance which was obvious to any one looking at him, and apparent to the agent who made out and received the proposal. The proposal contained a statement that the applicant had no physical infirmity and that there was no circumstance known to him which rendered him peculiarly liable to accidents; and yet the policy was held not to be voidable in respect of that statement being untrue. The ground of decision, as in *Follette*, was that the Company must be taken to have entered into the contract with the knowledge possessed by their agent, and to have consented to accept the risk in the knowledge that the assured was a one-eyed man.

The authority of *Bawden's* case has been considerably shaken by several recent decisions. It has been successfully "distinguished" in all the subsequent cases, and has been openly doubted in at least one case in England and another in our own courts. *Cruikshank* v. *The Northern Accident Assurance Company*,[2] though at first sight apparently following *Bawden*, does not really do so. The proposal in that case contained the following question and answer:—"*Q.* Are there any circumstances which render you peculiarly liable to accident?—*A.* Slight lameness from birth." It also contained a statement by the assured that he had not had paralysis and had no physical infirmity. The assured was in fact very lame, but the extent of his lameness was evident to the agent when he made out the proposal. In the Outer House Lord Low held that the knowledge of the agent on the matter was the knowledge of the Company, and the Company could not take advantage of any misdescription of the lameness in the proposal to avoid the policy. The Inner House, while adhering to the Lord Ordinary's judgment, proceeded rather on the ground that the statement as to lameness was not untrue. They further held that the statement as to absence of paralysis or physical infirmity must be taken along with the disclosure of lameness, and that paralysis in this statement meant paralysis arising from an organic cause, "that graver form of attack arising from lesion of the brain."

It is quite true that representations made to the agent are in law made to the Company, and that his knowledge is equivalent to the Company's knowledge. But it is equally well settled that the assured will be held responsible for written statements made

[1] L. R. (1892), 2 Q. B. 534.

[2] (1895), 23 Rettie, 147.

in the proposal which he has signed. He will not be responsible for the fraud or mistake of the agent of the insurance company in communicating to the company verbal statements which he has made to such agent. But he will not be excused from responsibility for the truth or accuracy of written statements in the proposal merely because he has not read these statements as filled in by the agent, or has otherwise negligently failed to assure himself of their accuracy before signing the proposal. Nor will he be excused from the written statements in the proposal because he has made other statements to the company's agent, inconsistent with those in the proposal. In the case of *Biggar* v. *The Rock Life Assurance Company*,[1] a proposal form was filled in by the company's canvasser, several of the answers being false in material respects. The false answers were inserted by the canvasser at his own hand and without the knowledge or authority of the applicant, who signed the proposal form without reading it. The policy as issued proceeded on the basis of the truthfulness of the answers contained in the declaration attached to the proposal. In an action on the policy it was held by Mr. Justice Wright (1) that it was the duty of the applicant to read the answers in the proposal before signing it, and he must be taken to have read these answers and adopted them, and (2) that the canvasser in filling in the false answers was acting as his agent and not as that of the insurance company. He therefore found that the policy was void. The same conclusion was arrived at and the judgment of Mr. Justice Wright was approved in a precisely similar case decided a few months ago in the Second Division of the Court of Session.[2] In the case of Reid[3] above referred to, the plea that an alleged false statement made in the proposal had been filled in by the agent before the proposal was signed was held by the Court not to free the assured from responsibility for the statement to which he had appended his signature. By signing it, he made the proposal and all statements contained in it his own. In *Levy* v. *The Scottish Employers Insurance Association*,[4] Mr. Justice Phillimore, in dealing with the argument founded upon *Bawden's* case that the incorrectness of certain statements in a proposal was known to the agent and could not be pled by the insurance company in defence to an action on the policy, observed that the decision in *Bawden* turned upon the special terms of the contract, and the utmost it came to was this, that it was the agent's duty to put into the proposal patent and obvious facts.

The most recent case on this subject is *M'Millan* v. *The Accident*

[1] L. R. (1902), 1 K.B., 516.
[2] *Life and Health Assurance Company* v. *Yule* (1904), 6 Fraser, 437.
[3] *Reid and Company* v. *The Employers Accident Assurance Association*, 1 Fraser, 1031.
[4] (1901), 17 Times Law Reports, 229.

Assurance Company Limited.[1] In that case Lord Salvesen questioned the decision in *Bawden* on two grounds, (1) that it was not clear that the agent there had any authority from the company to negotiate and settle the terms of the proposal, and (2) that the judgment overlooks the fact that the contract sued on was a contract in writing. In *M'Millan's* case the policy contained a clause that "the Company shall not be held liable in respect of any knowledge of or notice to an agent which shall not have been communicated to and have been acknowledged in writing by the Company at its registered office." But the decision did not proceed upon this clause, which was absent from the policy in *Bawden's* case. The ground of decision in *M'Millan*, following *Biggar* and *Levy*, was that one of the statements in the written proposal signed by the assured was untrue, and that it made no difference whether that untrue statement was written by the proposer himself or by the insurance company's agent for him. Lord Salvesen said, "I think it cannot be too strongly impressed on the minds of intending assurers that they must see to it that the proposal form is correctly filled up, and that no verbal statements made to the agents or servants of the company, who may have an interest of their own to serve by having the proposal accepted, are of any avail as correcting material misstatements in the proposal itself, the truth of which is the basis of the whole contract."[2] *A fortiori* it would appear that it is not open to the assured to correct material misstatements in the proposal by an appeal to the knowledge of the company's agent. It must of course be kept in view that if the assured's omission to read the proposal is due to circumstances not inferring negligence on his part, such as blindness or inability to read, the rule above stated will not apply; and similarly, if the applicant were to sign the form in blank, on the representation of the company's agent that the duty of filling it up correctly might be left to the agent, it would be a question of circumstances whether he would be held to be responsible for the statements made by the agent.[3]

The American decisions are quite in line with *Biggar* and *M'Millan*. The agent obtaining the insurance may become the agent of the applicant with reference to the part he takes in adjusting the proposal, but whether he is so or not is a question to be determined from the whole circumstances of the particular case, and not merely from a statement in the policy that he shall be deemed to be the agent of the assured.[4] The same rule has

[1] (1906), 13 Scots Law Times, 858.
[2] Cf. *The Fidelity Mutual Life Assurance Association* v. *Harris* (1900), 86 Am. St. R. 813.
[3] See cases quoted by Mr. Harvey, pp. 136 *et seqq.*
[4] *Lumbermen's Mutual Insurance Company* v. *Bell* (1897), 57 Am. St. R. 140.

been laid down with reference to the doctor examining the applicant, and the answers filled in by him in his report.[1] It has further been held that if an insurance company is misled by a false statement intentionally made by an applicant as to his occupation, the knowledge of the agent taking the application does not bar the company from pleading the misrepresentation. The agent must in such cases be presumed to be in collusion with the applicant.[2]

II. Agency.

(*a*) *Authority of Agent to Bind the Company.*—In the United States insurance agents have as a rule much wider powers than in this country, and accordingly the American cases are unsafe guides upon this branch of the subject. But both in America and in this country it is clearly settled that it is not within the ordinary duty of an insurance agent to undertake to grant a policy, and that such an undertaking will not bind the company unless the agent was specially authorised to make it.[3] Accordingly a statement made by an insurance company's agent to an intending insurer that he might hold himself as insured from the date of the proposal was held to be of no effect where such assurance was in conflict with the terms of the proposal itself or was not specially authorised.[4] Where, however, the agent is acting within the limits of his authority, his action will bind the company. Thus the acceptance of premiums by an agent, duly authorised to collect the same, in the knowledge that an assured is using intoxicating liquors to excess in violation of the conditions of his policy, is a waiver of these conditions, even although the agent failed to communicate his knowledge to the company.[5] In order to meet this difficulty it is common for American companies to provide that no condition of a policy shall be waived by an officer or agent of the company unless such waiver be endorsed upon the policy. But notwithstanding such a condition, the issuance and delivery of the policy by the company with knowledge of facts which would render the policy void is a waiver of such ground of forfeiture.[6]

Two cases may be referred to in connection with the right to recovery of premiums on policies which have been entered into in reliance upon a representation made by the agent of an insur-

[1] *Royal Neighbors of America* v. *Boman* (1898), 69 Am. St. R. 201; *Sternaman* v. *The Metropolitan Life Insurance Company* (1902), 88 Am. St. R. 625.
[2] *Triple Link Mutual Indemnity Association* v. *Williams* (1898), 77 Am. St. R. 34.
[3] *Linford* v. *Provincial Horse and Cattle Insurance Company* (1864), 34 Beavan, 291.
[4] *Levy* v. *The Scottish Employers Insurance Association*, supra; *Chamberlain* v. *The Prudential Insurance Company* (*of America*) (1901), 83 Am. St. R. 851.
[5] *North-Western Masonic Aid Association* v. *Bodurtha* (1899), 77 Am. St. R. 414.
[6] *Phœnix Insurance Company* v. *Flemming* (1898), 67 Am. St. R. 900.

ance company that the policies would be good, and which have afterwards been avoided as illegal. Where such a representation was made innocently and in good faith by an insurance agent, and the policy was afterwards avoided on the ground of want of insurable interest, the insured was found not to be entitled to recover the premiums.[1] "Unless there can be introduced the element of fraud, duress or oppression, or difference in the position of the parties which would create a fiduciary relationship to the insured so as to make it inequitable for the company to insist upon the bargain they had made with the insured, he is in the position of a person who has made an illegal contract and has sustained a loss in consequence of a misstatement of law, and he must submit to that loss." But where the statement on which the assured acted was fraudulently made by the agent of the insurance company the premiums can be recovered, for the company is not entitled to benefit by its agent's fraud.[2]

(*b*) *Agent's Lien.*—In England an agent or third party who advances the premiums on a policy at the request of the parties interested therein acquires a lien over the policy for the amount of such premiums, but he will have no lien if he does so merely at his own hand and for his own purposes.[3] In Scotland no lien will be acquired in either case without a bond or an assignation of the policy. A solicitor advanced premiums on an insurance policy belonging to a client who was his debtor. The advances were made at first at the debtor's request, and afterwards at the solicitor's own hand, with a view to keeping the policy alive as a security for his debt. He had all along possession of the policy, but had no assignation thereto. In a multiplepoinding raised to determine the right to the policy moneys, it was held that the solicitor had no lien upon the policy or right to a preferential ranking for any of the premiums paid by him.[4]

III. Conditions in Life Policies.

(*a*) *Insurable Interest.*—In this country the defence of want of insurable interest can be pled only by the insurer. If he does not plead it "the question who is entitled to the money must be determined as if the statute[5] did not exist,"[6] and our courts

[1] *Harse* v. *The Pearl Life Assurance Company*, L.R. (1903), 2 K.B. 92; (1904), 1 K.B. 558.

[2] *British Workman's and General Assurance Company* v. *Cunliffe* (1902), 18 Times Law Reports, 425, 502.

[3] In re *Leslie* (1883), 23 Ch. D. 552; *Falcke* v. *The Scottish Imperial Insurance Company* (1887), L.R. 34, Ch. D. 234.

[4] *Wylie's Executrix* v. *M'Jannet* (1901), 4 Fraser, 195.

[5] 4 Geo. III. c. 48.

[6] Lord Justice Mellish in *Worthington* v. *Curtis* (1875), 7 Ch. 424.

have accordingly refused to give weight to the argument that where the ownership of a policy is in doubt that view must be preferred which accords with the policy being a legal contract, having regard to the statute. A policy had been taken out on the life of a lad of eighteen and taken possession of by his father, who kept it, and paid all the premiums upon it till the death of the assured. In an action to determine the right to the policy moneys, no question being raised by the insurance company as to insurable interest, the court refused to consider that question and held that the policy was the property of the father.[1] It seems to have been assumed in this case that the father had no insurable interest in the son's life; but, while this would have been so in England,[2] it is not necessarily so in Scotland where the father has by law a direct claim against his child to maintenance. The rule is that relationship, apart from the legal obligation to support, does not confer an insurable interest. It has accordingly been recently held that the fact that a son will at some future date be under a moral, though not a legal, obligation to pay the funeral expenses of his mother, is not sufficient to confer on him an insurable interest in her life.[3]

The rule in our law is that the interest of the assured in the life must be a pecuniary interest and enforceable legally. It must therefore depend either on contract or on such relationship as entails an obligation to support. In the American courts the same rule has generally obtained.[4] Recently, however, there has been a tendency to relax the rule and to hold that neither relationship nor a legal right to support is required, and that it is sufficient if the termination of the life would result in pecuniary loss to the insured. Thus it has been held that a girl in whom an elderly man had taken an interest so far as to provide her with means to obtain an education and also to give her employment, and who, from his conduct and expression of intention, had a right to expect him to continue his quasi-parental care toward her, had an insurable interest in his life.[5] It is safe to say that the decision would have been the other way in this country.

The question of insurable interest is one to be decided by reference to the time when the policy is issued and not when the event insured against happens. So a policy, valid when effected, is not rendered invalid by the subsequent cessation of the interest of the assured. The decision in *Turnbull* v. *The Scottish Provident*

[1] *Hadden* v. *Bryden* (1899), 1 Fraser, 710.
[2] *Shilling* v. *Accidental Death Insurance Company*, 1 F. & F. 116.
[3] *Harse* v. *The Pearl Life Assurance Society*, L.R. (1903), 2 K.B. 92; (1904), 1 K.B. 558.
[4] *United Brethren Mutual Aid Society* v. *Macdonald* (1888), 9 Am. St. R. 111; *Prudential Insurance Company (of America)* v. *Jenkins* (1896), 57 Am. St. R. 228.
[5] *Carpenter* v. *The United States Life Insurance Company* (1894), 41 Am. St. R. 880.

Institution[1] presents this interpretation of the statute in rather an interesting form. A firm of merchants made a proposal to an insurance company for a policy on the life of an agent in Iceland through whose means they carried on a lucrative business. The contract of agency (which was exhibited to the company) was terminable by either party on 1st March of each year on giving three months' notice. The policy as issued contained a note that as the insured had stated they had an insurable interest in the life of the agent no further proof of their interest would be required when the policy became a claim. After the proposal was made, but before the policy was issued, the agent gave notice that he proposed to terminate his contract on 1st March of the ensuing year. This notice was not communicated to the insurance company, and the policy was issued. On the death of the agent the company refused payment of the policy on the ground of want of insurable interest at the date of the policy. Lord Kincairney held that, as the contract of agency was not actually terminated at the date of the policy, the insured had an interest, the sufficiency of which the company was precluded from denying.

It is also settled in England that the Gambling Act does not affect subsequent dealings with a policy originally valid. Some of the American States follow the same rule, but in the majority, and also in the Supreme Court of the United States, it has been laid down that a person who has no interest in the life of another cannot take by assignment a policy upon the life of that other. But a policy taken out by a person on his own life payable to one who has no interest in that life is good if it is taken out in good faith and the premiums are paid by the insured.[2] The transaction must be a *bona fide* one. If the assured is only nominally the contracting party the beneficiary, in order to claim payment, must be able to prove an insurable interest.[3] As Mr. Harvey points out, with us such a policy, apart from statute, would be valid only if the direction to pay to the third party was of a testamentary nature. An example of such a policy is found in a recent English case.[4] A took out a policy on his own life for behoof of B, his wife's sister, B, her executors, administrators, and assigns being entitled to receive the policy moneys on A's death. A survived B. He retained possession of the policy and paid the premiums thereon till his death. B's representatives claimed payment of the policy and it was held that they

[1] (1896), 34 Scottish Law Reporter, 146.

[2] *Hill* v. *United Life Insurance Association* (1893), 35 Am. St. R. 807; *Heinlein* v. *Imperial Life Insurance Company* (1894), 45 Am. St. R. 409; *Prudential Insurance Company (of America)* v. *Hunn* (1899), 69 Am. St. R. 380.

[3] *Prudential Insurance Company (of America)* v. *Hunn*, supra.

[4] In re *Scottish Equitable Life Assurance Society*, L.R. (1902), 1 Ch. 282.

were entitled to payment, but only as trustees for the representatives of A. The destination to B, being by way of testamentary disposition, had fallen through by the predecease of B, and there was a resulting trust for the representatives of A in the proceeds of the policy.

Even in those States of America in which the English rule as to validity of assignments of policies is followed, a policy taken out by a person on his own life, and immediately assigned to another who has no insurable interest, will be held void, for the court will look to the substance of the transaction to see whether it is merely an evasion of the rule against wagering policies.[1]

(*b*) *Payment of first Premium.*—Until the decision in *Roberts* v. *The Security Company, Limited*,[2] it had been taken to be sound law that an insurance company which accepts a proposal for insurance, subject to the condition that no assurance shall be held to be effected till the premium due thereon has been paid, is not bound to issue the policy or make good a loss if, before the premium is tendered, the risk has materially altered or has determined.[3] In America it had been held that such a condition might be deemed to have been waived by the delivery of the policy unconditionally to the assured before payment of the premium whether the policy contained an acknowledgment of payment of the premium or not.[4] In *Roberts'* case, however, the doctrine of waiver was carried much further. A policy of insurance against burglary had been executed, signed and sealed by the company. The policy contained a recital of the proposal on which it proceeded, and a statement that the proposer had paid to the company the sum called in the margin thereof the first premium. Before the policy was delivered or the premium paid a loss occurred. One of the conditions of the policy as executed was that no assurance by way of renewal or otherwise should be held to be effected till the premium due thereon had been paid. In an action for delivery of the policy it was held that the policy was a completed one, that the company had in the policy treated the premium as paid, and that if it had not in fact been paid, they had by completing the policy waived the previous payment of the premium as a condition of the existence of the insurance. It was accordingly open to the assured to tender the premium and demand the policy. The decision proceeded upon two alternative grounds, either (1) that the insurance company were estopped from contradicting the statement in the policy which they had formally

[1] *Steinback* v. *Diepenbrock* (1899), 70 Am. St. R. 424.
[2] L. R. (1897), 1 Q. B. 41.
[3] *Canning* v. *Farquhar* (1886), 16 Q.B. 727; *Sickness and Accident Assurance Association* v. *General Accident Assurance Corporation* (1892), 19 Rettie, 977.
[4] *Farnum* v. *Phœnix Insurance Company* (1890), 17 Am. St. R. 233; *Griffith* v. *New York Life Insurance Company* (1894), 40 Am. St. R. 96.

and solemnly completed, or at all events (2) that the acknowledgment of payment in the completed policy was sufficient evidence of waiver by them of the term as to previous payment of the premium.

Had there been no express provision in the acceptance of the proposal that the insurers should not be liable till the first premium was paid, the decision in *Roberts'* case would have amounted to no more than an affirmance of the familiar rule that a contract is completed as soon as the contracting parties are agreed upon its essentials. But where it was a term of the contract that there should be no liability till a premium was paid, it is somewhat startling to find that that stipulation may be held to have been waived where there have been no further negotiations between the contracting parties and no undue delay has taken place, and where the policy has never left the office of the company. As we have seen, it had been held in America that delivery of the policy would operate a waiver of such a condition.[1] But where there had been no delivery and no payment of premium, the American courts have never gone so far as to hold that the mere execution of the policy estopped the insurer from showing that the condition as to payment of the premium had not been complied with. In a case in which the stipulation was made in the application for insurance that the insurers should incur no liability till the policy was issued and delivered, it was held that there could be no recovery in the absence of such issue and delivery—even though the first premium had been paid and the agent who solicited the application had informed the applicant that his insurance would take effect at once.[2] But it is sufficient compliance with such a condition if the policy be issued to the insurance agent who solicited the proposal for delivery to the insured, even though the agent never parts with the policy.[3]

I think it unlikely that the decision in *Roberts* would be treated as other than a very special one by our own courts. Where a condition such as was in question in that case was properly made part of the contract, I think it would require very strong evidence to enable them to hold waiver of the condition established while the policy was still undelivered. If, however, it is desired to avoid any such question, it could, I think, be done effectively by a condition inserted in the proposal-form that there should be no liability on the part of the insurer until the policy is actually delivered to the insured.

(*c*) *Payment of Renewal Premiums.*—When the insurance is conditional upon the payment by a fixed date of renewal premiums

[1] *Griffith* v. *New York Life Insurance Company*, supra.

[2] *Chamberlain* v. *Prudential Insurance Company (of America)* (1901), 83 Am. St. R. 851.

[3] *New York Life Insurance Company* v. *Babcock* (1898), 69 Am. St. R. 134.

the condition will be strictly enforced.[1] Compliance with this condition may, of course, be waived by the company unconditionally accepting the premium after its due date. But the acceptance must be unconditional. If it be made subject to conditions stipulated for by the company at the time, these new conditions form the basis of a new contract and must be complied with before that new contract will be binding on the company. A life policy was expressed to be conditional upon payment of the annual premiums within thirty days of their becoming due. The holder of the policy failed to pay a premium within the thirty days. He subsequently tendered payment to the company, and they sent him a receipt upon a printed form stating that the policy had lapsed and that payment was accepted subject to certain conditions printed on the back of the receipt. The assured got the receipt but did not read it. One of the conditions was that the assured should have been for the past twelve months in continuous good health and free from all disease. At the time the premium was tendered assured was in fact, and to his own knowledge, suffering from heart disease, of which he afterwards died. The other premiums due on the policy were paid punctually till the death of the assured. On his death the company refused payment, and in an action raised against the company by the executor of the assured it was held that the policy had lapsed by non-payment of the premium within its due date, and that as there was no evidence of any conduct on the part of the company to justify the assured in thinking that the policy had not lapsed or to estop the company from relying upon the lapsing of the policy, the company was not liable.[2]

It is interesting to contrast this case with that of *Hemmings v. The Sceptre Life Association, Ltd.*,[3] where the unconditional acceptance by the company of two renewal premiums after they had become aware of a material mistake in the statement of the age of the assured barred them from disputing the policy even to the extent of correcting the mistake. The policy was an endowment one effected when the assured was stated to be 41 years of age and payable at age 60. After the policy had been in force for some time it was discovered that the assured, who was a lady, had understated her age by three years. The company having accepted two premiums after they became aware of this mistake were held, by Mr. Justice Kekewich, to be barred from exercising the right they would otherwise have had of cancelling the policy and returning the premiums, and were found liable to pay the policy moneys to the assured when she actually attained 60 years of age, that is, three years earlier than was originally contemplated, and

[1] *Phœnix Company v. Sheridan* (1860), 8 H.L. C. 745.
[2] *Handler v. The Mutual Reserve Fund Life Association* (1904), 90 Law Times 192.
[3] L R. (1905), 1 Ch. 365.

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after 17 payments instead of 20. They had barred themselves from challenging the contract as it stood, and the court could not form a new contract for the parties in accordance with the equities of the case. Such a result might have been avoided by a provision in the policy such as that alluded to by Mr. Harvey, that if the age had been inaccurately stated the insurance should be valid only to a specified extent. But there being no such provision in the contract, the court could not insert it.

(*d*) *Days of Grace.*—In the case of *Pritchard* v. *The Merchants' Life Assurance Society*,[1] the provision that the policy should be void if the premiums for renewal were not paid within a specified period was interpreted to be a mere agreement to renew the policy, personal to the assured, and not enforceable if he should die, after a renewal premium became due, within the days of grace. Mr. Harvey points out that as a result of that case the insurance companies hastened to reassure the public that the benefit of their policies would not be lost if death occurred within the days of grace and before payment of the new premium; and he ventured the opinion that if such a condition is to be found in a modern policy, as was under interpretation in *Pritchard's* case, no company in the kingdom would take advantage of it. The same question arose, however, no later than three years ago in the case of *Stuart* v. *Freeman*.[2] In that case Stuart was assignee of a policy of insurance on the life of another person. The policy was a yearly one, but the premiums were payable quarterly—the first quarterly payment being made at the date of the policy. One of the conditions of the policy was that it should be of no effect if at the time of the death of the assured any quarterly premiums should be more than thirty days in arrear. The assured died during the year, after one of the dates fixed for payment of a quarterly premium, but within the days of grace, and this premium was paid after, but in ignorance of the death, by the assignee—also within the days of grace. The company refused payment of the policy on the ground that the assured having died before payment of the premium, they were not bound to renew the policy. Mr. Justice Ridley, following *Pritchard*, upheld their contention. But the Court of Appeal reversed this decision, and held the company liable upon the policy. The view of the court was that no question of renewal arose on the policy, but that at the time of the death of the assured it was a subsisting policy, subject to forfeiture in one event only—the failure timeously to pay the quarterly premium; and that event had not occurred. The Master of the Rolls (Collins) observed, "Within the course of the year for which the insurance was to run, some

[1] (1858) 3 C. B. (N. S.), 622.
[2] L. R. (1903), 1 K. B. 47.

instalments of the premium were paid. The provision as to payment by instalments does not affect the fact that the insurance was for a year, but attaches a condition of defeasance if any instalment is not paid within the days of grace." In *Pritchard's* case "that which was agreed on was a right to renew on certain terms; in the case before us there is a subsisting insurance which, if not forfeited, endures for a year." Lord Justice Matthew found himself unable to agree with the view expressed in *Pritchard's* case, that allowing days of grace means only that during those days there is to be a continuing offer to renew the policy, and that unless that offer is accepted and payment made while the life insured is in existence, the policy is void. Most offices now frame their conditions as to renewal premiums so as to provide that the insurance shall continue in force until the expiry of the days of grace, notwithstanding the previous death of the assured; and many also provide for revival by payment of the overdue premiums within a specified time thereafter, without further evidence of health, or, in the event of death within the days of grace, for payment of the policy under deduction of the unpaid premiums.

(*e*) *Conditions dealing with Excepted Risks—Suicide.*—Where the evidence is equally balanced as between accident and suicide, the presumption of law against crime will entitle the assured's representative to recover.[1] This result has also been arrived at in recent American cases, the ground of decision in these rather being that the onus of proving that the policy is void because of breach of the exception against suicide, lies upon the insurers.[2] This exception is now generally expressed by such a provision as that in question in *Ellinger* v. *The Mutual Life Insurance Company of New York.*[3] The application of the insured contained the following clause: "I warrant and agree that I will not commit suicide, whether sane or insane, during the period of one year, from the date" of the policy. In May 1902 a policy was issued "in consideration of the application." In February 1903 the assured committed suicide while temporarily insane. It was held by Mr. Justice Bigham, whose decision was affirmed by the Court of Appeal, that the clause was a condition of the contract, and a limitation of the company's liability—and that the court could not enter upon questions as to the degree of assured's insanity or his capacity to understand either the physical or the moral character of his act.

[1] *Harvey* v. *The Ocean Accident and Guarantee Corporation* (1905), 2 Irish Rep. 1 (C. A.).

[2] *Walcott* v. *Metropolitan Life Insurance Co.* (1891), 33 Am. St. R. 923; *Agen* v. *the same* (1899), 76 Am. St. R. 905; *Hale* v. *Life Indemnity and Investment Co.* (1895), 52 Am. St. R. 616; *Carnes* v. *Iowa State Travelling Men's Association* (1898), 68 Am. St. R. 306; but contrast *Travellers' Insurance Co.* v. *M'Conkey*, post.

[3] L. R. (1904), 1 K. B. 832.

The Supreme Court of the United States has decided that even if the policy is silent as to suicide, the insurers will not be liable where the insured, while of sound mind, deliberately takes his own life. The same court has expressed the view that any provision that the insurer should be liable in such a case would be void as against public policy.[1] In this country it was long ago decided that it is an implied condition of life assurance that the assured will not terminate his own life, and that any provision to the contrary is void as against public policy.[2]

The American courts have generally refused to apply the exception against suicide, where there is no "sane or insane" clause, to the case of one who at the time was unable through insanity to appreciate the moral quality of his act, even if he could be shown to have understood its physical nature and consequences.[3] The same tendency to limit this exception is apparent from recent decisions in cases where the "sane or insane" clause was included in the policy. In some States that clause has received the same interpretation as in *Ellinger*, and has been held to cover all acts of self-destruction irrespective of the existence or extent of insanity.[4] But in other States even this clause, while it has been held to cover the case of one who commits suicide understanding the physical nature and consequences of his act, though not conscious of its moral quality, has not saved the insurer from liability, where the assured did not know the physical nature of his act, or that it would result in self-destruction.[5] It has further been held in America that an exception against the death of the assured in consequence of, or in violation of, law does not enable the company to avoid the policy on the ground of suicide of the assured.[6]

(*f*) *Other Conditions precedent to Recovery.*—The well-known newspaper cases, *Law* v. *Newnes*,[7] and *Hunter* v. *Hunter*,[8] are authorities for the proposition that the decision even of the parties who are to be entitled to recover under an insurance policy may, by the terms of the contract, be left in the hands of the insurer; and in such an event the decision of the insurer will be a condition precedent to the right to recover. *Law* v. *Newnes* arose in the following circumstances. *Tit Bits* newspaper offered that in the event of any person being killed by an accident to a train in which he was travelling as a passenger with a copy of the

[1] *Ritter* v. *Mutual Life Insurance Company of New York* (1898), 169 U. S. 139.
[2] *The Amicable Society* v. *Bolland* (1830), 4 Bligh N. S. 194.
[3] *Connecticut Mutual Life Insurance Co.* v. *Akens* (1893), 150 U. S. 468.
[4] *Billings* v. *The Accident Insurance Coy.* (1891), 33 Am. St. R. 913; *Travellers' Insurance Co.* v. *M'Conkey* (1887), 127 U. S. 661; *Accident Insurance Co.* v. *Crandal* (1886), 120 U. S. 527.
[5] *Hart* v. *Modern Woodmen of America* (1899), 72 Am. St. R. 380.
[6] *Patterson* v. *The Natural Premium Mutual Life Insurance Co.* (1898), 69 Am. St. R. 899.
[7] (1894), 21 Rettie 1027.
[8] (1904), 7 Fraser 136.

current number of the newspaper in his possession, £100 would be paid "to the person adjudged by the editor to be the nearest relation of the deceased," and it was further stipulated in the offer that the person adjudged by the editor to be the nearest relation should be the only person entitled to receive and give a valid discharge for the money. The newspaper, after inquiry, paid a claim to the widow of an assured. In an action by the children and next of kin, the newspaper was assoilzied, the court holding that the decision of the editor was a condition precedent to the right to receive the policy moneys, and that he having made inquiry and decided in good faith, his decision could not be disturbed. This decision was followed in *Hunter* v. *Hunter*.

The English courts have refused to entertain, during the life of the assured, an action for declarator that a policy is valid, and for an injunction to restrain the insurance company from repudiating liability upon the policy. Where an insurance company had repudiated liability on a life policy on the grounds of want of insurable interest and of fraudulent misrepresentation, and had refused to receive further premiums, the court, on an undertaking by the company that they would not in any future proceedings rely as a defence upon non-payment of premiums at the due dates, dismissed such an action as premature.[1]

IV. Settlement and Trust Policies.

The very full discussion of settlement policies both at common law and under the Married Women's Policies of Assurance (Scotland) Act, 1880,[2] given by Mr. Constable in his paper before referred to, makes it unnecessary for me to do more than notice one or two decisions which have since occurred to elucidate the provisions of that Act and of the English Married Women's Property Acts of 1870 and 1882.[3]

A policy taken out under Section 11 of the Married Women's Property Act, 1882, by a husband on his own life while his first wife was alive, and expressed to be "for the benefit of his wife and children," was held to apply to a second wife, whom he afterwards married and who survived him, and to the children of both the first and second marriages.[4] Mr. Justice Kekewich treated the question as one of intention, and found that the intention of the husband was that his surviving wife, if any, and his surviving children, whether by his then living or any other wife, should

[1] *Honour* v. *The Equitable Life Assurance Society of the United States*, L.R. 1900, 1 Ch. 852.
[2] 43 and 44 Vict. c. 26.
[3] 33 and 34 Vict. c. 95, sec. 10; 45 and 46 Vict. c. 75, sec. 11.
[4] In re *Browne*, L. R. 1903, 1 Ch. 188.

take. A similar result was arrived at by Mr. Justice Swinfen Eady in *In re Parker's Policies.*[1] In that case the policy was under section 10 of the Married Women's Property Act of 1870, and for the benefit of widow and children, with a power of appointment to the husband. His wife predeceased him. He married again, and subsequently he executed a deed of appointment giving the whole policy-moneys to the second wife. It was held that whether the second wife was within or without the Act she took under the appointment. Mr. Justice Swinfen Eady was unable to distinguish the case from that of *Browne*, or find any difference between the Act of 1870 and the Act of 1882, in respect of the question before him. In a case which occurred between *Browne* and *Parker*, Mr. Justice Joyce arrived at a different conclusion as to the intention of the husband. A man with a wife and four children effected a policy on his life under the Married Women's Property Act, 1870. The policy was expressed to be for the benefit of his wife or, if she died, of his children in equal proportions. The wife died, having had four more children after the issue of the policy. The husband married again, and had one child by his second wife. He was survived by his second wife and the nine children. It was held that on the wording of the destination in the policy the widow was not entitled to participate in the policy-moneys, but that all the children took in equal shares.[2]

It will be remembered that in *Kennedy's Trustees* v. *Sharpe*,[3] the Second Division held that a widower was entitled to take out a policy under the Married Women's Policies of Assurance (Scotland) Act, 1880. In the course of that case it was suggested from the bench that the Act might possibly extend to a policy taken out by an unmarried man in anticipation of his marriage and for the benefit of his intended wife. A contrary opinion has been expressed in a later case by the Lord Justice-Clerk.[4] Unlike the English Act of 1882, the Scotch Act applies only to a "married man." There seems no reason why the English Act of 1882 should not apply to an unmarried man. For it would seem to follow from the decisions, that a man may make provision under the Act not only for an existing wife, but also for one whom he may subsequently marry; and not merely for children of an existing marriage, but for children of a marriage entered into subsequent to the date of the policy.

Two important cases with reference to Settlement Policies are noticed by Dr. A. E. Sprague in his recent paper on "Titles to Life Assurance Policies."[5] They are *In re Turnbull*[6] and *The Scottish*

[1] (1906), 22 Times Law Rep. 259.
[2] In re *Griffith's Policy*, L. R. 1903, 1 Ch. 188.
[3] (1895), 23 Rettie 146. [4] *Coulson's Trs.* v. *Coulson* (1901), 3 Fraser 1041.
[5] *Transactions of the Faculty of Actuaries*, vol. ii. p. 343.
[6] L. R. 1897, 2 Ch. 415.

Life Assurance Company v. *John Donald (Limited)*.[1] In the former case it was held by Mr. Justice Stirling that a settlement policy issued under the Married Women's Property Act, 1870, could not be discharged by the legal representative of the husband, but only by a trustee appointed by the Court, in terms of the Act. In the latter case, which was that of a policy under the Scotch Act of 1880, Lord Stormonth Darling held that a policy taken out under that Act, for the benefit of the assured's wife, could not be assigned by her, *stante matrimonio*, the policy being a provision for her of which she could not validly deprive herself during the subsistence of the marriage. The effect of the statute being to create an effectual trust for the wife free from her husband's control, she was in the same position as she would have been under a common law trust created by assignation and delivery of the policy to trustees. An illustration of such a trust is found in *Barras* v. *The Scottish Widows' Fund Society*.[2] Policies on the life of a husband had been assigned by him to trustees for his wife and children in a post-nuptial deed of provision and delivered to the trustees. Following *Low's Trustees*[3] and *Peddie's Trustees*,[4] the Court held that even the consent of the wife and children, who were all major, would not entitle the husband to revoke such a provision where the policies had been delivered. In this case Lord Kyllachy approved the dicta in *Schumann* v. *The Scottish Widows' Fund Society*,[5] that the company would have been in safety to accept a surrender or make advances against the policies in terms of their rules, since such transactions were not necessarily inconsistent with the trust, and might indeed be necessary for its protection.[6]

When a policy is taken out under the Married Women's Policies of Assurance (Scotland) Act, 1880, for the benefit of the wife alone, a common form in which to express the destination is to take it "on the life of *A B* (the husband) for the whole duration thereof, such assurance to be for the benefit of *C D*, his present wife, pursuant to the provisions of the Married Women's Policies of Assurance (Scotland) Act, 1880, provided she be alive at the death of the said *A B*, but otherwise for his own use and benefit," and to make the policy payable "to the said *C D*, if she survive the said *A B*, but otherwise to the said *A B*, his executors, etc."

[1] (1901), 9 Scots Law Times, 200.
[2] (1900), 2 Fraser 1094.
[3] (1877), 5 Rettie 185.
[4] (1891), 18 Rettie 491.
[5] (1886), 13 Rettie 678.
[6] Cf. also *In re Lavender's Policy*, L. R. Ir. 1898, vol. i. p. 175. A stipulation attached to property given to a married woman for her separate use that such property shall not be assignable is in England operative at common law during her marriage; and such a stipulation may be attached not only where the property is made the woman's separate estate by contract or declaration of the donor, but also where it becomes so by operation of the Married Women's Property Act, 1882. In re *Lumley* ex parte *Hood Barrs*, L. R. 1896, 2 Ch. 690.

The question has been raised whether this destination clause is a proper one; and it has been suggested that the clause should bear that the payment is made to the husband or wife in trust in terms of the Act. It appears to me quite clear from the decision in *Schumann's* case that this is not necessary in order properly to constitute the policy a trust for the benefit of the wife. All that is required is that the policy should, on the face of it, be expressed to be for her benefit, in order that it may come under the operation of section 2 of the Act. So doing, it becomes the property of the wife. She is the sole beneficiary, and I see no objection to payment direct to her. Lord Shand, in *Schumann's* case, expressed the view that the insurance company could safely take a receipt from a trustee nominated under the policy without the concurrence of the wife. But all the judges agreed that her receipt was absolute protection to the company. In my view, therefore, it is better to make the policy-moneys payable to the wife directly, than to make them payable to a trustee—especially if that trustee be the husband. In the case of policies for the benefit of wife and children—and notwithstanding Lord Shand's opinion that there is no difference between a father and any other trustee—I agree with the view expressed by Mr. Constable that it would be more prudent for the insurance company to insist upon the appointment of an independent trustee to give them a discharge.

V. Stamps.

An important decision is that of *The Prudential Insurance Company* v. *The Commissioners of Inland Revenue*,[1] in which the question was raised whether an endowment policy, providing for payment of a fixed sum on attaining a certain age, or a less sum in the event of death before that age, was an insurance policy upon a contingency depending on a life within the meaning of section 98 of the Stamp Act, 1891; and was properly stamped with the policy stamp of 1s. per cent on the amount payable at death. The Court decided that such a contract was a policy of assurance within the meaning of the section, and Mr. Justice Channell expressed the opinion that even if the endowment had stood alone it would come within the section, and be stampable as a policy. For a full discussion of the history and results of this case, I would refer you to Dr. A. E. Sprague's paper of last year on Titles to Life Assurance Policies.

[1] L. R. 1904, 2 K. B. 658.

DISCUSSION.

THE CHAIRMAN (Mr. G. M. LOW).—We have listened with great interest to Mr. Wark's paper, and although I am afraid that, not being legal experts, we can hardly offer him much discussion on it, yet I am sure both he and the meeting will be very pleased to hear any remarks that any one present may make.

Mr. ARCH. HEWAT.—We are much indebted to Mr. Wark for the trouble he has taken in preparing his paper. It will be valuable as a record to refer to on the points he has mentioned. The reference to agent's lien on p. 94 brings out one distinction between the law of Scotland and that of England—the mere possession of deeds giving no title here, whereas in England it does to a certain extent.

Mr. A. D. LINDSAY TURNBULL.—Without presuming to criticise the legal opinions of a lawyer, we may, I think, distinguish in the business of Life Assurance between what may be termed theoretical law and equitable practice. Any one not connected with our business, on reading papers like Mr. Wark's, might imagine that British Life Offices very often disputed claims on theoretical points of law. As a matter of fact, Life claims are now disputed very seldom, and in such cases the office is generally sure that the contract has been fraudulent throughout, but even then the Company's solicitors will only try to set the policy aside on some legal technicality easily proved. For instance, were a man to insure the life of another, and directly or indirectly bring about his death, it might be very difficult to prove murder, but comparatively easy to prove want of insurable interest. With regard to this question of insurable interest, I think most offices, at least most English offices, are willing to issue policies where there is a *bonâ fide* or moral, although not, strictly speaking, a legal insurable interest. For example, it is generally accepted that shopkeepers with stocks of coloured silks and fancy material have an insurable interest in the principal members of the Royal Family, although probably, strictly speaking, they have not a legal interest. A question arises as to what is the real effect of issuing a policy where there is not a good insurable interest. I take it that a solvent proprietary company must in all cases pay the claim, but, in event of the insurance company becoming bankrupt, the trustee in bankruptcy would set aside such a policy as an "honour" policy. In the event of the directors of a mutual society issuing an "honour" policy, the members might possibly insist that the directors should be personally liable. Mr. Wark did not refer to the practice of writing "policy proof of interest" on the face of a policy, where doubts as to insurable interest might be raised. Such a clause is probably of little legal value in an "honour" policy, but is only a promise that the directors will not dispute the claim on the ground of want of insurable interest. It might be taken as a statement that at date of insurance they admitted an insurable interest, but often means that they really think it is not technically in order. I take it that a re-assuring office is not expected to question, and generally speaking refrains from questioning the insurable interest of the assured under the principal policy, and that a re-assurance policy, as far as insurable interest

is concerned, is always a thoroughly legal contract. While admitting that Mr. Wark is more than justified in quoting American cases and disputes under Accident policies, I think it only right to point out that American Offices have in the past more often, rightly or wrongly, disputed claims than British Life Offices. In America, I understand, there are, or were till recently, men called "Life Claim Adjusters," who would, no doubt, act either for the representatives of the life assured or the insurance office as required. It is satisfactory to think that there is no opening for such men in Edinburgh. Again, in the case of Sickness or Accident policies there is no doubt that malingering frequently occurs and dishonest claims are often made. Such incidents would probably be more common, if all claims were not carefully scrutinised and in many cases disputed. As regards "indisputable" policies I doubt whether Mr. Wark is correct in giving the Americans credit for being the first to issue them. Nearly all first-class British Offices now, if they do not print "indisputable" or some such expression on their policies—a proceeding which probably has no great legal value,—issue policies which are free from all restrictions and are, for all practical purposes, indisputable. I will even go so far as to say that where a solicitor or accountant has to arrange an assurance for a client of formed habits and an ordinary occupation, he would not be doing his duty if he did not obtain a policy, world-wide and practically indisputable, from the outset. Few offices, for instance, nowadays would lend money to a life-renter with a policy as collateral security unless the policy and, of course, all the re-assuring policies could be made free from all restrictions and payable in the event of death from any cause whatever.

Mr. J. J. M'Lauchlan.—Officials of Life Insurance Companies have occasion to be acquainted with a good many legal matters, and their legal knowledge is more apt to get out of date, than that of those who are continuously engaged in legal work. It is therefore very useful to us to have a paper of this kind, in which recent decisions are recorded and their effect on the law explained. Referring to the subject of insurable interest, my recollection is that at one time, offices were not nearly so particular as they are now. If a reason were given for effecting the insurance which commended itself to common sense, the proposal was accepted, and the policy was probably endorsed to the effect that no further evidence of insurable interest would at any time be required. With reference to the practice of offices in this matter, and to the statement that the defence of want of insurable interest can only be pled by the insurer, I think that a case occurred a short time ago in one of the English County Courts, which raised the question of whether a person who had taken out a policy on the life of another without a legal insurable interest, might not be entitled, at any time during the currency of the insurance, to demand a return of his premiums, on the ground that there had never been a legal contract, and he had therefore had no consideration for his money. Perhaps Mr. Wark would be good enough to tell us whether, in his opinion, there is any real danger to insurance companies here, or whether, on the other hand, we may take it that if the company does not, in any particular case, raise this question of insurable interest, no one else can raise it.

Mr. W. A. Sim referred to the instructive and suggestive character of the paper, and to the usefulness of it for purposes of reference. With regard to the filling up of the proposal form, he thought something could be said in favour of it being an office rule that that should be done by the proposer himself. The question of agency was many-sided, and cropped up from time to time in a variety of ways, in illustration of which he instanced several cases that had come to his knowledge.

The Chairman.—Before Mr. Wark replies on the discussion, I might

just refer to one or two points which have occurred to me. I was very much impressed with the references in the paper to the cases of *Delahaye* and *Reid & Co.* The importance to us of these cases lies mainly perhaps in the attention they direct to the *wording* of our forms and policies, and to the advisability of reconsidering and revising these from time to time in view of decisions which illuminate our contracts and show the necessity for adjustments. Neither of the offices in the cases in question appears to have had sufficiently before it the points that came out in the course of litigation, and in consequence their forms had not been adjusted to meet those points. With regard to the decision in *Roberts's* case, which was at the time and to many of us in Scotland still continues to be more or less an enigma, I myself got some light at the time which may possibly help you also to understand it. In conversation with an eminent English solicitor it was pointed out to me that what the case really turned upon was the sacredness and conclusiveness in England of a deed under seal. A receipt for money, for example, was only *prima facie* evidence of payment, liable to be displaced by evidence to the contrary effect, but a deed once executed under seal debarred the granter from raising any question as to the facts contained in it, and upon that the decision turned, for you will remember the policy contained an acknowledgment that the premium had been paid. Of course in Scotland the sealing of deeds has not at all the same significance, and therefore Mr. Wark is no doubt entirely right in saying that here the case would have been looked upon in a very different light. Then in regard to Married Women's policies, I suppose nothing has given rise to greater trouble and difficulty in life assurance practice than those Acts of 1870, 1880, and 1882, however beneficial in their general effect they may have been. I have met with the suggestion before, and I observe Mr. Wark says here, that it would appear that an unmarried man may effect a policy for the benefit of a future wife under the Act [Mr. Wark—"the English Act"]. Yes, I observe you limit your observation to the English Act of 1882. But it would require a very authoritative decision indeed to convince me that when the Act provided that a policy should be deemed a trust for the benefit of the insured's wife, it did not presuppose that he had a wife, or that in providing that the policy should not form part of his own estate, the Act enabled a man who had and might have no wife, to put his policy under a trust and keep it outwith his estate, simply because effected under that particular Act. I cannot think that was the meaning, yet if, as appears from the decisions, a policy effected for the benefit of one wife can be carried forward for a subsequent wife, then possibly the result inferred by Mr. Wark may follow. As bearing on this, I may mention what took place in connection with the Scottish Act of 1880, the history of which I remember very well. When the Bill was being framed, it was suggested that the object of enabling a *widower* to effect a policy for the benefit of his children might be attained by simply dropping out the word "married," which appeared in the English Act of 1870, and making the clause read "a man may effect . . ." As, however, there was great difficulty in getting the Bill for Scotland carried through at all, it was thought best not to endanger it by any conspicuous deviation from the English Act, and the suggestion was not acted upon. Certainly at that time there was no idea that an unmarried man might take advantage of the Act on the chance of his marrying. Lastly, I should like to hear Mr. Wark a little more fully on the case which was put to him under the Scottish Act of 1880, where it was proposed to grant a policy that should be for the benefit of the wife in the event of her surviving the husband, and for the husband's own benefit if she should predecease him. My point is that Mr. Wark does not seem to deal with the question whether the alter-

native destination to the husband could be competently made in a policy effected under the Act. I should like to hear him on that point, and as to whether he would approve of issuing policies so drawn.

Mr. WARK.—I have to thank you for the kind reception you have given my paper, which is indeed little more than a digest of cases many of which are perhaps merely of academic interest. I quite agree that the number of cases in which Scottish offices dispute claims is exceedingly small. On that account I am all the more glad to have the opportunity of at least doing something in return for the honour you have done me in devolving upon me the duty of reporting decisions of the Scottish Courts. With regard to the question of insurable interest, I agree that nowadays the question is seldom raised. Steps are usually taken at the outset of a policy to ensure that the question shall not be raised, and, as I have said in the paper, unless the company itself raises the question, in this country at least no one else can. That is clearly laid down in more than one decision. Accordingly I do not see that the mere fact of the interest being a doubtful legal one is of much importance to the assured if he is dealing with a sound and respectable company. I agree that in the event of the company having to be wound up, the liquidator would probably be bound to raise the question. Regarding the question of a return of premiums raised by Mr. M'Lauchlan, I do not see how the assured could make such a claim unless it were shown that the company had acted fraudulently, securing the assurance and receiving premiums on the footing that the policy was a good one, while all along intending to repudiate liability. The judgment in *Harse's* case (p. 94), would seem to apply. With regard to *Roberts's* case, I agree with the Chairman's explanation of the decision and adhere to the view that it is very doubtful if it would be followed in Scotland. Coming now to Married Women's policies, the suggestion I made as to the possibility of an unmarried man effecting a policy under the 1882 Act, was really based upon this—that in the cases cited, the question of who was entitled to benefit by policies under that and the 1870 Act was treated by the Judge as one of intention, and where the language used showed that the husband intended to benefit a second wife, the court gave effect to that. Now I agree that it is a very strong thing to say that a man who is not married may set aside a fund for a wife he may possibly never have. If however he does marry a wife who may enjoy it, I do not see that it is a much stronger thing to hold that he could set it aside for her, than it was to hold, as in one of the cases cited, that the wife and children of a subsequent marriage could participate. I agree that before marriage takes place the policy does not come under the Act, but once it takes place, I think the position is not very different from that of a second marriage. With regard to the destination clause in a policy under the Act of 1880, I think the clause quoted (p. 105) is perfectly good. The alternative destination giving a contingent reversion to the husband simply expresses his interest at common law. It has no effect on the trust and only comes into operation should the proper objects of the trust fail.

On the motion of the CHAIRMAN a very cordial vote of thanks was accorded to Mr. Wark for his paper and for his replies on the various points raised.

Vol. III. Part V.

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THE FACULTY OF ACTUARIES.

EXAMINATION PAPERS, 26 AND 27 APRIL 1906.

FIRST EXAMINATION.

Examiners—MR. W. A. SIM AND DR. J. BUCHANAN.

FIRST DAY—PART I.

1. Calculate the value of

$$1\cdot8\dot{3} \text{ of } \frac{\frac{1}{2}+1\frac{1}{6}}{2\frac{1}{8}-1\frac{1}{2}} \times \frac{2\frac{1}{2}+\frac{\frac{3}{8}}{2\frac{3}{4}}}{\frac{3}{4\frac{1}{5}}+\frac{5}{4\frac{3}{8}}} \text{ of £52, 4s. 6d}$$

$$+\tfrac{3}{7}\times\tfrac{28}{45} \text{ of £}17\cdot5041\dot{6}.$$

2. Deduce with reasoning the rule for finding the *Least Common Multiple* of two or more integral numbers. Extend this to the case of vulgar and decimal fractions.

There are four numbers which have one common factor, and no other factor is common to any two. Their product is 775719252, and their *Least Common Multiple* is 63756. Find their common factor.

3. Resolve $2x^2-3xy-7xz-2y^2+4yz+6z^2$ into factors.

4. At what time between five and six o'clock are the hands of a clock $\frac{3}{14}$ of the distance apart that they were twelve minutes before?

5. Find the sum of the squares of the reciprocals of the roots of $x^3+10x^2-3x+2=0$.

6. Of the whole cost of constructing a railway $\frac{5}{7}$ is held in

shares, and the remainder, £400,000, was borrowed on a mortgage at 5 %. Find what amount of gross annual receipts—of which 55 % will be required for the working expenses of the line, and 5 % for a reserve fund—will yield to the shareholders a dividend of $4\frac{1}{2}$ % on their investment.

7. A student is examined in 3 papers with a maximum of n marks for each paper. In how many ways can he get a total of n marks?

FIRST DAY—PART II.

8. Find the greatest term in the expansion of $(1+x)^n$, n being a positive integer. What is the condition that the greatest term may have the greatest coefficient?

9. Show that the value of the fraction $\frac{a+b+c}{p+q+r}$ lies between those of the greatest and the least of the fractions $\frac{a}{p}$, $\frac{b}{q}$, and $\frac{c}{r}$.

10. If $\frac{1}{a}$ and $\frac{1}{b}$ are the roots of $x^2-px+q=0$, show that $\log_e(x^2+px+q)=\log_e q+(a+b)x-\frac{1}{2}(a^2+b^2)x^2+\frac{1}{3}(a^3+b^3)x^3-\ldots$, where x is such that neither ax nor bx is >1.

11. 20,000 tons of water per minute flow out of a reservoir through an opening 14 yards broad, and 4 yards high. Find, correct to two decimal places, at how many miles per hour the water flows, assuming that a cubic foot of water weighs $62\frac{1}{2}$ lbs.

12. Form an equation with rational coefficients of which one of the roots is $\sqrt{5}-\sqrt{2}$.

13. By means of two different expansions of e^{ex}, show (1) that the coefficient of x^3 is $\frac{5e}{\underline{|3}}$, and (2) that $5e=\frac{1^3}{\underline{|1}}+\frac{2^3}{\underline{|2}}+\frac{3^3}{\underline{|3}}+\ldots\ldots$

14. In how many different ways can 1000 be divided into two parts, so that one of them may be divisible by 7, and the other by 13?

SECOND DAY—PART I.

15. Find the sum of the squares of the first n natural numbers.

The odd integers are arranged in groups as follows:—1; 3, 5; 7, 9, 11; . . . Show that the sum of the numbers in the nth group is n^3.

16. The first term of a series is a, and each succeeding term is r times the sum of all that precede it. Find the sum of the first n terms.

17. Solve the equations

$$\text{(i)}\ (x+4)(x-1)+2\sqrt{x^2+3x-1}=0$$
$$\text{(ii)}\ x^2+4y=y^2+4x=1.$$

18. Define a logarithm, and prove that $\log_a b \log_b c=\log_a c$.

If $\log_a b=\log_b c$ and $\log_a c=\log_c d$, prove that $(\log_a d)^2=(\log_b d)^4$.

19. Find the expression for the number of combinations of n things taken r at a time, without assuming that for the number of permutations.

20. How many different arrangements of the letters of the word *actuary* can be made, in which the two *a*'s do not come together?

21. At a game of golf A has won the first n holes and there are $n+1$ holes to be played, each of which may be won, halved, or lost. Show that A may win the game in $(n+1)^2$ ways.

SECOND DAY—PART II.

22. What is the chance of throwing heads (1) n times exactly, (2) n times at least, in $2n$ spins of a coin?

23. Of three dice one is marked with the numbers 1 to 6; another with the odd numbers 1, 3, 5, each repeated twice; and the third with the even numbers 2, 4, 6, each repeated twice. Show that the chance of throwing 9 is one ninth.

24. Ten cards, numbered with the digits 1 to 10, are turned face downwards on the table, and three are selected at random. Show that it is 5 to 1 against the number on one of the selected cards being equal to the sum of the numbers on the other two.

25. Sum to n terms the series

$$\frac{2}{1\cdot3\cdot5}+\frac{4}{3\cdot5\cdot7}+\frac{6}{5\cdot7\cdot11}+\ \ldots$$

26. Find the nth difference of the following

$$\text{(i)}\ e^{ax}$$
$$\text{(ii)}\ \frac{1}{x(x+1)(x+2)\ \ldots\ (x+m-1)}.$$

27. Prove that

$$u_n=u_0+n\Delta u_0+\frac{n(n-1)}{1\cdot2}\Delta^2u_0+\frac{n(n-1)(n-2)}{1\cdot2\cdot3}\Delta^3u_0+\ \ldots$$

Find the simplest algebraical expression in x, which assumes the values 1, 4, 15, 58, 181 by giving to x the values 0, 1, 2, 3, 4.

28. State and prove Lagrange's interpolation formula.

Interpolate a value midway between u_2 and u_3, having given

$$u_1=92815:\ u_3=91890$$
$$u_2=92414:\ u_4=91247$$

SECOND EXAMINATION.

Examiners—Mr. John Nicoll and Mr. Hugh W. Brown.

FIRST DAY—PART I.

1. What rate of interest has been employed if $\pi_{15} = \cdot 01123$ and $a_{15} = 21 \cdot 200$?

2. A policy, originally effected for the whole term of life, has been in force for n years. The holder desires to have it converted into an Endowment Assurance payable m years hence or at death; but instead of paying an increased premium he wishes the sum assured to be reduced, the annual premium being continued at the same amount as before. Determine the reduced amount of the policy.

3. Certain trustees are owners of terminable leasehold property yielding a rent which varies from year to year. They wish to protect the beneficiaries from loss of capital on the termination of the leases, and ask you to prepare a schedule showing what proportion of each year's rent must be held back as principal. Explain the nature of the required schedule.

4. What do you understand by the expression "Differential Coefficient"?

5. Show that the value of a Double Endowment policy effected at age x, to mature at age $x+t$, which has been n years in force, may be represented by ${}_nV_{x\overline{t}|}(1+{}_tE_x)+({}_{t-n}E_{x+n}-{}_tE_x)$.

6. Show that $\frac{du}{dx}$ is approximately equal to $\frac{1}{2}(u_{x+1}-u_{x-1})$.

7. If $\Sigma_{\overline{n}|}$ denote $u_0+u_1+u_2+\ \ldots\ +u_{n-1}$ and $\Sigma^{(m)}_{\overline{n}|}$ denote $\frac{1}{m}(u_0+u_{\frac{1}{m}}+u_{\frac{2}{m}}+\ \ldots\ +u_{n-\frac{1}{m}})$

prove that
$$\Sigma^{(m)}_{\overline{n}|}=\Sigma_{\overline{n}|}\times\frac{\Delta}{m\{(1+\Delta)^{\frac{1}{m}}-1\}}$$
where Δ represents the operation of differencing, the interval being unity; and show that the expression
$$s^{(m)}_{\overline{n}|}=s_{\overline{n}|}\times\frac{i}{m\{(1+i)^{\frac{1}{m}}-1\}}$$
is a particular case of the general formula.

FIRST DAY—PART II.

8. Express ${}_{n|m}a_x$ in terms (1) of temporary annuities, and (2) of deferred annuities; and prove the two expressions equal.

9. Show how you would arrive at the annual premium for an assurance payable only if the last survivor of two lives aged x and y die within n years, or if one at least of them survive that term.

10. Find expressions for the probabilities that of three lives x, y, and z, x will die—

(1) Second of the lives.

(2) In the same year as z, whether first, second, or third of the lives.

(3) At least t years after y, and at least t years before z.

11. Find an expression for A^1_{xy} on the assumption that the chance of both x and y dying in the same year may be neglected.

12. By entering Orchard's Tables with a certain life annuity, the Single Premium at 4 per cent. for an assurance of 1 is found to be ·56731. Given at 4 per cent. $v=$·961538, find the value of the annuity made use of in arriving at the foregoing Single Premium, and give also the Single Premiums in Orchard's Tables following ·961538 for the next two higher values of annuities.

13. Give the formula for the annual premium for a Capital Redemption policy payable at the end of n years on the basis of interest at the rate of 4 per cent. for the first k years, 3 per cent. for the next r years, and 2 per cent. for the remainder of the term.

What should be the net value of such a policy at the end of $k+r$ years?

14. An Assurance Association is started with l_x members all aged x. Each succeeding year l_x members join, also all aged x. Assuming that there are no discontinuances except by death, show how you would arrive at the number of years that will elapse until the number of members in the Association becomes stationary.

SECOND DAY—PART I.

15. Find an expression for the value of a perpetuity payable by quarterly instalments.

16. Define shortly the following terms, namely—(1) Expectation of Life; (2) Vie probable; (3) Mean age at death; (4) Most probable afterlifetime; (5) possible lifetime; (6) Complement of life; (7) limiting age; (8) Radix of a Table; (9) Truncated Table.

17. An Annuity of £k is to be paid to z so long as he and x and y are all alive. At the first death of the three lives the Annuity

is to be shared equally by the survivors; and, at the second death, it is to be continued for n years certain to the last survivor of the lives or to his heirs. Express the share of z in the annuity.

18. Give the formula for the annual premium by t payments (t being less than n) for an Endowment on x payable at age $x+n$, the premiums paid to be returned without interest in the event of death occurring before age $x+n$. Allow for a loading of ten per cent. Express, by the Prospective Method, the Surrender Values of the policy at the end of k years (k being less than t), and at the end of $t+1$ years.

19. In a certain Annuity Society the Annuities instead of being payable in advance during life are to be forborne and improved at interest until the deaths of the various members occur. Give expressions for (1) the *equal* sum that can be paid at each death, and (2) the *average* of all the sums that may become payable if the amounts are to vary with the term of the accumulation. In ordinary circumstances which of these expressions will be the larger, and under what conditions will they be equal to one another?

20. Give the formula for the Single Premium for an Assurance payable at the death of the last survivor of x and y, if that occur in the lifetime of z, or within t years after the death of z.

SECOND DAY—PART II.

21. A debt is to be discharged by n equal annual payments to include (1) principal, (2) interest, and (3) the premium for an assurance to provide for the cessation of the annual payments and the extinction of the debt if the debtor, aged x, die before the expiration of the n years. What should be the amount of the annual payment?

22. A temporary annuity during the life of x is purchased on 27th April 1906, to be payable half-yearly in equal portions until 20 payments have been made, the first half-year's payment becoming due on 15th May, with a proportion in event of death within the period. Express its value.

23. Given $\left.\begin{array}{l} l_{45}=79267 \\ l_{46}=78627 \\ l_{47}=77807 \\ l_{48}=76837 \end{array}\right\}$ What is the annual premium at 3 % for a Temporary Insurance for three years on a life aged 45?

24. Show how to construct a table of Whole Life Policy-Values. Give the actual arrangement of the work, and explain what checks you would introduce to secure accuracy.

25. State the bases of the English Life Tables, and give your reasons for any opinions you may hold as to their applicability or inapplicability to the affairs of Life Assurance Institutions.

26. Prove that $\int_{-n}^{n} u_x dx = \frac{n}{3}(u_{-n} + 4u_0 + u_n)$ approximately.

FINAL EXAMINATION.

Examiners—Dr. A. E. Sprague and Mr. Alex. Fraser.

FIRST DAY—PART I.

1. Describe fully the various columns you would employ in calculating, for a friendly society, valuation factors based on its own experience, allowing for secessions; the rates of mortality, sickness, and secession being given for each age. State clearly your assumptions as to the principles on which the various rates have been tabulated.

2. Describe Mr. Moore's investigation into the rates of mortality among total abstainers from alcoholic liquors and non-abstainers. State the results of the investigation, and point out the causes which tend to vitiate the conclusions drawn from them.

3. Describe the method of graduation of the $O^{[M]}$ tables, and state what relations hold between the various constants used in the process.

4. In order to graduate a series of select mortality tables by Makeham's law it is necessary to alter the constants each year during which "selection" remains. Prove that this alteration of the constants does not interfere with the property which is known as "Uniform Seniority" provided the lives in question have been insured for the same number of years.

5. Having given a series of equidistant terms, demonstrate formulas (working to fourth differences) to subdivide each interval into m parts so as to produce a series of terms distant from each other by only $\frac{1}{m}$ of the interval between the given terms. Apply the formulas to insert four terms between the first two terms of the following series, 40,000; 38,125; 30,000; 23,125; 40,000.

6. If the force of mortality be constant ($=c$) prove by direct integration that $\bar{A}_x = \frac{c}{c+\delta}$.

FIRST DAY—PART II.

7. Find the value of $\bar{A}_{\overline{30.30}:\overset{70}{\underset{3}{}}}$ by the O^{M} Table at 3% interest, using Woolhouse's "Nugget" formula. The following are the logarithms of the constants in the formula :—

u_0 $\bar{1}$·44859
u_t 0·20924
u_{2t} 0·33943
u_{3t} 0·26254

8. Give a short account of the investigations that have been made into the rates of mortality in tropical Africa, and state what extra premiums have been suggested for various localities there. What reasons are there for supposing that these rates should be modified?

9. It is proposed to adopt Herr Altenburger's method of valuation for Limited Payment Policies in an office where the premium income is due on the average eight months after the close of the year. Show in detail how you would arrive at the necessary constants in the case of yearly premiums. Assume lives born between 1st July of one year and 30th June of the following year to be grouped together.

How would you make adjustments for half-yearly and quarterly cases (included amongst the yearly cases), where a full year's premium did not fall due in the year of last payment?

10. If you were asked to value a large number of endowment assurances without profits, the data to be supplied on cards, what information would you require to be given on the cards? Explain the various processes by which you would make the valuation, and sketch the forms of books or schedules you would employ.

11. Describe a method of valuing (approximately) whole life assurances by the O$^{[M]}$ (select) tables, and give a short explanation of the principles underlying the method.

12. How is the increase in endowment assurance business likely to affect the future rates of bonus?

SECOND DAY—PART I.

13. How do the usual systems of simple and compound bonuses compare with the results of the contribution method under endowment assurance policies effected with an office which charges

average premiums, earns an average rate of interest, and experiences average rates of mortality and expense?

14. An office gives to limited payment policyholders the option of applying the reversionary bonuses towards making the sum assured payable during lifetime, and it is desired to arrange the books so that policies where the option has been exercised may still be valued in the limited payment class. Show in what ratio the declared bonuses must be increased at subsequent valuations to allow of this (*a*) when the valuation basis remains constant; (*b*) when the basis is altered. Assume that the original number of premiums will have been paid before the sum assured finally becomes due by application of the bonuses.

15. A life office purchases Convertible Preference Stock of a British Railway shortly before the stock is quoted *ex. div.*, receives payment of the dividend, immediately thereafter exercises the option to convert, and sells a few months later, making a profit on the transaction. Draft all the cash-book, journal, and ledger entries necessary.

16. Give a short statement of the principle of "Double-Entry" book-keeping; and explain how it facilitates the drawing up of a Balance Sheet and Revenue Account. How does the latter differ from a Cash Account?

Describe the reconcilement which is necessary to prove that all of the entries in the Journal have been posted to the Ledger.

17. Give an account of the course for the past ten years of the market for American Railway securities, and state shortly your opinion of these as an investment at the present day.

18. Give your opinion, with reasons therefor, as to the suitability for Life Assurance Offices of the following mortgages, and state what special inquiries you would make in each case:—

(*a*) On leasehold property, with residential suites of apartments, in London.

(*b*) On freehold property in one of the chief provincial cities, let at small weekly rents.

(*c*) On shop and house property in a good position in a fashionable seaside resort.

SECOND DAY—PART II.

19. What is an "Equitable Mortgage," and how does the position of the holder differ from that of an ordinary mortgagee as regards

(*a*) surrendering a policy; (*b*) borrowing upon security thereof; and (*c*) giving a receipt for the policy money?

Can a valid security be constituted by the mere deposit of a policy?

20. Under what conditions can a proprietary Assurance Company safely accept (*a*) a Colonial or (*b*) a foreign grant of representation as proof of title to (1) a policy, and (2) shares in the Company?

21. What would you give for the purchase of a well-secured reversionary life interest of £500 per annum to a life aged 40 after the death of one aged 65, power of redemption being reserved for 5 years; and on what terms would you grant an extension of the redemption period for other 5 years at the expiry of the first term?

22. Draft a form of world-wide indisputable policy for use by a proprietary Life Office, the policy to be worded so that it may be used for any of the principal classes of assurance by means of entries in an appended schedule. Draft the schedule also.

23. State the principal recommendations of the Committee recently appointed to investigate life assurance affairs in America, and give briefly your opinion on each.

24. Describe what are meant by the "Reversionary Annuity" and the "Varying Assurance" methods of valuing a Widows' Fund. Give your opinions as to the advantages and disadvantages of each, with reasons.

REPORT BY THE COUNCIL, 1906.

THE Council have pleasure in submitting their Annual Report.

At the Ordinary General Meeting held on 26th June 1905, Mr. THOMAS GANS ACKLAND, F.I.A., and Mr. RALPH PRICE HARDY, F.I.A., were elected Honorary Fellows.

The following six gentlemen, who passed the Final Examination this year, have been admitted Associates, viz. :—Messrs. W. BORLAND, W. FENWICK, T. FRAZER, Jun., R. M. HUNTER, J. M'INTOSH, and J. A. THOMSON.

The following gentlemen have been admitted Fellows in terms of Rule 3, viz. :—Messrs. H. F. COWAN, J. N. FORREST, W. E. GILLHESPY, R. M. HUNTER, J. M'INTOSH, C. C. NICHOLL, B.A., and J. A. THOMSON.

The Council have to report with regret the death of Mr. THOMAS GOLDIE DICKSON, C.A., an original member of the Faculty, and of Mr. PETER LORIMER CATTANACH, Jun., C.A., who was admitted a Fellow in 1903.

The total Membership is now 190, viz. :—7 Honorary Fellows, 179 Fellows, and 4 Associates.

Since 31st May 1905, 19 Students have been admitted, 6 have become Associates, 9 have resigned, and the names of 15 have been removed from the roll, in consequence of their having discontinued payment of the annual subscription. The number now on the roll is 139.

The Annual Examinations were held on 26th and 27th April last. Fifty-five Candidates entered for examination, viz. :—24 for the First Examination, 16 for the Second, and 15 for the Final. The following is a list of the successful Candidates arranged in order of merit :—

FIRST EXAMINATION.

1 and 2 (*Equal*) *ARTHUR COCKBURN. *ALEXANDER M. HOGG.
3. DAVID BEGG.
4. DAVID G. YOUNG.
5. WILLIAM MACFARLANE.
6. JOHN B. JACKSON.
7. ALEXANDER T. MACLEAN.
8. ALFRED W. BEWS.
9. ALEXANDER J. GREEN.
10. WILLIAM M. WISHART.
11. SYDNEY J. TAVENDER.
12. MALCOLM HALCROW.

* With distinction.

SECOND EXAMINATION.

1. DAVID N. STRATHIE.
2. JOHN S. B. WILSON.
3. STEVEN J. L. HARDIE.
4. JOHN P. P. OLIVER.
5. JOHN G. DAVIDSON.
6. JAMES P. M'NAUGHT.
7 and 8 (*Equal*) JOHN R. MOODIE. WILLIAM G. WALKER.
9. ARCHIBALD HARROWER.
10. JAMES N. ADAMS.
11. WILLIAM L. CRAWFORD.

FINAL EXAMINATION.

1. WALTER BORLAND.
2. THOMAS FRAZER, JUN.
3. CHARLES H. L. CATCHLOVE.
4 and 5 (*Equal*) ROBERT M. HUNTER. JAMES M'INTOSH.
6. WILLIAM FENWICK.
7. JAMES A. THOMSON.

The Honorary Treasurer's Accounts for the year ended 31st December 1905, with relative Statement of Funds, audited by Mr. Charles F. Whigham, C.A., are herewith submitted. The Income of the year was £637, 4s. 8d., and the Outgo was £694, 9s. 4d. The Funds at the close of the year amounted to £1966, 6s. 2d.

Donations of books for the Library have been received from The Census Office of the United States of America, Dr. T. B. Sprague, Mr. Hugh Blair, Mr. Henry Cockburn, Mr. M. M. Dawson, Mr. Archibald Hewat, Mr. Henry Moir, Messrs. Bradshaw and Sanderson, and others, to all of whom the thanks of the Council have been conveyed. From Mr. David Paulin has been received a donation of twenty guineas, to be employed in the purchase of books.

The Jubilee of the foundation of the Faculty was celebrated on 19th January last by a Dinner, at which one hundred gentlemen were present. A full report of the proceedings on that occasion has been published in the *Transactions*.

During the Session 1905-1906 the following Papers were read :—

11th December 1905. "An Analysis of the Profit from Endowment Assurances." By Mr. James Chatham.

12th March 1906. "Notes on Summation and Interpolation." By Mr. Austyn J. C. Fyfe.

12th March 1906. "Results of a Group Valuation of Joint-Life Policies." By Mr. George C. M'Laren.

4th June 1906. "Notes on Recent Decisions relating to Life Assurance." By Mr. John L. Wark, Advocate.

The Papers by Mr. Chatham and Mr. Fyfe, with abstracts of the relative discussions, have already been published in the *Transactions*, and Mr. Wark's Paper will soon be ready for issue.

The Council reappointed Mr. John L. Wark, Advocate, as Reporter to the Faculty on law cases connected with actuarial questions or the business of life assurance. It is hoped that members will assist in this matter by giving the Honorary Secretary of the Faculty notice of any cases of which they may have knowledge, with a view to their being brought before Mr. Wark.

N. B. GUNN,
President.

EDINBURGH, 6*th June* 1906.

RECEIPTS AND EXPENDITURE FOR YEAR 1905.

I.—General Fund.

FUND at 31st December 1904,					£1,995 3 10	
ENTRANCE FEES—						
Fellows,			£18 18 0			
Associates,			14 14 0			
Students,			25 4 0			
			£58 16 0			
ANNUAL SUBSCRIPTIONS—						
Fellows,	£326 11 0					
Associates,	14 14 0					
Students,	16 15 0					
			358 0 0			
Examination Fees,			49 7 0			
Interest and Dividends,			60 1 0			
Rent received,			15 0 0			
Transactions sold,			96 0 8			
					637 4 8	
					£2,632 8 6	

COST OF *Transactions*,			£243 10 6
LIBRARY—			
Purchase and Binding of Books, etc.,			46 19 1
FACULTY'S HALL—			
Rent, less Taxes,	£143 16 11		
Rates, Taxes, and Fire Insurance,	36 5 2		
Furniture (written off for depreciation),	12 5 11		
Sundries,	39 5 1		
			231 13 1
EXAMINATION, MANAGEMENT, AND MISCELLANEOUS EXPENSES—			
Examination Expenses,	£46 15 6		
Stationery, Printing, etc.,	8 18 2		
Secretary's Allowance,	51 2 8		
Miscellaneous Expenses, Postages, etc.,	13 0 4		
			119 16 8
Presentation to Mr. Gordon Douglas,			52 10 0
			£694 9 4
FUND at 31st December 1905,			1,937 19 2
			£2,632 8 6

II.—Commutation Fund.

FUND at 31st December 1904,	£28 7 0	FUND at 31st December 1905,	£28 7 0

EDINBURGH, 16*th March* 1906.—As instructed by the Council of the Faculty of Actuaries in Scotland, I have examined and audited the foregoing Account of the Intromissions of William G. Walton, Honorary Treasurer of the Faculty, for the year ending 31st December 1905, and have found the same to be correctly stated and sufficiently vouched and instructed. I have also examined the Security Writs for the Investments held at the close of the Account and found them in order.

CHARLES F. WHIGHAM, C.A., *Auditor.*

STATEMENT OF FUNDS, 31st DECEMBER 1905.

£750	Caledonian Railway 4 per cent Consolidated Preference Stock, No. 1 (Cost),	£892	17	9
£300	Highland Railway 4¼ per cent Debenture Stock (Cost),	389	2	9
£220	Victoria Government 3½ per cent Inscribed Stock (Cost),	196	13	0
£200	North British Railway 3 per cent Debenture Stock (Cost),	208	7	2
£150	Great Central Railway 4½ per cent Preference Stock (Cost),	195	12	9
	Value of Furniture in Faculty's Rooms,	110	12	9
		£1,993	6	2
	Less Balance due Hon. Treasurer,	27	0	0
		£1,966	6	2

General Fund,	£1,937	19	2
Commutation Fund,	28	7	0
	£1,966	6	2

NOTE.—In addition to the above Assets the Faculty has also (1) the Library, which contains a large and valuable collection of Books and Pamphlets; and (2) a considerable stock of *Transactions* in hand.

ANNUAL MEETING, 1906.

The Annual General Meeting of the Faculty was held in the Faculty's Hall on 14 June 1906, the President (Mr. N. B. GUNN) in the chair. The Report by the Council (see p. 121), which had been circulated among the members, was held as read.

The PRESIDENT said:—Gentlemen, the Report has been in your hands and I presume you have all read it. It is usual for the Chairman, in moving its adoption, to make a few remarks. I have, however, very little to add to what is detailed there. We have had a prosperous but, apart from our Jubilee celebration, an uneventful year. You will note the increase in our Membership —from 184 to 190. We have lost two members by death—Mr. T. G. Dickson, one of our oldest members, and Mr. P. L. Cattanach, one of our youngest. The Sessional Meetings have unfortunately been fewer in number than usual, the Secretary having had great difficulty in getting papers—probably on account of the forthcoming Congress, for which several of our members have been preparing papers. Some of these might otherwise have been contributed to our *Transactions*. Speaking of the *Transactions*—you are aware that Mr. Warden, who acted as Editor and did his work so admirably, and to whom the Faculty owes so much for this and other services, has found it necessary to resign. As his successor we have fortunately secured Mr. Armstrong, under whose care the papers of the past Session have been edited. I need hardly refer to the proceedings in connection with the Jubilee of the Faculty, a full Report of which appears in the *Transactions*. Thanks to the excellent arrangements made by the Honorary and Acting Secretaries the Dinner was most successful. I have referred to the International Congress of Actuaries—the fifth—which, as you are aware, is to be held in Berlin in September. A circular has been issued by the Organising Secretary of the Congress, specifying the proceedings generally, and indicating the hospitalities which our Actuarial brethren in Germany are preparing to bestow on their visitors. I trust all of us who can possibly do so will attend the Congress, and that we shall be largely represented there. Finally, I may refer to the appointment by

the House of Lords of a Select Committee to consider what steps should be taken for the protection of British policyholders in Foreign and Colonial Companies. In response to the Committee's invitation, the Council of the Faculty have requested the President to attend and give evidence on the Faculty's behalf. With these few remarks I now ask you to approve of the Report, to adopt the Honorary Treasurer's Accounts, and to pass a hearty vote of thanks to the Auditor, Mr. C. F. Whigham, for his services.

Mr. J. CHATHAM.—I rise for the purpose of seconding the motion for the adoption of the Report. It is no ordinary Report, since it refers, though in modest terms, to the celebration of our Jubilee. While I agree that our grateful recognition is due to the Secretaries for their work in connection with the Dinner, I think I am endorsing the opinion of every one when I say we are specially sensible of the admirable manner in which you, Sir, presided there. When we arrive at a landmark such as the Jubilee we are inclined to look back and see how far we have travelled, and in what measure the Faculty has realised the expectations of its founders. Without enlarging on this, I may take one gratifying point disclosed in the Report, viz. our membership. I feel sure the founders had no idea the Faculty would ever attain its present dimensions. We have now a great deal of good material to work upon. You, Sir, referred to the paucity of papers last Session. With the exception of the first in the list, I have no doubt that what they lacked in number they made up for in quality. One would like, however, to see the younger members coming forward and taking their share in the preparation of papers, which would not only be of advantage to themselves but would redound to the honour of the Faculty.

The Report was adopted unanimously.

The usual ballot was then taken for Office-bearers and Members of Council, and the following gentlemen were elected for the year 1906-7, as recommended by the Council:—

PRESIDENT.

ARCHIBALD HEWAT.

VICE-PRESIDENTS.

NIEL BALLINGAL GUNN.
GEORGE MACRITCHIE LOW.
ANDREW HUGH TURNBULL.

HONORARY SECRETARY.
GORDON DOUGLAS.

HONORARY TREASURER.
JAMES CHATHAM.

ORDINARY MEMBERS OF COUNCIL.

THOMAS WALLACE.
WILLIAM RAE MACDONALD.
JOHN NICOLL.
ALEXANDER FRASER, JUN.
ROBERT M. M. RODDICK.
JAMES STIRLING.
ROBERT MURRIE.
VYVYAN MARR.
*R. HILL STEWART.
*CHARLES F. WHIGHAM.
*WILLIAM HUTTON.
*JAMES J. M'LAUCHLAN.

* New Members.

On the motion of Mr. G. M. Low a very hearty vote of thanks was accorded to Mr. Gunn for all he had done for the Faculty during the three years he had been President. Mr. Low referred to the special circumstances under which the Faculty had last year unanimously extended Mr. Gunn's term of Office. Nothing, he said, could have been better or more suitable than that the Faculty, at the period of its Jubilee, should have had Mr. Gunn as its President.

On the motion of Mr. PAULIN the services of Mr. J. J. M'Lauchlan, the retiring Honorary Secretary, and of Mr. W. G. Walton, the retiring Honorary Treasurer, were also heartily acknowledged.

LIST OF MEMBERS AND STUDENTS

OF THE

FACULTY OF ACTUARIES IN SCOTLAND

CONSTITUTED 1856

Incorporated by Royal Charter 21st September 1868.

HALL: 24 GEORGE STREET, EDINBURGH.

(*August* 1906)

EDINBURGH: PRINTED BY T. AND A. CONSTABLE.

1906

The Faculty of Actuaries in Scotland

OFFICE-BEARERS; MEMBERS OF COUNCIL AND COMMITTEES, 1906-1907.

PRESIDENT
ARCHIBALD HEWAT.

VICE-PRESIDENTS
NIEL BALLINGAL GUNN.
GEORGE MACRITCHIE LOW.
ANDREW HUGH TURNBULL.

HONORARY SECRETARY
GORDON DOUGLAS.

HONORARY TREASURER
JAMES CHATHAM.

COUNCIL
THE PRESIDENT, VICE-PRESIDENTS, HON. SECRETARY, HON. TREASURER,
AND

THOMAS WALLACE.
WILLIAM RAE MACDONALD.
JOHN NICOLL.
ALEXANDER FRASER, JUNIOR.
ROBERT M. M. RODDICK.
JAMES STIRLING.
ROBERT MURRIE.
VYVYAN MARR.
RALPH HILL STEWART.
CHARLES F. WHIGHAM.
WILLIAM HUTTON.
JAMES JOHN M'LAUCHLAN.

HALL AND LIBRARY COMMITTEE
THE PRESIDENT AND HON. SECRETARY (*ex officio*)
AND

NIEL BALLINGAL GUNN.
GEORGE MACRITCHIE LOW.
THOMAS WALLACE.
WILLIAM RAE MACDONALD.
ROBERT M. M. RODDICK.
CHARLES F. WHIGHAM.

HONORARY LIBRARIAN
ALFRED ERNEST SPRAGUE.

COMMITTEE ON EXAMINATIONS
THE PRESIDENT AND HONORARY SECRETARY (*ex officio*)
AND

JAMES JOHN M'LAUCHLAN.
JAMES CHATHAM.
WILLIAM HUTTON.
ROBERT MURRIE.
*ALFRED ERNEST SPRAGUE.
*ALEXANDER FRASER, JUNIOR.
*HUGH WYLIE BROWN.
*ALEXANDER G. DONALD.
*WILLIAM ABERNETHY SIM.
*JAMES BUCHANAN.

* *Examiners*

COMMITTEE ON SESSIONAL MEETINGS AND TRANSACTIONS
THE PRESIDENT AND HONORARY SECRETARY (*ex officio*)
AND

RALPH HILL STEWART.
ALEXANDER FRASER, JUNIOR.
VYVYAN MARR.
†HUGH WYLIE BROWN.
†STUART F. M. CUMMING.
†JOHN W. WINK.

† *Nominated by Council under Rule 24.*

HONORARY EDITOR OF THE TRANSACTIONS
JOHN ROBERT ARMSTRONG.

SECRETARY
JOHN EDGAR, EDINBURGH LIFE ASSURANCE COMPANY, 22 GEORGE STREET, EDINBURGH.

LIST OF MEMBERS AND STUDENTS

August 1906

HONORARY FELLOWS

Date of Admission.

1905 Ackland, Thomas Gans, F.I.A., F.S.S., F.A.S., 5 & 6 *Clement's Inn, Strand, London, W.C.*

1868 Blackburn, Hugh, M.A. (*Late Professor of Mathematics, University of Glasgow*), *Roshven, Moidart, Inverness-shire.*

1868 Fuller, Frederick, M.A., LL.D. (*Late Professor of Mathematics, University of Aberdeen*), 9 *Palace Road, Surbiton, Kingston-on-Thames.*

1856 Gregor, David Clunie (*Late Colonial Secretary, Standard Life Assurance Company*), 24 *Murrayfield Avenue, Edinburgh.*

Date of Admission.

1905 Hardy, Ralph Price, F.I.A., 61 *Addison Road, London, W.*

1902 Sprague, Thomas Bond, M.A. (Cantab.), LL.D., F.I.A., F.R.S.E. (*Late Manager and Actuary, Scottish Equitable Life Assurance Society*), 29 *Buckingham Terrace, Edinburgh.*

1904 Thomson, Spencer Campbell, B.A. (Cantab.), F.R.S.E. (*Late Manager and Actuary, Standard Life Assurance Company*), 10 *Eglinton Crescent, Edinburgh.*

FELLOWS

*Those marked thus * have passed the Faculty's Examinations.*

Date of Admission.

1902* Adamson, William, A.I.A., *Scottish Accident Life and Fidelity Insurance Company, Ltd.*, 115 *George Street, Edinburgh.*

1901* Adamson, William Livingstone, *Standard Life Assurance Company*, 3 *George Street, Edinburgh.*

1896* Anderson, David Leslie, C.A., *Secretary, The Linen Thread Company, Ltd.*, 180 *Hope Street, Glasgow.*

1864 Anderson, James John Philip (*Late Secretary, Scottish Widows' Fund and Life Assurance Society*), 51 *Melville Street, Edinburgh.*

Date of Admission.

1903* Anderson, Thomas Frederic, F.I.A., *Royal Exchange Assurance Corporation, Royal Exchange, London, E.C.*

1898* Armstrong, John Robert (HONORARY EDITOR), *Scottish Provident Institution, 6 St. Andrew Square, Edinburgh.*

1902* Bannatyne, William, *Scottish Temperance Life Assurance Company, Ltd.*, 105 *St. Vincent Street, Glasgow.*

FELLOWS

*Those marked thus * have passed the Faculty's Examinations.*

Date of Admission.

1898* Blount, Edward Thomas Joseph, A.I.A., F.S.S., *Standard Life Assurance Company, Shanghai.*

1878 Bloxsom, William Gibson, *Managing Director, The Profits and Income Insurance Company, Ltd., 9 Fleet Street, London, E.C.*

1893 Blyth, Robert, C.A., *General Manager, Union Bank of Scotland, Limited, Glasgow.*

1902* Bowie, John, *Scottish Temperance Life Assurance Company, Ltd., 105 St. Vincent Street, Glasgow.*

1905* Boyd, James Colville, *North British and Mercantile Insurance Company, 64 Princes Street, Edinburgh.*

1886* Bremner, Thomas William, A.I.A., *Secretary, Australasian Department, Mutual Life Assurance Company of New York, Martin Place, Sydney.*

1904* Brodie, Robert Raynal, A.I.A., *Scottish Provident Institution, 6 St. Andrew Square, Edinburgh.*

1897* Brown, Hugh Wylie, F.I.A., *Actuary, Scottish Union and National Insurance Company, 35 St. Andrew Square, Edinburgh.*

1902* Buchanan, James, F.I.A., D.Sc. (Glasgow), M.A. (Cantab.), *Scottish Widows' Fund and Life Assurance Society, 9 St. Andrew Square, Edinburgh.*

1904* Cameron, Finlay James, A.I.A., *Friends' Provident Institution, 45 Darley Street, Bradford, Yorkshire.*

1885* Cameron, George, *Actuary, Standard Life Assurance Company, 3 George Street, Edinburgh.*

1904* Cameron, John Campbell, M.A., *Scottish Metropolitan Life Assurance Company, Ltd., 25 St. Andrew Square, Edinburgh.*

Date of Admission.

1893* Campbell, Neil, *Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.*

1871* Carment, David, F.I.A., F.A.S., *Assistant Actuary, Australian Mutual Provident Society, Sydney.*

1890* Carment, David Malcolm, *Assistant Manager, Law Guarantee and Trust Society, 49 Chancery Lane, London, W.C.*

1904* Cathles, Lawrence Maclagan, A.I.A., *Actuary, Franklin Life Insurance Company, Springfield, Illinois, U.S.A.*

1886* Chatham, James, F.I.A. (Honorary Treasurer), *Secretary and Actuary, Scottish Life Assurance Company, Ltd., 19 St. Andrew Sq., Edinburgh.*

1882* Chisholm, James, F.I.A., F.A.S., (*Late Manager and Actuary, Imperial Life Insurance Company*), *Crossfield, Alberta, N.W.T., Canada.*

1900* Clapperton, David Alexander, C.A., *6A George Street, Edinburgh.*

1874* Cockburn, Henry, F.I.A., F.A.S., *Manager of Life Dept. and Actuary, North British and Mercantile Insurance Company, 61 Threadneedle Street, London, E.C.*

1878* Cousin, John William, (*Late Assistant Actuary, Edinburgh Life Assurance Company*), *58 Newbattle Terrace, Edinburgh.*

1906* Cowan, Hugh Francis, *Edinburgh Life Assurance Company, 22 George Street, Edinburgh.*

1902* Cumming, Stuart Ferrier Macdonald, *Life Association of Scotland, 82 Princes Street, Edinburgh.*

FELLOWS

*Those marked thus * have passed the Faculty's Examinations.*

Date of Admission.

1901* Denham, Walter, F.I.A., *City of Glasgow Life Assurance Company, 30 Renfield Street, Glasgow.*

1905* Denmead, John Charles, M.A. (Cantab.), F.I.A., *Estate-Duty Office, Somerset House, London, W.C.*

1876* Deuchar, John James Walker, F.I.A., *General Manager and Actuary, Norwich Union Life Insurance Society, Norwich.*

1901* Donald, Alexander Graham, M.A., A.I.A., *Scottish Provident Institution, 6 St. Andrew Square, Edinburgh.*

1884* Douglas, Gordon, F.I.A. (Honorary Secretary), *Actuary, Life Association of Scotland, 82 Princes Street, Edinburgh.*

1879* Dovey, William Roadley, A.I.A., F.A.S., *62 Weston Park, Crouch End, London, N.*

1889* Duff, Campbell Kerr, C.A., *65 Bath Street, Glasgow.*

1891 Duncan, Alexander (*Late General Manager, Scottish Union and National Insurance Company*), *Edinburgh.*

1900* Edgar, John (Secretary), *Edinburgh Life Assurance Company, 22 George Street, Edinburgh.*

1877* Evans, William, A.I.A., F.R.S.E. (*Late Assistant Actuary, Scottish Widows' Fund and Life Assurance Society*), *38 Morningside Park, Edinburgh.*

1902* Falconer, Herbert Francis, *21 West Mayfield, Edinburgh.*

1896* Fenton, Francis Kerr, *Scottish Amicable Life Assurance Society, 1 Threadneedle Street, London, E.C.*

Date of Admission.

1891* Fenton, James, *North British and Mercantile Insurance Company, 64 Princes Street, Edinburgh.*

1898* Ferguson, John, *Assistant Actuary, Standard Life Assurance Company, 3 George Street, Edinburgh.*

1906* Forrest, James Nairne, *Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.*

1888* Fox, Charles Edward, A.I.A., *Standard Life Assurance Company, 83 King William Street, London, E.C.*

1897* Fraser, Alexander, Jun., F.I.A., *Assistant Actuary, Scottish Life Assurance Company, Ltd., 19 St. Andrew Square, Edinburgh.*

1904* Fyfe, Austyn James Claude, *Northern Assurance Company, 1 Union Terrace, Aberdeen.*

1897* Gaff, William Robertson, C.A., A.I.A. (Messrs. Gaff, Harper & Company), *53 New Broad Street, London, E.C.*

1899* Gibb, James Burnett, A.I.A., *Penn: Mutual Life Insurance Company, 923 Chestnut Street, Philadelphia, U.S.A.*

1891* Gibson, William, *Assistant Actuary, Caledonian Insurance Company, 19 George Street, Edinburgh.*

1906* Gillhespy, William Edward, *Yorkshire Coalowners Mutual Indemnity Company, Foster' Buildings, Sheffield.*

1884* Gillison, John Brotch, F.I.A., *Manager for the United Kingdom, National Mutual Life Association of Australasia, Limited, 5 Cheapside, London, E.C.*

1891* Glen, Ninian, M.A., B.Sc., C.A., *107 St. Vincent Street, Glasgow.*

FELLOWS

*Those marked thus * have passed the Faculty's Examinations.*

Date of Admission.

1875* Gordon, Charles, F.I.A., *Actuary, South African Mutual Life Assurance Society, Darling Street, Cape Town.*

1892* Govan, John, *Norwich Union Life Insurance Society, Norwich.*

1883* Graham, James, F.I.A., *Actuary, Australian Widows' Fund Life Assurance Society, Melbourne.*

1905* Graham, George, *New York Life Insurance Company, 346 and 348 Broadway, New York, U.S.A.*

1871* Gunn, Niel Ballingal, F.I.A. (VICE-PRESIDENT), *Manager and Actuary, Scottish Widows' Fund and Life Assurance Society, 9 St. Andrew Square, Edinburgh.*

1895* Guthrie, Charles, *Assistant Secretary, Scottish Amicable Life Assurance Society, 1 Threadneedle Street, London, E.C.*

1892* Halliday, William Ross, *34 Old Broad Street, London, E.C.*

1905* Hamilton, William, *North British and Mercantile Insurance Company, 64 Princes Street, Edinburgh.*

1904* Harrison, Edward John, C.A., *1 Broad Street Place, London, E.C.*

1877* Hewat, Archibald, F.I.A. (PRESIDENT), *Manager and Actuary, Edinburgh Life Assurance Company, 22 George Street, Edinburgh.*

1856† Howden, James, C.A., *8 York Place, Edinburgh.*

1893* Hunter, Arthur, A.I.A., F.A.S., *Actuary, New York Life Insurance Company, 346 and 348 Broadway, New York, U.S.A.*

Date of Admission.

1906* Hunter, Robert Marshall, *Scottish Provident Institution, 6 St. Andrew Square, Edinburgh.*

1895* Hutcheson, William Anderson, F.I.A., F.A.S., *Associate Actuary, Mutual Life Insurance Company of New York, 32 Nassau Street, New York, U.S.A.*

1895* Hutton, William, F.I.A., *Manager and Actuary, Scottish Amicable Life Assurance Society, 35 St. Vincent Place, Glasgow.*

1904* Imrie, John Hamilton, M.A., A.I.A., *Life Association of Scotland, 82 Princes Street, Edinburgh.*

1905* Jackson, Samuel, *Resident Secretary, Scottish Widows' Fund and Life Assurance Society, 48 Castle Street, Liverpool.*

1903* Jamieson, Charles William Steele, A.I.A., *Scottish Amicable Life Assurance Society, 35 St. Vincent Place, Glasgow.*

1899* Kilpatrick, Hugh Grainger, *Liverpool and London and Globe Insurance Company, 1 Dale Street, Liverpool.*

1885 King, George, F.I.A., F.A.S., *15 Walbrook, London, E.C.*

1877* Kyd, Thomas, F.I.A., *Resident Manager, Northern Assurance Company, 1 Union Terrace, Aberdeen.*

1905* Laing, James Murray, A.I.A., *National Mutual Life Association of Australasia, 5 Cheapside, London, E.C.*

1885* Lamb, John, C.A., *Secretary, Scottish Provident Institution, 6 St. Andrew Square, Edinburgh.*

1896* Latta, Alexander, A.I.A., *Guardian Assurance Company, Ltd., 28 King Street, Covent Garden, London, W.C.*

† Original Member.

FELLOWS

*Those marked thus * have passed the Faculty's Examinations.*

Date of Admission.

1896* Laughton, Alexander Millar, F.I.A., *National Mutual Life Association of Australasia, Limited, Melbourne.*

1898* Lees, Mungo Mackenzie, *Assistant Secretary, Norwich Union Life Insurance Society, Norwich.*

1895* Lewis, John Norman, F.I.A., *London Assurance Corporation, 7 Royal Exchange, London, E.C.*

1887* Lisle, George, C.A., *5 North St. David Street, Edinburgh.*

1874* Low, George Macritchie, F.I.A., F.R.S.E. (VICE-PRESIDENT), *Manager and Actuary, Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.*

1888* Lugton, Hugh, A.I.A., *Assistant Actuary, North British and Mercantile Insurance Company, 61 Threadneedle Street, London, E.C.*

1856 Macandrew, James M'Lean, C.A., *16 York Place, Edinburgh.*

1870* Macdonald, William Rae, A.I.A., *Secretary and Actuary, Scottish Metropolitan Life Assurance Company, Ltd., 25 St. Andrew Square, Edinburgh.*

1904* M'Ilvenna, William Richard, *Scottish Equitable Life Assurance Society, 14 Westmoreland Street, Dublin.*

1903* M'Intosh, Archibald, *Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.*

1906* M'Intosh, James, *North British and Mercantile Insurance Company, 64 Princes Street, Edinburgh.*

1903* Mackay, Robert, *Secretary, Northern Assurance Company, 1 Union Terrace, Aberdeen.*

Date of Admission.

1902* M'Kean, Philip Campbell, *Scottish Widows' Fund and Life Assurance Society, 9 St. Andrew Square, Edinburgh.*

1879* Mackenzie, Alexander George, F.I.A., *29 Chester Terrace, Regent's Park, London, N.W.*

1894 Maclagan, Philip Robert Dalrymple, F.R.S.E., *Manager, North British and Mercantile Insurance Company, 64 Princes Street, Edinburgh.*

1905* M'Laren, George Cunningham, *South African Mutual Life Assurance Society, Darling Street, Cape Town.*

1877* M'Lauchlan, James John, *Secretary, Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.*

1893* Macphail, Donald, F.I.A., *Manager for South Africa, Yorkshire Insurance Company, 122 St. George's Street, Cape Town.*

1889* Manson, John, M.A., *Scottish Widows' Fund and Life Assurance Society, 9 St. Andrew Square, Edinburgh.*

1898* Marr, Vyvyan, F.I.A., *Edinburgh Life Assurance Company, 22 George Street, Edinburgh.*

1899* Maxwell, Benjamin Bell, A.I.A., *Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.*

1897* Meikle, Henry George Watson, *Actuary, Oriental Government Security Life Assurance Company, Bombay.*

1892* Miller, Robert Duncan, *Actuary, Mutual Life Association of Australasia, George and Wynyard Streets, Sydney.*

1876* Milligan, James Macmillan, *5 Royal Terrace, Edinburgh.*

FELLOWS

*Those marked thus * have passed the Faculty's Examinations.*

Date of Admission.

1893* Moir, Henry, F.I.A., F.A.S., *Actuary, Provident Savings Life Assurance Society, 346 Broadway, New York, U.S.A.*

1884* Murrie, Robert, *Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.*

1906* Nicholl, Charles Carlyon, B.A. (Cantab.), *Royal Exchange Assurance Corporation, Royal Exchange, London, E.C.*

1890* Nicol, William Smith, *General Manager, City of Glasgow Life Assurance Company, 30 Renfield Street, Glasgow.*

1886* Nicoll, John, A.I.A., *Life Association of Scotland, 82 Princes Street, Edinburgh.*

1885* Nisbet, Robert James, *North British and Mercantile Insurance Company, 64 Princes Street, Edinburgh.*

1880* Notman, John, *Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.*

1888* Orr, Lewis Potter, A.I.A., *Joint Secretary, Scottish Life Assurance Company, Ltd., 19 St. Andrew Square, Edinburgh.*

1899* Padday, Percy King, A.I.A., *Actuary, Scottish Metropolitan Life Assurance Company, Ltd., 8 King Street, Cheapside, London, E.C.*

1873* Park, David Francis, C.A., A.I.A., *Crédit Foncier of Mauritius, 39 Lombard Street, London, E.C.*

1893 Paulin, David, F.R.S.E., *Manager, Scottish Life Assurance Company, Ltd., 19 St. Andrew Square, Edinburgh.*

1897* Pearce, Henry John, Jun., A.I.A., *Assistant Secretary, Scottish Amicable Life Assurance Society, 35 St. Vincent Place, Glasgow.*

Date of Admission.

1862* Pearson, David, C.A., *64 Queen Street, Edinburgh.*

1894* Prentice, Alexander, *Secretary, Life Association of Scotland, 32 Mosley Street, Newcastle-on-Tyne.*

1904* Price, Edmund Forbes, *Standard Life Assurance Company, 3 George Street, Edinburgh.*

1874* Pringle, James, C.A., A.I.A., *42 Drumsheugh Gardens, Edinburgh.*

1886* Pullar, James, A.I.A., *Secretary and Actuary, Colonial Mutual Life Assurance Society, 419 Collins Street, Melbourne.*

1906* Ramsay, William Livingstone, M.A., *Edinburgh Life Assurance Company, 11 King William Street, London, E.C.*

1903* Rankin, John Adam, A.I.A., *Edinburgh Life Assurance Company, 22 George Street, Edinburgh.*

1886* Reid, Walter Alexander, C.A., *Messrs. James Meston & Co., 6 Golden Square, Aberdeen.*

1896* Rennie, William John Martin, *North British and Mercantile Insurance Company, 64 Princes Street, Edinburgh.*

1891 Richardson, Josephus Hargreaves, A.I.A., F.A.S., *Government Insurance Commissioner, Wellington, N.Z.*

1892* Robertson, Archibald, *Secretary, Caledonian Insurance Company, Caledonian Buildings, Tithebarn Street, Liverpool.*

1904* Robertson, Frederick William, A.I.A., *Caledonian Insurance Company, 19 George Street, Edinburgh.*

FELLOWS

*Those marked thus * have passed the Faculty's Examinations.*

Date of Admission.

1904* Robertson, James Leask, A.I.A., *Edinburgh Life Assurance Company, 22 George Street, Edinburgh.*

1889* Robertson, William, A.I.A., *29 Stafford Street, Edinburgh.*

1900* Robertson, William Alexander, *Scottish Union and National Insurance Company, 35 St. Andrew Square, Edinburgh.*

1901* Robertson-Durham, Alexander Weir, C.A., *Messrs. A. & J. Robertson, C.A., 33 Charlotte Square, Edinburgh.*

1874* Robertson-Durham, James Alexander, C.A., *Manager, Scottish Reversionary Company, Ltd., 33 Charlotte Square, Edinburgh.*

1898* Roddick, Robert Murray M'Cheyne, M.A., *Actuary, Century Insurance Company, Limited, 18 Charlotte Square, Edinburgh.*

1884 Rolland, John Henry William, C.A., *General Secretary for England, Standard Life Assurance Company, 83 King William Street, London, E.C.*

1872* Ronaldson, Peter, C.A., *Actuary, Edinburgh Reversion Company, Limited, 3A North St. David Street, Edinburgh.*

1903* Ross, Frederick Alexander, C.A., *Messrs. Moncreiff & Horsbrugh, C.A., 46 Castle Street, Edinburgh.*

1902 Sanderson, Frank, M.A., A.I.A., F.A.S., *Actuary, Canada Life Assurance Company, Toronto, Ontario.*

1889* Sewell, Richard, C.A. (C. F. Burton, Sewell & Co., C.A., 35 Copthall Avenue, London), *General Manager and Actuary, British Widows' Assurance Company, Limited, 1 Old Street, London, E.C.*

Date of Admission.

1891* Shearer, Gilbert Edward, *Scottish Provident Institution, 17 King William Street, London, E.C.*

1896* Sim, William Abernethy, F.I.A., *Scottish Union and National Insurance Company, 35 St. Andrew Square, Edinburgh.*

1891* Slimon, William James, *North British and Mercantile Insurance Company, 64 Princes Street, Edinburgh.*

1885 Smith, John Turnbull, LL.D., C.A., *Manager, Life Association of Scotland, 82 Princes Street, Edinburgh.*

1886* Smith, Joseph Brown, C.A., *Prell's Buildings, 60 to 70 Queen Street, Melbourne.*

1899* Smith, Randolph Gordon, F.I.A., *Assistant Actuary, Scottish Amicable Life Assurance Society, 35 St. Vincent Place, Glasgow.*

1884 Smith, Walter Arthur, *Manager and Secretary, English and Scottish Law Life Assurance Association, 41 Charlotte Square, Edinburgh.*

1877* Sorley, James, F.I.A., C.A., F.R.S.E., *32 Onslow Square, London, S.W.*

1894* Sprague, Alfred Ernest, F.I.A., D.Sc. (Edinburgh), M.A. (Cantab.), (HONORARY LIBRARIAN), *Assistant Secretary, Edinburgh Life Assurance Company, 22 George Street, Edinburgh.*

1905* Steell, Graham, *Standard Life Assurance Company, 3 George Street, Edinburgh.*

1886* Stenhouse, George Crichton, *Assistant Actuary, Scottish Widows' Fund and Life Assurance Society, 9 St. Andrew Square, Edinburgh.*

FELLOWS

*Those marked thus * have passed the Faculty's Examinations.*

Date of Admission.

1887* Stewart, John, F.I.A., *Actuary, City of Glasgow Life Assurance Company, 30 Renfield Street, Glasgow.*

1893* Stewart, Ralph Hill, *Secretary and Actuary, Caledonian Insurance Company, 19 George Street, Edinburgh.*

1892* Stirling, James, *Sub-Manager, Life Department, Law Union and Crown Insurance Company, 126 Chancery Lane, London, W.C.*

1890* Stirling, Robert, F.I.A., *Secretary, Rock Life Assurance Company, 15 New Bridge Street, London, E.C.*

1884* Stuart, John Moody, F.I.A., *Manager and Actuary, Leeds Permanent Benefit Building Society, 18 Park Lane, Leeds.*

1866* Surenne, David John, (*Late Secretary, Caledonian Insurance Company*), *29 Inverleith Row, Edinburgh.*

1881* Tait, Edwin, *North British and Mercantile Insurance Company, 91 Union Street, Aberdeen.*

1901* Tait, William, *Secretary, Imperial Paper Mills of Canada, Ltd., 62 London Wall, London, E.C.*

1893 Teece, Richard, F.I.A., F.A.S., *General Manager and Actuary, Australian Mutual Provident Society, Sydney.*

1893 Thiselton, Herbert Cecil, F.I.A., F.A.S., *Life Manager and Actuary, Commercial Union Assurance Company, Ltd., 24 Cornhill, London, E.C.*

1906* Thomson, James Allan, *Scottish Widows' Fund and Life Assurance Society, 9 St. Andrew Square, Edinburgh.*

Date of Admission.

1904* Thomson, Gordon William, *African Life Assurance Company, Johannesburg.*

1904* Thomson, John Walter, A.I.A., *Scottish Life Assurance Company, Ltd., 19 St. Andrew Square, Edinburgh.*

1895* Turnbull, Andrew Dalzel Lindsay, A.I.A., C.A., *Assistant Secretary, Scottish Widows' Fund and Life Assurance Society, 9 St. Andrew Square, Edinburgh.*

1862* Turnbull, Andrew Hugh, F.I.A., F.R.S.E. (VICE-PRESIDENT), (*Late Manager and Actuary, Scottish Widows' Fund and Life Assurance Society*), *The Elms, Whitehouse Loan, Edinburgh.*

1902* Turnbull, Phipps, *1 Royal Circus, Edinburgh.*

1885* Walker, Davidson, *Secretary, Norwich Union Life Insurance Society, Norwich.*

1897* Wallace, David Eckford, C.A., *105 Hanover Street, Edinburgh.*

1869* Wallace, Thomas, F.I.A., *Actuary, North British and Mercantile Insurance Company, 64 Princes Street, Edinburgh.*

1904* Wallace, Thomas, Jun., *Guardian Assurance Company, Ltd., 27 Cross Street, Manchester.*

1881* Walton, William Gandy, A.I.A., *Actuary, Scottish Provident Institution, 6 St. Andrew Square, Edinburgh.*

1884* Warden, John Maben, *Assistant Actuary, Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.*

1891 Watson, James Graham, C.A., *Manager, Scottish Provident Institution, 6 St. Andrew Square, Edinburgh.*

FELLOWS

*Those marked thus * have passed the Faculty's Examinations.*

Date of Admission.

1897* Watt, James, A.I.A., W.S., *28 Charlotte Square, Edinburgh.*

1896* Webster, George Rae, C.A., *Messrs. Price, Waterhouse & Company, Mills Building, San Francisco, U.S.A.*

1898* Whigham, Charles Frederick, A.I.A., C.A., *46 Castle Street, Edinburgh.*

1899* Wilkie, William Thom, *North British and Mercantile Insurance Company, 64 Princes Street, Edinburgh.*

Date of Admission.

1905* Wilson, William Maxwell Gunn, *Standard Life Assurance Company, 3 George Street, Edinburgh.*

1904* Wink, John Whyte, *Scottish Metropolitan Life Assurance Company, Ltd., 25 St. Andrew Square, Edinburgh.*

1888* Wood, William Alexander, C.A., *4 Melville Street, Edinburgh.*

1898* Young, William, F.A.S., *New York Life Insurance Company, 346 and 348 Broadway, New York, U.S.A.*

ASSOCIATES

All of whom have passed the Faculty's Examinations.

Date of Admission.

1906 Borland, Walter, *Scottish Temperance Life Assurance Company, Ltd., 105 St. Vincent Street, Glasgow.*

1906 Catchlove, Charles Hamilton Leyland, F.I.A., *Australian Mutual Provident Society, Sydney, New South Wales.*

Date of Admission.

1906 Fenwick, William, *Life Association of Scotland, 82 Princes Street, Edinburgh.*

1906 Frazer, Thomas, Junior, *Scottish Union and National Insurance Company, 35 St. Andrew Square, Edinburgh.*

STUDENTS

The figures indicate the number of Examinations to be passed for qualification as Associate.

Date of Enrolment.

1904 [1]Adams, James Napier, *Caledonian Insurance Company, 132 Union Street, Aberdeen.*

1893 [1]Anderson, James Gibson, M.A., *University Life Assurance Society, 219 St. Vincent Street, Glasgow.*

1900 [2]Andrew, Alexander, *Scottish Accident Life and Fidelity Insurance Company, Ltd., 115 George Street, Edinburgh.*

Date of Enrolment.

1895 [1]Arnott, George, *Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.*

1902 [1]Barnett, John, *Messrs. Lindsay, Jamieson & Haldane, C.A., 24 St. Andrew Square, Edinburgh.*

1906 [3]Barron, Albert Gibson, *23 Sciennes Road, Edinburgh.*

STUDENTS

The figures indicate the number of Examinations to be passed for qualification as Associate.

Date of Enrolment.

1906 [2]Begg, David,
Messrs. Hardie & Grieve, C.A., 157 St. Vincent Street, Glasgow.

1897 [1]Bell, Laurence Bruce,
42 Castle Street, Edinburgh.

1902 [2]Bews, Alfred William,
City of Glasgow Life Assurance Company, 30 Renfield Street, Glasgow.

1903 [1]Blackwood, George Glendinning,
Edinburgh Life Assurance Company, 22 George Street, Edinburgh.

1898 [2]Blair, Patrick Alexander,
4 Grosvenor Gardens, Edinburgh.

1905 [2]Blyth, Robert Oswald,
Scottish Amicable Life Assurance Society, 35 St. Vincent Place, Glasgow.

1905 [3]Bolton, James Butler,
Messrs. Romanes & Munro, C.A., 50 Frederick Street, Edinburgh.

1905 [3]Bonnar, James Norman,
Scottish Life Assurance Company, Ltd., 19 St. Andrew Square, Edinburgh.

1888 [1]Boyd, Henry Norris,
Secretary, City of Glasgow Life Assurance Company, 21 St. Andrew Square, Edinburgh.

1902 [3]Bowerman, Judah Philip,
472 Lafayette Avenue, Brooklyn, New York, U.S.A.

1900 [1]Bradshaw, Thomas, F.I.A., F.A.S.,
3 Spadina Road, Toronto, Ontario.

1885 [1]Brown, George,
Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.

1893 [1]Caldwell, David Livingstone,
Scottish Widows' Fund and Life Assurance Society, 21 Albert Square, Manchester.

Date of Enrolment.

1906 [3]Campbell, Samuel Wilson,
Scottish Provident Institution, 6 St. Andrew Square, Edinburgh.

1904 [2]Clark, George Hutchison,
Scottish Union and National Insurance Company, 35 St. Andrew Square, Edinburgh.

1906 [2]Cockburn, Arthur,
Scottish Metropolitan Life Assurance Company, Ltd., 25 St. Andrew Square, Edinburgh.

1901 [1]Colam, Stanley d'Eyncourt,
Yorkshire Insurance Company, York.

1898 [1]Collier, Charles St. John,
36 Walbrook, London, E.C.

1897 [1]Cownie, Thomas,
Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.

1902 [3]Crawford, Andrew West,
City of Glasgow Life Assurance Company, 30 Renfield Street, Glasgow.

1899 [1]Crawford, William Lindsay,
National Mutual Life Association of Australasia, Limited, 5 Cheapside, London, E.C.

1887 [1]Crow, John Reid,
Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.

1896 [2]Cumming, Robert, B.A.,
Manager, Scottish County and Mercantile Insurance Company, Ltd., 10 North St. David Street, Edinburgh.

1903 [1]Davidson, John George,
Scottish Metropolitan Life Insurance Company, Ltd., 25 St. Andrew Square, Edinburgh.

1899 [1]Davidson, William,
Scottish Amicable Life Assurance Society, 35 St. Vincent Place, Glasgow.

1895 [1]Dick, Alexander,
Northern Assurance Company, 20 St. Andrew Square, Edinburgh.

STUDENTS

The figures indicate the number of Examinations to be passed for qualification as Associate.

Date of Enrolment.

1905 [3]Dick, Alexander Blaikie, *Messrs. Alexander Sloan & Co., C.A.*, 190 *West George Street, Glasgow.*

1895 [1]Dick, James, *Scottish Accident Life and Fidelity Insurance Company, Ltd.*, 115 *George Street, Edinburgh.*

1902 [1]Douglas, David Lawson Scott, *Standard Life Assurance Company*, 102 *Hope Street, Glasgow.*

1905 [2]Drake, John William, Jun., *Britannic Life Assurance Company, Ltd., Fern Bank, Wisewood, Sheffield.*

1899 [2]Elgin, George Eric, *Scottish Equitable Life Assurance Society*, 28 *St. Andrew Square, Edinburgh.*

1896 [1]Fairman, Harry Albert, *Scottish Equitable Life Assurance Society*, 28 *St. Andrew Square, Edinburgh.*

1889 [3]Farnworth, Edward James, F.S.S., F.S.A.A., 26 *Winckley Square, Preston.*

1891 [2]Ferguson, Walter Wolston, *Gas Commissioners' Office*, 25 *Waterloo Place, Edinburgh.*

1897 [1]Flett, John Edmund, *Scottish Provident Institution*, 6 *St. Andrew Square, Edinburgh.*

1901 [1]Forrest, Douglas, *Scottish Equitable Life Assurance Society*, 28 *St. Andrew Square, Edinburgh.*

1904 [2]Fyfe, Alexander Adair, *Messrs. J. and C. Logan and Steven, C.A.*, 82 *Gordon Street, Glasgow.*

1900 [1]Geoghegan, John Michael, C.A., *Messrs. Barstow & Millar, C.A.*, 30 *York Place, Edinburgh.*

1905 [1]Gerrish, Frank Wilfrid, B.A. (Cantab.), *Minerva Villa, Albert Road South, Buckhurst Hill, Essex.*

Date of Enrolment.

1906 [2]Gibb, John Francis Pillans, *North British and Mercantile Insurance Company*, 64 *Princes Street, Edinburgh.*

1902 [3]Goldie, William, Jun., *City of Glasgow Life Assurance Company*, 30 *Renfield Street, Glasgow.*

1901 [2]Gordon, James Alison, *Messrs. John Mann & Son, C.A.*, 142 *St. Vincent Street, Glasgow.*

1902 [1]Granger, Charles Keith, *City of Glasgow Life Assurance Company*, 30 *Renfield Street, Glasgow.*

1905 [3]Grant, Frederick John, *Edinburgh Life Assurance Company*, 12 *King Street, Manchester.*

1906 [2]Green, Alexander John, *Scottish Provident Institution*, 6 *St. Andrew Square, Edinburgh.*

1905 [2]Halcrow, Malcolm, *Scottish Union and National Insurance Company*, 35 *St. Andrew Square, Edinburgh.*

1902 [1]Hardie, Steven James Lindsay, *Messrs. Hardie & Grieve, C.A.*, 157 *St. Vincent Street, Glasgow.*

1903 [1]Harrower, Archibald, *Northern Assurance Company*, 20 *St. Andrew Square, Edinburgh.*

1899 [1]Hart, William Frederick, 13 *Balmoral Crescent, Queen's Park, Glasgow.*

1905 [2]Heron, David, M.A., *Viewbank, New Scone, Perth.*

1870 [2]Hobbs, Samuel James Symons, *Scottish Amicable Life Assurance Society*, 35 *St. Vincent Place, Glasgow.*

1906 [2]Hogg, Alexander Mackintosh, *Liverpool and London and Globe Insurance Company*, 119 *St. Vincent Street, Glasgow.*

STUDENTS

The figures indicate the number of Examinations to be passed for qualification as Associate.

Date of Enrolment.

1893 [1]Honeyman, David Livingstone, C.A., 18 *St. Swithin's Lane, London, E.C.*

1898 [1]Hope, Francis Moffat, *Caledonian Insurance Company*, 19 *George Street, Edinburgh.*

1905 [2]Jackson, John Bell, *North British and Mercantile Insurance Company*, 64 *Princes Street, Edinburgh.*

1904 [2]Jeffrey, Robert, Jun., *Scottish Provident Institution*, 6 *St. Andrew Square, Edinburgh.*

1906 [3]Jones, Henry Edwin, *Scottish Union and National Insurance Company*, 35 *St. Andrew Square, Edinburgh.*

1887 [1]Kay, Thomas Chalmers, *English and Scottish Law Life Assurance Association*, 41 *Charlotte Square, Edinburgh.*

1902 [3]Kirkpatrick, Robert, *Norwich Union Life Insurance Society, Norwich.*

1904 [2]Knox, George Cruden, *Northern Assurance Company*, 1 *Union Terrace, Aberdeen.*

1903 [1]Kyd, James Gray, *Northern Assurance Company*, 1 *Union Terrace, Aberdeen.*

1900 [1]Lang, John Charles, *Assistant Secretary, City of Glasgow Life Assurance Company*, 5 *Lyons Range, Calcutta.*

1896 [1]Leggatt, Henry James, *Alliance Assurance Company, Ltd., Cape Town.*

1904 [2]Linzmeyer, Louis, F.A.S., *Manhattan Life Insurance Company*, 66 *Broadway, New York, U.S.A.*

1893 [1]Little, John Robert, *Assistant Secretary, Century Insurance Company, Limited*, 18 *Charlotte Square, Edinburgh.*

1906 [3]Lord, Wilfrid, *Messrs. Ronaldson & Gordon, C.A.*, 3A *North St. David Street, Edinburgh.*

Date of Enrolment.

1896 [1]Lorimer, Norman William, *North British and Mercantile Insurance Company*, 64 *Princes Street, Edinburgh.*

1903 [2]Macfarlane, John Miller, *North British and Mercantile Insurance Company*, 102 *St. Vincent Street, Glasgow.*

1906 [2]Macfarlane, William, *City of Glasgow Life Assurance Company*, 30 *Renfield Street, Glasgow.*

1891 [1]Macgibbon, James Morrison, *Alliance Assurance Company, Bartholomew Lane, London, E.C.*

1898 [1]Maclachlan, William Macneille, C.A., *Pelican and British Empire Life Office*, 70 *Lombard Street, London, E.C.*

1901 [2]M'Laren, Alexander, *South African Mutual Life Assurance Society, Darling Street, Cape Town.*

1901 [3]Macnair, Donald John, *Edinburgh Life Assurance Company*, 22 *George Street, Edinburgh.*

1905 [2]Maclean, Alexander Tweedie, *City of Glasgow Life Assurance Company*, 30 *Renfield Street, Glasgow.*

1904 [1]M'Naught, James Provan, *Scottish Temperance Life Assurance Company, Ltd.*, 105 *St. Vincent Street, Glasgow.*

1903 [2]MacPhail, Angus Donald, *Scottish Life Assurance Company, Ltd.*, 19 *St. Andrew Square, Edinburgh.*

1903 [2]MacPherson, John Reid, *English and Scottish Law Life Association*, 41 *Charlotte Square, Edinburgh.*

1895 [3]Malloch, John Smith, *Edinburgh Life Assurance Company*, 22 *George Street, Edinburgh.*

STUDENTS

The figures indicate the number of Examinations to be passed for qualification as Associate.

Date of Enrolment.

1905 [3]Marshall, John Cooke,
Scottish Life Assurance Company, Ltd., 19 *St. Andrew Square, Edinburgh.*

1905 [2]Meiklem, William Halliday,
City of Glasgow Life Assurance Company, 30 *Renfield Street, Glasgow.*

1905 [3]Millar, Alexander Mitchell,
Caledonian Insurance Company, 19 *George Street, Edinburgh.*

1898 [2]Millar, William,
City of Glasgow Life Assurance Company, 30 *Renfield Street, Glasgow.*

1890 [2]Miller, Robert Mitchell,
Edinburgh Life Assurance Company, 22 *George Street, Edinburgh.*

1906 [3]Milne, Francis James,
Caledonian Insurance Company, 19 *George Street, Edinburgh.*

1905 [1]Moodie, John Robert,
City of Glasgow Life Assurance Company, 30 *Renfield Street, Glasgow.*

1899 [2]Morham, Malcolm,
Liverpool and London and Globe Insurance Company, 1 *Dale Street, Liverpool.*

1894 [1]Muir, Arthur Henry, C.A.,
Scottish Provident Buildings, 2 *Wellington Place, Belfast.*

1889 [1]Niven, John Ballantine, C.A.,
30 *Broad Street, New York, U.S.A.*

1902 [1]Oliver, John Pringle Pattison,
North British and Mercantile Insurance Company, 64 *Princes Street, Edinburgh.*

1902 [1]Paterson, George Watson,
General Accident Fire and Life Assurance Corporation, General Buildings, Perth.

Date of Enrolment.

1906 [2]Patrick, Walter Sydney,
Britannic Assurance Company, Limited, Broad Street Corner, Birmingham.

1899 [1]Pearson, Joseph Gilmour,
Scottish Union and National Insurance Company, 35 *St. Andrew Square, Edinburgh.*

1904 [3]Pulford, William Henry Edward,
16 *Spottiswoode Road, Edinburgh.*

1906 [3]Ranken, George,
Scottish Equitable Life Assurance Society, 28 *St. Andrew Square, Edinburgh.*

1896 [1]Revell, Charles John, Mus. Bac. (Oxon.),
10 *Waterloo Place, Edinburgh.*

1888 [1]Rintoul, Peter, C.A.,
212 *West George Street, Glasgow.*

1899 [2]Robertson, David M'Intosh,
Scottish Metropolitan Life Assurance Company, Ltd., 25 *St. Andrew Square, Edinburgh.*

1901 [1]Robertson, John Howard, M.A.,
University Life Assurance Society, 219 *St. Vincent Street, Glasgow.*

1901 [1]Russell, James Wemyss,
Liverpool and London and Globe Insurance Company, 1 *Dale Street, Liverpool.*

1899 [2]Scott, Peter Martin,
Norwich Union Life Insurance Society, Norwich Union Chambers, Glasgow.

1901 [1]Sellar, Alexander Smith, M.A.,
City of Glasgow Life Assurance Company, 30 *Renfield Street, Glasgow.*

1906 [3]Simpson, James Dyer,
Liverpool and London and Globe Insurance Company, 119 *St. Vincent Street, Glasgow.*

1893 [2]Sinclair, Thomas Ord, C.A.,
Finnart Road, Greenock.

STUDENTS

The figures indicate the number of Examinations to be passed for qualification as Associate.

Date of Enrolment.

1901 [3]Smith, Arthur William,
Edinburgh Life Assurance Company, 126 *St. Vincent Street, Glasgow.*

1902 [1]Smith, John Tasker,
119 *Plantation Street, Accrington, Lancashire.*

1904 [2]Spence, John Dick White,
Messrs. Dawson & Macgregor, C.A., 34 *York Place, Edinburgh.*

1906 [3]Strachan, Benjamin,
15 *Wellington Street, Edinburgh.*

1903 [1]Strathie, David Norman,
8 *Bowmont Terrace, Kelvinside, Glasgow.*

1902 [2]Sutherland, William Robertson,
Life Association of Scotland, 82 *Princes Street, Edinburgh.*

1905 [2]Tavender, Sydney John,
Scottish Amicable Life Assurance Society, St. Andrew Square, Edinburgh.

1898 [3]Walbank, John Arthur, C.A.,
34 *Grey Street, Newcastle-on-Tyne.*

1903 [1]Walker, William Gray,
Liverpool and London and Globe Insurance Company, 119 *St. Vincent Street, Glasgow.*

1894 [1]Watson, Thomas Jackson,
Messrs. Chalmers & Watson, C.A., 3 *North St. David Street, Edinburgh.*

1892 [1]Webster, Thomas,
617 *Birkbeck Bank Chambers, London, W.C.*

Date of Enrolment.

1889 [1]Weston, Wilfred Walter,
Life Association of Sco[illegible]
Princes Street, Edinburg[illegible]

1893 [1]Wilson, James Hugh Ell[illegible]
P.O. Box 771, *Cape Tow[illegible]*
Africa.

1905 [1]Wilson, John Stark Ben[illegible]
49 *Balgreen Road, Murr[illegible]*

1890 [1]Wilson, Robert Hodgson,
Resident Secretary, Edi[illegible]
Life Assurance Company,
St. Vincent Street, Glasgow.

1897 [1]Wink, Frank Adam,
Scottish Equitable Life Assur[illegible]
Society, 28 *St. Andrew Sq[illegible]*
Edinburgh.

1896 [3]Winning, James Wills,
Messrs. John Wilson & Sto[illegible]
154 *St. Vincent Street, Glas[illegible]*

1905 [2]Wishart, William Muir,
Scottish Metropolitan Life A[illegible]
surance Company, Ltd., [illegible]
Andrew Square, Edinburg[illegible]

1899 [1]Wood, Robert,
Northern Assurance Com[illegible]
1 *Union Terrace, Aberdeen.*

1898 [1]Wybar, John,
Scottish Amicable Life Assur[illegible]
Society, 35 *St. Vincent P[illegible]*
Glasgow.

1906 [3]Wyse, William Edwin,
Edinburgh Life Assurance Co[illegible]
pany, 22 *George Street, [illegible]*
burgh.

1906 [2]Young, David Goldie,
Scottish Life Assurance Co[illegible]
pany, Limited, 19 *St. Andr[illegible]*
Square, Edinburgh.

Vol. III. Part VI.

TRANSACTIONS

OF THE

FACULTY OF ACTUARIES.

CONSTITUTED 1856. INCORPORATED 1868.

No. 30.

THE ACTUARY IN SCOTLAND:

Inaugural Address, Session 1906-7.

By the President, Archibald Hewat.

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FARRINGDON STREET.
1908.

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TRANSACTIONS

OF THE

FACULTY OF ACTUARIES.

VOL. III.—PART VI.

No. 30.

THE ACTUARY IN SCOTLAND:

Inaugural Address, Session 1906-7,

By the President, Archibald Hewat.

LONDON:
CHARLES & EDWIN LAYTON,
FARRINGDON STREET.
1906.

[*Entered at Stationers' Hall.*]

TRANSACTIONS

OF THE

FACULTY OF ACTUARIES.

THE ACTUARY IN SCOTLAND: *Being the Inaugural Address—Session 1906-7—by the President,* ARCHIBALD HEWAT, *Manager and Actuary of the Edinburgh Life Assurance Company.*

[Delivered 5th November 1906]

IN presiding at the opening meeting of a new Session I avail myself of this, the earliest, opportunity of thanking you for having called me to occupy the highest position in this Faculty to which one may aspire. While I greatly appreciate the honour thus conferred I assure you I do none the less realise the responsibility involved. It shall be my earnest endeavour to discharge, as best I can, the duties appertaining to the position and to guard the interests, advance the objects, and promote the welfare of the Faculty, as opportunities may arise. In this I know I may confidently rely upon the loyal support of every Fellow and, more particularly, upon that of every Member of Council.

In entering upon the duties of the high office to which you have elected me I naturally think of those who have preceded me in this Chair. I have personally known each one of them, and it has been my good fortune to have served under four of them. While, in much else, I readily yield to them the palm, I would be untrue to myself and to my grateful sense of the honour you have now done me if I did not claim at least as ardent an affection for, as devoted a loyalty to, and as earnest a desire to further the interests of the Faculty as the best of them. "As in a theatre, the eyes of men, after a well-graced actor leaves the stage, are idly bent on him that enters next, thinking his prattle to be tedious" I crave your indulgence as I now enter on the stage,

and I ask your forbearance as I now prattle during the time at my disposal for this Address, which I shall endeavour to make as little tedious as the limitations of my subject will allow. Perhaps it will be found, ere I close, that I may have only "broken old ground with a new spade"—but even that is something in these busy days, when, alas! it would almost seem as if there were "no shelter to grow ripe, no leisure to grow wise."

You could not have chosen a time more agreeable to myself than the present at which to enter upon my duties as President; for, to-day, we stand in a peculiarly interesting position. The echoes of our jubilee celebration of ten months ago seem hardly to have yet died away, reminding us that we have closed the first half-century of our history as a constituted Faculty. To me the forward look is always much more stimulating and inspiring than the backward. "Forgetting those things which are behind"—except their precious lessons—we now press forward.

As a Faculty we have a past of which we may well be proud; full of valuable experience, the result of much good work accomplished, which cannot fail to give impetus to our efforts, stimulating enterprise and giving good hope for still greater and better things in the future.

Standing thus on the threshold of a new half-century I take for my theme—THE ACTUARY IN SCOTLAND. In dealing with it I shall give a retrospective glance—tracing, through his work, something of his origin, rise, and progress—so that, like the athlete with his momentary step backward, we may the better take the leap forward towards what he may yet become under the fostering care of the Faculty of Actuaries in Scotland, and with ever-increasing demands for his professional advice and guidance. The life of incorporations, as of individuals, is but "a series of new beginnings."

By sheer force of circumstances the Scot is, and has long been, a citizen of the world, a traveller, a pioneer, a founder of empires, and a successful administrator in the State and in the Church, in the industries and in the finances of nations, as well as of the Insurance Companies of the world. These last, in their multifarious forms, their enormous resources, and the vastness of their influence for good, have taken an admittedly high place among the civilising agencies of the world.

It was a Scotsman who projected the Bank of England, which received its charter of incorporation in 1694; one year before the Bank of Scotland was instituted by a charter of incorporation from the Scots Parliament.

John Coutts, Lord Provost of Edinburgh (1742-1744), founded in this city the famous banking-house of Coutts & Company—"the earliest private banking-house in Scotland"—which, on the

separation of the Edinburgh Office and the London Office, the latter having long been a branch, was succeeded here by Sir William Forbes' banking-house, in Parliament Close (*now* Square), under the shadow of St. Giles' Cathedral; William Forbes having himself served his apprenticeship under John Coutts, in whose bank he had become a partner. Sir William's bank, in turn, was succeeded by what is now known as the Union Bank of Scotland.

We have this year been celebrating the quater-centenary of George Buchanan, who has been called "the type and exemplar of the wandering Scot"; and of whom it has been said "he was of the adventurous band which made its country's name glorious for arms and arts from one end of Europe to another." "The hard life of an [then] inhospitable land sent forth the many profound scholars and brave soldiers who taught and fought wherever there was a professor's rostrum or a field of battle." It was also said that "the Scots abroad proved themselves as nimble with their swords as with their brains"; of one of whom, in those bygone days, it was said he "fought mathematically and professed mathematics like a soldier." Possibly this was an early type of the Actuary—be he Manager or Branch Secretary—who "in the crowded haunts where man encounters man in daily strife" fights hard in these days of keen competition for business for his Company, at the same time professing mathematics as he must do holding the Faculty's Certificate of Competency.

We are also this year remembering the two hundredth anniversary of the Treaty of Union * between England and Scotland. These two neighbour—if not always neighbourly—kingdoms had a common interest in union; to the one it gave security and to the other a wider and freer commerce. It was said by a writer in the year of the union that "the English are so bent upon securing the backdoor against enemies and the Scots so bent upon opening the foredoor for an outlet into England"—of which latter the Scottish Actuary has not failed to avail himself freely, while at the same time—emulating his "brither Scots" in past times as in the present, and in other spheres of labour—he has gone forth a wandering Scot to the ends of the earth to practise his profession and to add lustre to the Faculty through whom he has qualified; whose imprimatur or hall-mark he has earned and has duly received. Lord Chesterfield, in one of his Letters, pointed out that "education, more than nature, is the cause of that difference you see in the character of men"; and Lyttleton has truly observed that "the object of all education is to endow the mind of youth with strength, accuracy and elevation, and ability to express thought with clearness and attractiveness."

Whether because of the poverty of the land, in former days, or

* which was signed on the 22nd July 1706.

because of the geographical and numerical smallness of his kingdom—like Palestine of old to the Jew—the Scot has long had the credit of being a frugal person, shrewd and thrifty, who husbands resources for future eventualities, cautiously enterprising withal. All this and more I suppose we must attribute to the "plain living and high thinking" on which he is said to be reared.

It is to the prudent and thrifty Scot, who has realised that "the bedrock of prosperity in nations and municipalities is the due consideration of finance," that the world owes much that is best and soundest in the development and management of Insurance, Banking, and Railway Companies; not to mention other joint-stock concerns, and commerce generally. Our Scottish-trained bankers, insurance and railway officials, as well as our Scottish-trained merchants, are to be found in all the habitable parts of the earth.

The Scot has largely developed in him the gift of finance—using that term in its widest sense. He has devoted much time, attention, and skill specially to institutions founded for the encouragement of thrift in its many forms, and to the successful development of schemes of provision for the future, for himself and his dependants—especially to Friendly Societies, Widows' Funds, and Life Assurance Companies. Few know better than the Scot that the family, love of kith and kin, is the true foundation of prosperous national life.

In the year 1810 the Church of Scotland minister in the rural parish of Ruthwell, in Dumfriesshire, established a "Parish Bank Friendly Society," which, however, was more akin to our Savings Banks, as we now know them, than to what are known as Friendly Societies. "A minute account of its organisation and mode of operation drew so much attention to it that, by 1817, there were 78 establishments resembling it in the United Kingdom." In Scotland alone there are now 59 Trustee Savings Banks, with deposits amounting to nearly £18,250,000.

FRIENDLY SOCIETIES, as we all know, are of very ancient origin—how ancient it is not my intention to investigate on this occasion—and have long found a congenial home in Scotland and among Scotsmen "furth of the kingdom." In the year 1771 the actuary-clergyman, Dr. Price, of *Northampton* Tables' celebrity, wrote, in his *Observations on Reversionary Payments*, that "the lower part of mankind are objects of particular compassion when rendered incapable by accident, sickness, or age, of earning their subsistence," adding that "this has given rise to many very useful Societies among them for granting relief to one another out of little funds supplied by weekly contributions." In that same year a Society of that kind was founded in Glasgow—by no means the earliest in Scotland—setting forth in its "preamble"

the following pious sentiment; "endowed by infinite wisdom and goodness with dispositions and principles which prompt mankind more especially in a Christian community to associate together for their mutual relief and support to promote the comfort and alleviate the sufferings of their fellow-creatures by lightening the pressure of sickness and the infirmities of age, a number of persons . . . agreed . . . to unite themselves for the above purposes."

The first attempt to collect data by which to estimate the sickness experienced in Friendly Societies and to frame tables thereon was, I believe, that made by the *Highland Society of Scotland*, in 1824. A similar attempt was made in England, in 1835, by the *Society for the Diffusion of Useful Knowledge*, but both of those tables were found to be unreliable and have long since been disused, better having been provided.

It was my intention to have instituted a comparison between the number of Friendly Societies in Scotland and the amount of their Funds when our Faculty was constituted, fifty years ago, with the same at the present time; but I regret to find that the information relating to the former period is not available in any useful form. I have, however, ascertained that the number of registered Societies in Scotland (including Branches of registered Orders) is now over 2000, their Funds amounting to nearly £3,500,000.

These Societies afford a considerable amount of interesting work for the Actuary in Scotland, in advising as to rules and rates, investigating sickness, mortality, and other "experience," and in conducting periodical valuations required by the Friendly Societies' Acts. The two Public Valuers in Scotland, appointed by the Lords of His Majesty's Treasury, are Fellows of our Faculty, and, besides these, fourteen of our Fellows are "actuaries authorised to certify annuity tables," under the *Friendly Societies' Act*, 1896. These, with our other Fellows, are the modern representatives of what in a Bill, in 1818, were called "persons skilled in arithmetical calculations," but who, when that Bill was passed in 1819, had their designation enlarged to "persons known to be professional actuaries, or persons skilled in calculation"—the village schoolmaster, in early days, frequently coming in under the latter and less definite part of the description.

As Dr. Price truly remarked, Friendly Societies were intended more particularly for "the lower part of mankind"—now sometimes known as "the masses"—and to be maintained by such out of their *little* funds supplied by *weekly* contributions. These Societies have long occupied, and perhaps never more so than now, a most important and useful place among the provident institutions of our country. While serving a good purpose in the performance of their more immediate functions—aliment in

sickness and in old age, and provision for defraying funeral expenses—among the numerous wage-earning members of the community, for whose benefit they were specially founded, they have also served in great measure as an incentive to the larger and better provision which has been found in the Annuity Schemes and Funds of our forefathers for the benefit of Widows and Orphans, and now, in still greater fulness, in Life Assurance in all its varied and convenient forms. This process of education in thrift and prudence is going on steadily year by year; the wage-earner, as he prospers and rises in the social scale, naturally seeking the further protection and the many advantages so readily available to him by means of modern Life Assurance as he can now obtain it in all its attractiveness of stability, elasticity, certainty, and profit.

This brings me now to refer to the WIDOWS' FUNDS in connection with which also the services of the Actuary have been so necessary, and, let me add, so beneficial. Most of the best known of those in Scotland were founded towards the close of the eighteenth century and in the early part of the nineteenth. Their foundations were, for the most part, "well and truly laid" by shrewd, far-seeing men, who made the best use of the comparatively few and ill-assorted materials then available for their purpose. The founders had regard to the sturdy independence of those who desired to make some provision for their possible widows, consequently they based their schemes, not on vague eleemosynary ideas, but on as sound financial lines as were then possible. The earliest of them was, I suppose, that of the *Fund for a Provision for the Widows and Children of the Ministers of the Church and Professors in the Universities of Scotland*, founded in this City so far back as the year 1744. There being no Actuaries in those days, as we now know them, the Doctor of Divinity who conceived the idea of establishing such a fund called to his aid, in laying the foundations, the combined skill of a mathematician and of an accountant—a blend now known as an actuary—which he found in the persons, respectively, of the Professor of Mathematics in the University of Edinburgh and of the Accountant-General of the Board of Excise for Scotland, who was also Accountant to the City of Edinburgh. Those two "persons skilled in calculations" were sandwiched in between the Lord Provost of this City and the Edinburgh minister referred to—a foursome of such weight and influence that you will not be surprised to know that this ancient Fund has always been in a prosperous condition during its useful career of one hundred and sixty-two years, and never more so than now. Not only so, but, profiting by its successful record of more than a century, it served as a model, in 1846, for the *Ministers' Widows' and Orphans' Fund of the Free Church of Scotland*,

including, as that one does, not only the Ministers of that Church and the Professors in its Colleges, but also, in recent years, its Missionaries abroad. Each of these Funds is, like some of the others, managed under the regulations and the protection of Acts of Parliament. When this Faculty was constituted, fifty years ago, these Funds had between them 1950 Contributors and 350 Annuitants, the latter receiving Annuities ranging from £22 to £46 each; the Funds then amounting to £295,000 or thereby. Now the Contributors number almost 3250, the Annuitants rather more than 750, and the Annuities ranging from £29 to £53—the Funds amounting to nearly £930,000. Fifty years ago these two Funds were paying nearly £13,500 to the Annuitants, while now they are paying them so much as almost £30,500 per annum.

The close connection which had in the past subsisted between the Church and the School in Scotland naturally suggested a Scheme for Schoolmasters also, and, accordingly, the *Fund for the Relief of the Widows and Children of Burgh and Parochial Schoolmasters in Scotland* was founded in 1807. But there came before it, in 1792, the *Widows' Fund of the Faculty of Physicians and Surgeons in Glasgow*; and there came, in 1813, the only other one in the medical profession which I can trace, that of the *Royal College of Surgeons*, Edinburgh. The legal profession came next with that of the *Writers to His Majesty's Signet* [W.S.] in 1803; the *Faculty of Procurators in Glasgow* in 1812; the *Solicitors before the Supreme Courts in Scotland* [S.S.C.] in 1817, and the *Faculty of Advocates*, Edinburgh, in 1830. The Funds which I have now named have reference, it will be observed, to what were anciently known as "the learned professions; law, medicine, and divinity"—that of the Actuary not having yet arisen as a distinct and separate "learned profession," as our searching examinations and our extensive literature now entitle us to claim it to be.

The employees of the Scottish Banks having their Head Offices in Edinburgh have also their Widows' Funds—that of the *British Linen Bank* dating from 1808, of the *Royal Bank of Scotland* from 1820, of the *Bank of Scotland* from 1821, of the *National Bank of Scotland* from 1851, and of the *Commercial Bank of Scotland* from 1866.

The *Merchant Company of Edinburgh*—a corporation dating from 1681—commenced its Widows' Fund in 1827. Besides this, and a comparatively small and limited, almost semi-private, Fund of less than £30,000, I do not know of any important Widows' Fund connected with the mercantile community in Scotland.

Having already given the figures relating to the Ministers' and Professors' Widows' Funds, I shall now give those of the other Funds which I have named. The Schoolmasters' Widows' Fund had, when our Faculty was constituted, upwards of 1250 Con-

tributors and over 200 Annuitants; the Funds amounting to nearly £76,000, and the Annuities ranging from £6 to £30. Now the Contributors number only 260, while the Annuitants have increased to 272—the Annuities ranging from £9 to £45, and the Funds amounting to nearly £92,500. Fifty years ago a little over £3000 was being paid in Annuities, now nearly £5750 per annum is being so paid. As this Fund was closed to new entrants many years ago, it is gradually working itself out; the Contributions becoming smaller year by year, while the sum payable in Annuities still goes on increasing, as it will do until a maximum is reached, when, like the Contributors, the Annuitants also will steadily decrease in number.

The Medical Widows' Funds, above referred to, had, fifty years ago, 140 Contributors and 66 Annuitants—the Annuities ranging from £45 to £60; the Funds being a little over £82,000. The one ceased to exist when it was taken over, in 1890, by an Edinburgh Insurance Office, and the other ceased to exist when it was taken over, in 1897, by a Glasgow Insurance Office.

The Lawyers' Widows' Funds referred to had among them, fifty years ago, 1024 Contributors and 219 Annuitants, the Annuity ranging from £30 to £85, and the Funds amounting to nearly £380,000. Now they have 1367 Contributors and 282 Annuitants, the Annuity ranging from £52 to £115, and the Funds amounting to more than £791,000. The Faculty of Procurators' (Glasgow) Fund was closed to new entrants many years ago and is now gradually working itself out. While these Lawyers' Funds were paying among them £13,785 in Annuities, fifty years ago, they are now paying upwards of £19,250 per annum.

The Banks' Widows' Funds referred to had among them, fifty years ago, 430 Contributors and 45 Annuitants, the Annuities ranging from £12, 10s. to £50, and the Funds amounting to nearly £64,000. Now they have nearly 2800 Contributors and 325 Annuitants, the Annuities ranging from £25 to £100, and the Funds amounting to almost £780,000. While they were paying Annuities at the rate of nearly £1900 per annum, fifty years ago, they are now paying almost £20,000 per annum.

The Merchant Company's Widows' Fund amounted to a little over £46,000 fifty years ago, the Contributors then numbering nearly 230 and the Annuitants about 95, the Annuity then being £25. Now the Fund amounts to fully £220,000, the Contributors having increased to nearly 600, the Annuitants to over 180, and the Annuity to £42.

These Widows' Funds have now accumulated, among them, almost £2,850,000—an increase of nearly £2,000,000 during the half-century. They have all along been periodically investigated

and reported upon by as qualified Actuaries as the times and circumstances have provided; always by Fellows of this Faculty since it was constituted. The utmost care has all along been taken to maintain the solvency of these Funds and to safeguard the interests of all connected with them. Their sound and prosperous condition to-day reflects great credit upon all who have been responsible for their management during the now many decades of their existence. The early efforts of which these still-subsisting Widows' Funds and the existing Friendly Societies are the excellent results were, as it has proved, but the groping after a larger, more comprehensive, and more acceptable scheme of provision for all sorts and conditions of dependants—mothers and sisters, for example, as well as widows and children, not to mention estate-duties, creditors, and other beneficiaries; including even the assured himself, who can now by means of Annuity or Endowment-Assurance — in their many attractive forms —facilitate his own retirement from the more active duties of business life.

The rise and progress of the Life Offices, to which I am about to refer, with their doors open to all classes, and with their Assurance, Endowment, and Annuity schemes to meet the varying circumstances of all, practically put a stop to the founding of Widows' Funds, a more excellent way having thus been discovered. Not only did this check the founding of Widows' Funds, but it caused some of the Funds to remove, during the past half-century, the regulation which made it obligatory on the members of their respective Corporations or Societies to become Contributors to their Widows' Fund. Where this relaxation of rule has been given effect to growth of the Funds has ceased and a process of gradual extinction has commenced.

I come now to the great business of LIFE ASSURANCE, with which the Actuary is more immediately associated. In the year 1855 there were fifteen Scottish Insurance Offices transacting Life business—eleven in Edinburgh, two in Glasgow, and two in Aberdeen; six of these transacting Fire business also. Three of those fifteen Offices amalgamated with other three of them, thus reducing the number to twelve. To those, however, there fall to be added six new Scottish Offices founded since 1855—the earliest of them dating from 1876—making now eighteen Scottish Offices; thirteen in Edinburgh, three in Glasgow, one in Aberdeen, and one in Perth, transacting Life Assurance, Endowment, and Annuity business. But, unlike the majority of those in existence fifty years ago and still flourishing, none of these are purely Life Offices—fire, accident, sickness, burglary, fidelity-guarantee, and other departments of insurance business being carried on by them along with the life business. The paid-up Capital

of the Scottish Offices fifty years ago slightly exceeded £750,000, while now it somewhat exceeds £5,600,000. The Life Assurance and Annuity Funds were then a little more than £8,900,000, while now they only slightly fall short of £93,000,000. The Premium Income was then rather less than £1,300,000, while now it is not far short of £6,600,000. The Interest Income was then under £360,000, while now it exceeds £3,550,000. A truly marvellous growth during half a century—and, be it observed, I am referring only to Offices having their headquarters in Scotland, the population of which is less than five-sevenths of that of London with its suburbs. The eighteen Scottish Offices are now paying in claims under Life Assurance and Endowment policies nearly £5,500,000 per annum and considerably over £750,000 per annum to Annuitants. They also possess among them "other funds" belonging to the Life Department, amounting to upwards of £500,000 beyond the £93,000,000 already referred to.

Friendly Societies, Widows' Funds, and Life Assurance Offices in Scotland with funds reaching, as we have seen, to the large sum of, say, £100,000,000, not only afford sufficient justification for the existence of the Faculty of Actuaries but, were there nothing else, they also provide ample employment and a very honourable sphere of usefulness to the Fellows thereof. It must not be supposed that it was the Faculty which called the Actuary into existence: it was rather the Actuary who created the Faculty. If it was not the Actuary who, in some primitive form, first called Friendly Societies, Widows' Funds, and Life Assurances Offices into existence, it is undoubtedly to him that they owe, for the most part, their development, their continued existence, and their promise of lasting stability.

He was a boatbuilder who, with the use of heated stones, hollowed out the trunk of a tree, trimmed it, and set it afloat to carry him and his few goods across a creek or along a lee shore. Primitive and humble though the effort was, that canoemaker was ancestor to the modern shipbuilder who now launches his powerful ironclads and his palatial liners to do battle in and to cross over oceans, in calm or storm, of which the early savage was utterly incapable of dreaming. While primæval man constructed his boat all by himself in its every detail our modern shipbuilder long ago learned the meaning and power of that greatest of modern aids to industrial development which we know as Division of Labour. So also the Actuary. We must not, however, allow ourselves to think of *him* having ever been in a savage state—even when lacking tables of logarithms and the many other tables he now has, besides his equipment of arithmometers, tabulating and sorting machines, card systems, typewriters, and suchlike mechanical aids to calculation, as well as the better educated and better

trained human agency he now has to assist him in his labours, and the larger mass of reliable statistical data now available to him.

There is one great Scotsman to whom the actuarial profession is much indebted. I refer to John Napier, Baron of Merchiston, the inventor of logarithms, who was born in Merchiston Castle—then "near Edinburgh," but now well within the City boundaries—fifty-three years before our Scottish King James VI. ascended the English throne as James I. of England—and who died in that same castle in the year 1617, leaving large estate. He came of an ancient Scottish family. A Napier ancestor of his was Lord Provost of this City in 1438, whose son—Comptroller of Scotland under James II. in 1450—was also Provost between 1469 and 1471. John Napier's father, Sir Alexander Napier of Merchiston and Edinbellie, was Master of the Mint to James VI.

I would here remind you of the scholarly work—*The Construction of Logarithms with a Catalogue of Napier's Works*—by a member of our Council, Mr. William Rae MacDonald, one of His Majesty's Officers of Arms, being a member of the Court of the Lord Lyon, the heraldic establishment of Scotland. He has translated from Latin into English the posthumous work whose title-page is as follows:—*The Construction of the Wonderful Canon of Logarithms; And their relations to their own natural numbers; with An Appendix as to the making of another and better kind of Logarithms, to which are added Propositions for the solution of Spherical Triangles by an easier method: with Notes on them and on the above-mentioned Appendix by the learned Henry Briggs. By the Author and Inventor, John Napier, Baron of Merchiston, &c., in Scotland. Printed by Andrew Hart, of Edinburgh; in the Year of our Lord*, 1619.

Those who have perused that volume of Mr. MacDonald's will, I am sure, agree with me when I say it is most interesting and well repays careful study. Baron Napier's mortal remains were buried, nearly three centuries ago, in St. Cuthbert's churchyard in this City. An elaborately carved marble mural tablet was erected to his memory, in 1842, in the vestibule of that Church. In 1898 the Fellows of this Faculty subscribed for a statue of him, which adorns one of the niches of the Scottish National Portrait Gallery. There he stands—in close proximity to most of our Scottish Insurance Offices—among Scottish kings and queens, nobles, statesmen, and ecclesiastics; representative makers of much of our Scottish history. A replica of the sculptor's model has since had a place in this Hall.

We Scottish Actuaries have a mixed ancestry, none the less but rather the more of which to be proud. We are evolved from many useful and dignified professions. In early days the accountant, the lawyer, the banker, the insurance official, the

schoolmaster, and the professional mathematician performed, among them, practically all that was then expected of or was possible to an Actuary. I have in the course of my professional reading and work been much struck with the care and skill of those men, as exhibited in their reports upon Funds and Schemes which they had investigated—before the Faculty came into being; and I have marvelled at the results, remembering the paucity and comparatively unwieldy character of the tools which alone they then possessed. It well becomes us of this generation to look back with much respect and gratitude to those early pioneers and ornaments of our profession.

Not to wander too far back I would now merely remind you that about sixty years ago the professions in Scotland began to specialise—the actuary sorting himself out from the lawyer, the banker, the accountant, the insurance official, and the "business man." I am happy to say we still have among our Fellows to-day those who belong to each of those professions and who have added that of Actuary to their other professional qualifications.

I hope you have been able to follow me as I have endeavoured to trace or evolve the Actuary in Scotland through what has been his principal work. The work and the Actuary have grown up and developed together, the one acting and re-acting on the other. But there are here in Scotland, as elsewhere, wider fields which he is well qualified to occupy, and which he ought to occupy sooner than later. A too common mistake has been made, for which I fear the Actuary himself is somewhat to blame, in narrowing or limiting his sphere; which has a much wider horizon than some of us have yet realised. To enlarge the view and deepen the interest one has only to be present at one or more of our great INTERNATIONAL CONGRESSES OF ACTUARIES, which, meeting triennially, have already had great and increasing gatherings and discussions in Brussels, London, Paris, New York, and Berlin; the next having been appointed to be held in Vienna, and the following one probably in Amsterdam. I had the honour, with Mr. Chatham, our Honorary Treasurer, and five of our Fellows, of representing, as delegates, the Faculty at the Congress which held its meetings in Berlin in the Reichstag Building, which was kindly placed at our disposal by the President of the Reichstag during the week from 10th to 15th September last—the German Government also, I understand, giving a substantial grant in money towards the expenses of the Congress. There were altogether seventeen Fellows of this Faculty who were members and attended the meetings of that Congress. Six of our Fellows contributed papers to the Congress and, besides these, others of us took part in the discussions. These Papers and our speeches may be read in the volume of the *Proceedings*, which will in due time

be found in our library here. I commend these to the careful study of our members and students. We were most cordially welcomed and very hospitably entertained. The Emperor sent us a gracious message of welcome through the Under-Secretary of State, who, recognising us as exponents of actuarial science as well as the international character of the Congress, referred to us as seekers after truth who, in our intercourse and discussions, would know no frontiers. The General Secretary, Professor Manes, remarked to me how very pleased he was to find such a large representation from the Faculty; and the President of the Congress, Dr. Hahn, paid the Faculty the compliment of inviting your President to occupy the Chair at one of the afternoon meetings. Not only so, but I was elected a Vice-President of the Congress, Mr. Chatham being at the same time elected a Secretary; both of us being also elected members of the Permanent Committee. The membership of the Berlin Congress was the largest on record, the number having reached upwards of five hundred and fifty—including delegates from almost every Government. The actuarial adviser of our Board of Trade, an Honorary Fellow of this Faculty, was one of the members of Congress who was present at the meetings. There were members and delegates from the Argentine Republic, Austria, Australia, Belgium, Canada, Chili, Denmark, England, France, Finland, Germany, Holland, Hungary, Italy, Japan, Ireland, Mexico, Norway, Portugal, Russia, Scotland, Servia, Spain, Sweden, Switzerland, the United States of America, etc.

But, to return to Scotland, the Actuary of our Life Offices here has generally held the official position of Manager of his Company, or, when that has not been so, he has always been in close relationship with the management. As "knowledge is power" the Actuary of the right sort—skilful, independent, tactful, a man of character—ought to be, and usually is, a trusted officer of his Company, able to guide the management, holding his own, respectfully but firmly, with Directors and Manager. When he combines in himself the duties at once of an executive and of an advisory officer, as is so frequently the case in Scotland, the most gratifying results have ensued. In America, where these two positions had unfortunately been divorced and where the Actuary has been too frequently relegated to a back room, as if he were a mere calculating machine or automaton, the business of Life Assurance has suffered. In the recent exposures of American scandalous mismanagement no blame, so far as I know, has been attributed to the Actuaries, who, to the credit of our profession be it said, have come well out of the ordeal.

Apart from our official duties as Actuaries to Life Offices, Widows' Funds, and Friendly Societies, we are frequently called upon, in a professional capacity, to assist in legal and other cases

and to appear as expert witnesses in our Supreme Courts and in Arbitrations—the highly technical and involved questions and calculations in connection with the Entail Acts, for example, having required in a special manner the skilled advice and guidance of the Actuary. We are also called in to advise in financial matters in which our municipal and other corporations are interested. It might be better for such corporations if we were more frequently called in, and at an earlier stage than sometimes happens. I have seen—we have all seen—the mistake made of calling in the Actuary at too late a stage; not to prepare Schemes on a sound financial basis, but to try to remedy the effects of mistakes which had been made by amateur, unqualified advisers, or mere arithmeticians. The Actuary we know is a scientific financier, who sees much further ahead than some others who pose as "financiers"—a much-abused term. As well might a man set about building a house without employing an Architect as, without first consulting an Actuary, embark upon Schemes which can only be safely reared upon a sound actuarial foundation. The man who thinks he saves an Architect's fee runs the risk of being entombed with his household in the ruins of his own folly.

An important and increasing department of the Actuary's work, in Scotland as elsewhere, is the advising upon PENSION FUNDS and SUPERANNUATION SCHEMES, with their periodical investigation and valuation. If there is one department of scientific finance more than another where the public need to be reminded of "the seven years of plenty and the seven years of famine" it is in this domain of superannuation—where the Contributions are present and immediate while the Benefits are future and remote. It is here, if anywhere, that the Actuary requires consummate skill, equalled only by the absolute firmness with which he must hold and drive home his views. It is better for the Actuary to stand abuse at the outset than for him to weakly make concessions which he ought to know must end in disaster—and that perhaps long after he is out of the way. The Actuary who lacks moral courage is a dangerous adviser, however much he may be esteemed as a "person skilled in calculation."

Let us Actuaries be always looking at the foundations and at the building in its every part and as a whole, ever making permanent stability our aim at all costs. Let us see to it that our principles are sound and that they are applied with judgment, tact, and firmness in daily practice. It may be truly said of us Actuaries that we are the widow's shield and the orphan's stay; a responsibility to which may we ever rise.

The Faculty, in its corporate capacity and through its individual members, has assisted in most of the LEGISLATION affecting the business of Life Assurance in the United Kingdom during the past half-century. I need not here enumerate or enlarge. I

would merely mention the greatest and most useful of all the Acts which have come into force during that period, and one where the work of the Actuary occupies a very prominent position. I refer to the *Life Assurance Companies' Act*, 1870, which has satisfactorily stood the test of upwards of thirty-five years. In submitting evidence on behalf of the Faculty to a Select Committee of the House of Lords, in July last, I availed myself of the opportunity of endeavouring to impress upon their Lordships the growing importance of the Actuary in relation to the business of Life Assurance, and I drew their attention to an obvious defect in the Act of 1870, viz. the absence of any definition of the term "actuary" referred to in the Act; as to which there need now be no difficulty, seeing that the Faculty of Actuaries, with its Royal Charter of 1868, and the Institute of Actuaries, with its Royal Charter of 1884, have been, for upwards of half a century, educating, training, examining, and turning out fully qualified Actuaries. It is on their Investigations and Valuations that the stability of British Life Assurance rests, and may confidently be trusted to rest in the future as in the past. We have no cause to regret the inquiry recently made by that Select Committee of the House of Lords to which I have referred. On the contrary, it has afforded much useful information to the public as to the great stability of our British Life Offices—Scottish and English alike—and as to the status of the Actuary in this country, to whom they owe so much. What we in this country have contended for, and that I think successfully, is summed up in two words—freedom and publicity—which have amply proved to have been not only the best means of protecting the interests of our policyholders but also the best means of developing, on the soundest lines, the great and beneficial business of Life Assurance. Under the ablest actuarial guidance the fullest benefits and advantages of which the institution is capable are to be had through our Home Offices. Neither they nor their Actuaries profess to give gold rings for the price of brass ones. The public should be content, as a guide, with the past results of good management and the present conditions, and have done with their foolish demands for "estimates" as to future results. These so-called "estimates"—generally based upon results arrived at under conditions which no longer exist—have done much to mar the fair fame of Life Assurance.

Thinking it might be useful and not without interest at this period in our history I have prepared a CLASSIFIED CATALOGUE of the published works of the members of our Faculty during the past half-century. It has necessarily cost me some labour to prepare this, and I think it will be found to be about as complete a list as it is possible for any one person to make. I have ranged far and wide for my information, and I have added the

results by way of an Appendix to this Address, leaving the list to bear its own testimony to the literary work of our Scottish Actuaries, alike in its volume and in the comprehensiveness of the variety of its subjects and the skilful treatment thereof. It must afford satisfaction to us all to find how much this Faculty has thus contributed to actuarial science in its many branches. I hope this may prove a stimulus to our younger members to come forward and follow the good example that has thus been shown them. "Experience is by industry achieved and perfected by the due course of time." There is no finality in actuarial science, nor any limit to its application. Huxley has defined Science as "organised common sense"; and, some one else, Genius as "the power of taking pains."

If in the past the work of the Faculty and its members has been formative, let us see to it that in the future it shall also be florescent.

Jointly with our friends of the Institute, the Faculty has brought out volumes showing the results of the recent investigation, by these incorporations, into the MORTALITY EXPERIENCE of assured lives and of annuitants in our British Life Offices, with Tables based thereon. With such a quarry to work in there is much which our younger members could do to still further build up that actuarial science upon which rests the stability of our Life Offices.

Seventeen of our members hold among them twenty-two University degrees (LL.D., D.Sc., B.Sc., M.A., and B.A.), and thirty are Chartered Accountants (C.A.); two are Directors, twenty-eight are (or were) Managers, twenty-two are (or were) Secretaries,* and fifty-two are (or were) Actuaries* of their Companies; some of those last holding that post conjointly with that of Manager or of Secretary. One hundred and fifteen of our members are resident in Scotland, forty-six in England, one in Ireland, nine in Australia, one in New Zealand, eight in the United States of America, two in Canada, four in South Africa, one in India, and one in China. Two of our Fellows who entered when the Faculty was constituted in 1856 are, we rejoice to know, still with us.

I am glad to find seventeen are engaged in Branch Offices. It would be better for our Fellows, for their Offices, for the public, and for Life Assurance generally, if more of them could be so fortunate as to spend several years there, and so acquire—what cannot be had from books, or study, or even in a Head Office—a practical knowledge of what is sometimes called "outside" work, or the commercial department of our business. It would broaden their views and deepen their sympathies to thus come in contact with agents, policyholders, and the public, and so find out

* including Assistant Secretaries and Assistant Actuaries.

in what form the benefits of Life Assurance are most desired, guiding them to the best and most suitable—and exposing the empirical. It would greatly simplify matters at Head Office, especially the correspondence, if we had such skilled experts at our Branch Offices; men who understand the theory and the practice of the business in such a way as to act as interpreters between the public and the offices in such technical matters as those with which we have to do. As Cicero said: "Man has been born for two things—thinking and acting."

Boswell said: "I often had occasion to remark, Johnson loved business, loved to have his wisdom actually operate on real life." So also the fully equipped Actuary. Nothing should please him better than to find his actuarial science so applied as to "actually operate on real life," on the real business of his professional life. This can best be done when he is in touch with the public—in the market-place rather than in the study.

The nature of my subject has necessarily limited me to our native land. I have therefore spoken, as in duty bound, of the Actuary in Scotland and of him who has gone forth from us to other lands. This, however, must not be taken as an indication of narrowness, any more than it would be were a mathematician in discoursing upon algebra to omit, for the time being, any reference to geometry. We have a much wider outlook, and can enjoy it to the full; for, having an independent position and forming an integral part of the International Congress of Actuaries, we are in fraternal relationship with the profession all the world over. We are on the happiest and most cordial terms with our nearest neighbours, our good friends of the Institute of Actuaries, whose headquarters are in London. Thirty-six Fellows of the Faculty are also Fellows of the Institute, and seven Fellows of the Institute are also Fellows of the Faculty. We and they are rather proud than otherwise of the double qualification. The aims and objects of these two British corporations are the same; and each in its own sphere—larger or smaller—is doing its best to further these, to the great advantage not only of our Life Assurance Offices but of all concerns, ever increasing in number, to which scientific finance can be applied.

Nor would I omit a passing reference to our friends of the Insurance and Actuarial Society of Glasgow, founded in 1881; of the Insurance Society of Edinburgh, founded in 1901; and of the many Insurance Institutes in England and Ireland, which are doing good work of a kind, in their own way, in training and encouraging young men for that wider insurance profession which includes the great and growing departments of life, fire, marine, accident, burglary, fidelity-guarantee, and others. These Societies and

Institutes have on their Rolls many members of our Faculty. So long as they confine themselves to their own sphere, and assist in passing on young men in the Life department for the qualification and diploma of the Faculty of Actuaries, or of the Institute of Actuaries, they will always find these two incorporations among their best friends.

In closing, let me remind you of the motto of our Faculty, which should always keep alive in us a sense of the great responsibilities which lie upon us. Though expressed in what is now to many a dead language they are living words which can be known and read of all men, and which our *alma mater* expects each one of us ever to bear on his heart and to practise in his life—AD FINEM FIDELIS.

On the motion of Mr. N. B. GUNN, the Senior Vice-President of the Faculty, a hearty vote of thanks was accorded to Mr. Hewat for his address. Mr. GUNN said:—'As most of you are aware, it is the duty of the Senior Vice-President to move the usual vote of thanks to the President for his Opening Address. On this occasion I may say it is not only a duty but a privilege. We have all listened to Mr. Hewat's remarks with great interest. He has given a very full and complete history of the rise of the Actuarial Profession, and, as we would all expect from our knowledge of the vein of patriotism which runs in Mr. Hewat's disposition, he has done thorough justice to the Actuary in Scotland. I would therefore ask you to accord a hearty vote of thanks to Mr. Hewat for his very excellent address.'

Before delivering his Address the President said:—Since last we met we have been called upon, during the past three months, to mourn the loss of three well-known Fellows of our Faculty:—

MR. HUGH BLAIR,

after becoming a Member of the Society of Accountants in Edinburgh, extended his studies and, passing the required examinations of the Faculty, was admitted a Fellow in 1872. Mr. Blair entered the Council and served on it, for more terms than one, between the years 1880 and 1898. He was our Honorary Treasurer during the three years 1885-1888. In addition to his extensive business as an Accountant Mr. Blair enjoyed a not inconsiderable practice as an Actuary also. He always took a lively interest in the affairs of the Faculty; and we shall all miss his genial presence at our meetings and his kindly manner in our business and social intercourse.

MR. PHILIP R. D. MACLAGAN

came of a well-known family closely identified with our profession. His grandfather and his father, successively Managers of the Edinburgh Life Assurance Company, were among the first Members of our Faculty, and of which Company he was himself at one time Assistant Secretary. Mr. Maclagan was elected a Fellow in 1894. While he did not turn his attention so much to actuarial matters as others have done he was well known in the Life and Fire Offices, having held important official positions in more than one of them. He will be remembered by us all here as the courteous Manager of the North British and Mercantile Insurance Company, and especially by his brother Managers, who knew his worth as Honorary Secretary of our Associated Scottish Life Offices. As Treasurer of the Royal Society of Edinburgh, and otherwise, he was well and favourably known in still wider circles.

MR. DAVID CLUNIE GREGOR,

who passed away a fortnight ago in his eighty-sixth year, was known to comparatively few of our younger Members, seeing that he retired about sixteen years ago, after forty years' service, from the active duties of the Colonial and Foreign Secretaryship of the Standard Life Assurance Company—a post which he filled with conspicuous success. Mr. Gregor was one of our Original Members, having entered in 1856 when the Faculty was constituted. His name appears in our Royal Charter, and he was elected an Honorary Fellow early in 1894. Entering his office in 1869 I had the pleasure of serving under him for nearly three years. I have always looked back upon that time with much satisfaction, for I found Mr. Gregor a model official; himself one of the hardest workers in the office, he was always ready to encourage and help on younger men. He commanded the respect and esteem of us all. I found in him a true friend—a friendship which grew and deepened as the years rolled on and which had been closely maintained up to the last. As the oldest and one of the best of my friends in the profession I mourn his loss; his removal breaks for me the last link with those under whom I received my early training.

APPENDIX

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Training for the Insurance Profession.—A. HEWAT.
The Actuary in Scotland.—A. HEWAT.
Some Suggestions regarding the Education of Actuarial Students.—A. HUNTER.
The Actuarial Profession from a Branch Man's Point of View.—J. N. LEWIS.
The Actuary.—J. N. LEWIS.
The History and Work of the Actuarial Society of Edinburgh.—G. M. LOW.
The History and Present Position [1901] of the Faculty of Actuaries in Scotland.—G. M. LOW.
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Administration: Notes by an Old Hand.—J. M. M'CANDLISH.
Personal Character and Business Life.—J. M. M'CANDLISH.
The Duties of an Actuary in his Professional Relationships.—J. M. M'CANDLISH.
The Philosophy and Ethics of Business.—J. M. M'CANDLISH.
The Education of an Actuary.—D. MACLAGAN.
Practical Notes on the Training and Duties of an Actuary.—T. MARR.
Life Branch Work.—A. H. MORGAN.
The Relation of the Actuarial Profession to the State.—J. NICOLL.
On the Position and Objects of the Profession of an Actuary.—S. RALEIGH.
On the Instruction given in Canadian Universities on Actuarial Subjects.—F. SANDERSON.
Outside Business Training.—J. T. SMITH.
Remarks on the Exercise of Independent Thought by the Head of an Office and his Subordinates.—T. B. SPRAGUE.
Remarks on the Position and Prospects of the Actuarial Profession and of the Business of Life Assurance.—S. C. THOMSON.
Notes on the Work of an Actuarial Department.—W. G. WALTON.

Annuities

On a General Expression for the Value of an Annuity on the last r Survivors of m Lives.—G. KING.
On Reversionary Annuities.—G. KING.
On Family Annuities.—G. KING.

On Benefits payable fractionally throughout the Year and on Complete Annuities.—G. KING.
On Joint-Life Annuities.—J. J. M'LAUCHLAN.
On the Formulas for the Approximate Determination of the Rate of Interest of an Annuity.—J. J. M'LAUCHLAN.
Terminable Annuity Tables.—T. MARR.
An Approximation of the Rate of Interest in a Life Annuity.—J. MEIKLE.
Annuities (*Ency. Brit.* article).—T. B. SPRAGUE.
On the Value of Annuities payable Half-yearly, Quarterly, etc.—T. B. SPRAGUE.
On the Value of Reversionary Annuities, payable Half-yearly, etc.—T. B. SPRAGUE.
On the Value of Apportionable Annuities, or of Annuities in which a proportionate part is payable up to the day of death.—T. B. SPRAGUE.
Life Annuities.—J. STEWART.

Bonuses, Profits, etc.

Bonuses in Model Office Valuations and their Relations to Reserves.—J. BUCHANAN.
An Analysis of the Profit from Endowment Assurances.—J. CHATHAM.
Bonuses: How Earned and How Distributed.—A. HEWAT.
Bonus, as Applied to Limiting Term or Number of Payments.—J. R. MACFADYEN.
On the Conversion of Ordinary Assurances for the Whole Term of Life with Endowment Ones by Application of Bonus.—T. MARR.
On Determination and Distribution of Profit.—J. MEIKLE.
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On a Method of Distributing the Surplus among the Assured in a Life Assurance Office.—T. B. SPRAGUE.
On certain Methods of dividing Surplus among the Assured in a Life Assurance Company, and on the Rates of Premium that should be charged to render them equitable.—T. B. SPRAGUE.
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On Life Premium Book-keeping.—J. CHATHAM.
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The Tendency towards Equalisation in the Rate of Interest.—R. BLYTH.
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A brief Review of the Silver Question.—E. C. JOASS.
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On the Value in Practice of a Life Interest, allowing for Income Tax.—G. KING.
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Compound Interest and Annuity Tables, with Rules for determining the Amount of Principal and Interest in any Payment of Annuity, and for the Construction of Tables showing the same.—A. H. TURNBULL.
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On the Grant of Settlement Policies under the Provisions of the "Married Women's Property Acts, 1870 and 1882."—T. B. SPRAGUE.

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On the Measure of Expenses in Life Assurance Companies.—D. DEUCHAR.
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Notes on Theory and Practice in Life Assurance.—C. GUTHRIE.
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Notes on Summation and Interpolation.—A. J. FYFE.
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Translation of Napier's Canonis Logarithmorum Descriptio, with Notes and Catalogue of Napier's Works.—W. R. MACDONALD.
On some Problems in the Calculus of Finite Differences.—W. R. MACDONALD.
On the Interpolation of Logarithmic Series.—J. MEIKLE.
On the Development of Tables derived from an Abridged Expression for the Calculated Expectation of Future Life.—J. MEIKLE.
On a New Method of performing approximately certain Operations in Multiplication and Division.—J. A. ROBERTSON.
On Probability and Chance, and their connection with the Business of Life Assurance.—T. B. SPRAGUE.
On the Usefulness of Mathematical Studies to the Actuary.—T. B. SPRAGUE.
On the Application of Lubbock's Formula of Summation to find the Value of Annuities and Assurances.—T. B. SPRAGUE.
Explanation of a New Formula for Interpolation.—T. B. SPRAGUE.

Medical

Height and Weight in Relation to Life Assurance.—D. CARMENT.
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On the Alleged Increase of Cancer.—G. KING (*collab.* Dr. NEWSHOLME).
The Variations in Masculinity under different Conditions.—J. N. LEWIS (*collab.* Dr. LEWIS).
Natality and Fecundity. A Contribution to National Demography.—J. N. LEWIS (*collab.* Dr. LEWIS).
Gout as a Factor in Life Assurance.—J. MEIKLE.
Life Assurance without Medical Examination.—J. NICOLL.
Medical Handbook on Life Assurance.—J. CHISHOLM (*collab.* Dr. POLLOCK).
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Vaccination: Does it afford any Protection against Smallpox.—T. B. SPRAGUE.
The Increase of Cancer.—R. TEECE.
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Mortality and Marriage

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A few Remarks on the Practice of Colonial Life Offices in regard to the Comparison of Actual and Expected Mortality.—D. CARMENT.

A Comparison of the Rates of Mortality among Insured Lives and Government Annuitants.—J. CHATHAM.

On the Construction and Use of Combined Marriage and Mortality Table for Widowers.—J. CHATHAM.

On the relative Age of Husbands and Wives whose Marriages are fruitful.—J. CHATHAM.

An Examination of the published Experience of several Life Offices and Groups of Life Offices, for the Purpose of determining with the greatest accuracy obtainable: (1) The Rate of Mortality among recently Selected Lives; (2) The Rate at which the Mortality increases during the ten years after Insurance; and (3) Whether such Rate of Increase depends upon the Rate of Discontinuance or not; with a general Outline of the best Methods to be pursued in the future Collection and Compilation of any Experience.—J. CHATHAM.

On the Graduation of the British Offices' Annuity Experience, 1863-1893.—J. CHATHAM.

Notes on a Mortality Experience in connection with Reversionary Annuity Business in Great Britain.—S. F. M. CUMMING.

A Sketch of the History of the Science of Life Contingencies, with special Reference to the Origin and Construction of Mortality Tables.—J. J W. DEUCHAR.

Statistics as to the Mortality Experience among Assured Lives engaged in the Liquor Trade.—G. DOUGLAS.

On the Influence of Selection on the Mortality from different Classes of Diseases amongst Assured Lives.—W. R. DOVEY.

The Rates of Mortality in New South Wales and Victoria.—W. R. DOVEY.

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An Investigation of the Marriage and Mortality Experience of the Widows' Funds of the Scottish Banks, with relative Tables and Diagrams, and Notes on other Marriage and Mortality Tables.—A. HEWAT.

An Investigation of the Marriage and Mortality Experience of a Scottish Ministers' Widows' and Orphans' Fund, with Relative Statistical and Monetary Tables.—A. HEWAT.

On the Mortality during the first Policy-Year under Endowment Insurances.—A. HUNTER.

Mortality among Non-Caucasian Races.—A. HUNTER.

Mortality among Athletes and other Graduates in Yale University.—A. HUNTER.

Mortality Experience among Annuitants resident in the United States of America and Canada.—A. HUNTER.

On the Mortality among Assured Lives, and the Requisite Reserves of Life Offices.—G. KING.

Brief Notes on the British Offices' Mortality Experience, 1863-1893.—G. KING.

On the Rate of Mortality amongst the Female Nominees of the General Annuity Trust Fund during the Period 1869-1895.—G. KING.

Longevity: The various Influences affecting it, and its Relation to Life Assurance.—A. M. LAUGHTON.

On an Investigation to show the Rates of Mortality and Marriage among Daughters of Peers and of Heirs-Apparent; with Values of certain Benefits depending on Marriage and Death.—M. M. LEES.

On Extra Risk, with some Particulars of the recent Investigation of the Mortality of Persons engaged in the Sale of Intoxicating Liquor.—G. M. LOW.

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Notes on War Mortality in recent Campaigns, with special reference to the German Experience in the War of 1870-71.—A. G. MACKENZIE.

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The Practice of different Countries in the Arrangement of Census Returns and of Official Mortality Statistics, and how these may be arranged so as to be useful to the Actuary.—J. J. M'LAUCHLAN.

On the Rate of Mortality among Adult Government Emigrants on the Voyage to Australia, during the Years 1847-1861.—J. J. M'LAUCHLAN.

On the Construction of a Table of Mortality.—J. MEIKLE.

On a Mode of Graduating a Table of Mortality.—J. MEIKLE.

On the Preparation of a Table of Mortality from Observations of various Magnitudes.—J. MEIKLE.

On the Official Publication of the Mortality of Assured Lives.—J. MEIKLE.

The Relation of Tables of Mortality to Values of Policies—Policy Life-Lines.—J. MEIKLE.

Observations on the Rate of Mortality of Assured Lives as experienced by Ten Assurance Companies of Scotland.—J. MEIKLE.

On a Mode of Tabulating the Facts, for the Purpose of ascertaining the Numbers exposed to Risk, and calculating the Rate of Mortality experienced by Assurance Companies.—J. MEIKLE.

First Year's Risk.—H. MOIR.

The new Combined Mortality Experience of British Life Assurance Companies.—W. S. NICOL.

Observations on the Graduation of Mortality Tables, with Special Reference to the Conditions under which certain Methods are to be preferred.—J. SORLEY.

On a Method of Calculating the Force of Mortality in Select Life Tables.—A. E. SPRAGUE.

Note on the Rate of Mortality in Sierra Leone.—A. E. SPRAGUE.

On the Rates of Mortality in Certain Parts of Africa.—A. E. SPRAGUE.

On the Construction and Use of a Series of Select Mortality Tables to be employed in Combination with the $H^{M(6)}$ Table.—T. B. SPRAGUE.

On the Construction of a Combined Marriage and Mortality Table, from Observations made as to the Rates of Marriage and Mortality among any Body of Men; and on the Calculation of the Values of Annuities and Assurances that depend on the Contingency of Marriage as well as of Death, and their Application to determine the Rate of Premium for an Insurance against the Contingency of a Bachelor of a given Age leaving Issue: Illustrated by Various Tables calculated from the Experience of the British Peerage Families.—T. B. SPRAGUE.

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On the Rate of Marriage among Widows and Spinsters.—T. B. SPRAGUE.

On the Probability that a Marriage entered into by a man above Forty will be fruitful.—T. B. SPRAGUE.

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On the Rate of Mortality in the Navy.—T. B. SPRAGUE.

The Graphic Method of adjusting Mortality Tables, a Description of its Objects and Advantages as compared with other Methods, and an Application of it to obtain a Graduated Mortality Table from A. J. Finlaison's Observations on the Mortality of the Female Government Annuitants, four Years and upwards after Purchase.—T. B. SPRAGUE.

On the Tabulation of the Facts extracted from the Records of a Life Office for the Purpose of investigating its Mortality Experience.—T. B. SPRAGUE.

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On the Rate of Mortality among Europeans in Tropical Africa.—T. B. SPRAGUE.

Note on the Mortality among Europeans resident in India.—T. B. SPRAGUE.

On the Rate of Mortality prevailing among Assured Lives as influenced by the length of time for which they have been assured.—T. B. SPRAGUE.

Note on Extra Risk attaching to the Occupation of Pilot.—T. B. SPRAGUE.

The Mortality among Assured Lives viewed in relation to the Sum at Risk.—G. C. STENHOUSE.

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A Comparison of the Rates of Mortality of the City and Suburbs of Sydney.—R. TEECE.

A Comparison of American and Australian Mortality.—R. TEECE.

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TRANSACTIONS

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Some Notes on the Death Duties from an Actuarial Point of View. By John Charles Dommett, M.A., F.F.A., F.I.A., Barrister-at-Law.

Abstract of Discussion thereon.

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Some Notes on the Death Duties from an Actuarial Point of View.

By JOHN CHARLES DENMEAD, M.A., F.F.A., F.I.A., *Barrister-at-Law, Actuary to the Estate Duty Office, Somerset House, London.*

[Read before the Faculty, 3rd December 1906.]

INTRODUCTORY.

Object of the Paper.

WHEN some time ago I was flattered with an invitation to read a paper before the Faculty of Actuaries, it was suggested that I should deal with the question of the amount to be deducted, in valuing reversions, for duties which will be payable out of the funds in reversion.

This is a somewhat difficult question to deal with directly, and I fear it will be thought that in the following pages the *raison d'être* of my paper has been forgotten. I may say, however, that while not overlooking the ostensible object in view, I have felt that the best way of giving effect to it is to write a short account of the various death duties which will have to be considered in making such valuations. The only alternative method of dealing with the subject, which occurred to me, was to give a series of examples, but this would involve such a large number of instances to meet even a small fraction of the possible cases that may arise in practice, that I had to abandon all idea of treating the subject in this way. I must fall back upon general principles and not rely on concrete instances, although examples of the general principles are given in a few cases.

In view of the fact that the subject suggested for this paper is only indirectly dealt with, for the reasons stated, I have thought

that it should be described merely as "Some Notes on the Death Duties from an Actuarial Point of View," and not by any more ambitious title.

Its limitations.

The accounts given below of the various death duties are not by any means exhaustive. While endeavouring to make them as complete as possible for the purpose in view, I have tried to avoid overloading them with unnecessary details. Exceptions of infrequent occurrence are not mentioned.

As an instance of an omission, there is no mention of the one per cent. temporary Estate duty imposed by the Customs and Inland Revenue Act of 1889, because the cases in which that duty can now be payable are so rare that it does not seem necessary to give an account of it. Also advowsons, gifts to charities, gifts of pictures, books, etc., to be enjoyed in kind, and other special subjects, are not dealt with. For these things, and, generally, for a complete account of the complicated subject of the death duties, the recognised text-books must be consulted. I have, naturally, availed myself of these text-books, particularly of Mr. A. W. Soward's work on the Estate Duty, to which every writer on this subject must necessarily be indebted. With this acknowledgment of my indebtedness to the existing authorities, I hardly think any further reference in the following pages is necessary.

It is almost unnecessary to add that no one is responsible for anything in this paper but myself.

There are practically five death duties with which we are likely to be concerned, viz.:—

1. The Probate (and Administration) Duty.
2. The Account-Stamp Duty.
3. The Legacy Duty.
4. The Succession Duty.
5. The Estate Duty (including Settlement Estate Duty).

THE PROBATE DUTY.

The Scale.

When the personal estate of the deceased is—

Not above the value[1] of £100, . . .			nil.	
Above £100 and not above £150	£3	0	0	
„ 150 „ 200	4	0	0	

[1] By the value is meant the net and not the gross value, and it should be the value at the date of grant, and should include all accretions (*Partington* v. *Attorney-General*, L.R. 4 H.L. 100).

Above £200	and not above	£250	£5	0	0
„ 250	„	300	6	0	0
„ 300	„	350	7	0	0
„ 350	„	400	8	0	0
„ 400	„	450	9	0	0
„ 450	„	500	10	0	0
„ 500	„	550	13	15	0
„ 550	„	600	15	0	0
„ 600	„	650	16	5	0
„ 650	„	700	17	10	0
„ 700	„	750	18	15	0
„ 750	„	800	20	0	0
„ 800	„	850	21	5	0
„ 850	„	900	22	10	0
„ 900	„	950	23	15	0
„ 950	„	1000	25	0	0
„ 1000	„	1100	33	0	0

and so on, £3 being added for every £100 (and £3 for the fractional part of £100) above £1000.

Where the whole estate of any person dying on or after 1st June 1881, without deduction of debts, etc., does not exceed £300 in value, a stamp duty only, of 30s., is payable.

History.

The Probate Duty is the oldest of these five Death Duties, and was first imposed in 1694, and although it is now displaced in cases of deaths occurring on or after 2nd August 1894 by the Estate Duty, it is still payable upon Personal Estate falling into the estates of persons who died before that date.

Description.

It is an *ad valorem* stamp duty on an instrument of title, viz.: the grant of Probate or Letters of Administration; its incidence depends upon the local situation of the property, and its amount on the value of the property.

Property liable.

The property liable to Probate duty must be impressed with the character of personal estate and such as the executor or administrator has the right to receive or recover *virtute officii.* It includes not only such obviously personal property as stocks, shares, money, furniture, accrued and apportioned rents, etc., but also leaseholds, real estate contracted to be sold during the lifetime of its deceased owner, and the deceased's share of freeholds comprised in partnership property, because all these are in equity 'personal' estate. But real estate (freehold and copyhold) is not liable to Probate duty.

The persons liable for the duty are the executors or administrators of the deceased.

Local situation of property, not owner's domicile, the test of liability.

The duty does not depend upon the domicile of the deceased owner, but is payable on all personal estate belonging to him locally situate in the United Kingdom at his death, or capable of being dealt with here or disposed of by some act done within the jurisdiction of the Court of Probate.

The property must also actually belong to the deceased, or, in the case of any one dying after 3rd April 1860, be personal property disposed of by will by virtue of a general power of appointment.

Exceptions.

It may be pointed out that the shares of some companies are not personal estate, and are therefore not liable to Probate duty; such are

New River Shares, prior to the establishment of the Metropolitan Water Board in 1902.
Aire and Calder Canal Shares.
Kennet and Avon Canal Shares.
Leeds Cloth Market Shares.
Droitwich Canal Shares.
Shares of two or three Welsh railways.

The tolls of a lighthouse are real estate; and although estates *pur autre vie,* of which there is no special occupant other than the executor, are applicable by law as personal estate, they are not liable to Probate duty. The Probate duties now in force are those which were imposed by the Customs and Inland Revenue Act of 1881 (44 and 45 Vict. c. 12, ss. 27-43). They are applicable to the estates of persons who died before 2nd August 1894, and to whom grants of representation are obtained after 1st June 1881.

The value of the estate is, in all cases where the deceased died domiciled in the United Kingdom (except the £300 estate), the net value to be ascertained by deducting from the gross value the debts and funeral expenses. If the deceased died domiciled out of the United Kingdom, his debts and funeral expenses may not be deducted from his personal estate in this country.

The Act of 1881 transferred the stamp from the grant itself to the affidavit of value. It also provided that no grant should be made unless it bears a certificate in writing that the affidavit has been delivered, and is duly stamped, and mentions the gross value of the estate as shown by the affidavit. In Scotland the stamp is attached to the inventory.

The methods of dealing with the case of additional property, not previously accounted for, falling into the estate of a deceased person, to whom a grant of representation had been obtained

whether prior to the 1881 Act, or subsequently to it, should perhaps be mentioned. If the grant is still in force the additional Probate duty must be accounted for by increasing the stamp duty originally paid according to the scale in force when the grant was obtained. If, however, the grant is no longer operative, the additional duty may be accounted for either (*a*) by increasing the stamp duty according to the original scale or (*b*) by paying the *ad valorem* stamp duty on the accretion under the 1881 Act, according as the persons liable find cheaper or more convenient. If method (*a*) is adopted a duty paid stamp is placed on the affidavit to lead to the new grant *de bonis non administratis.* If method (*b*) is adopted the payment of the *ad valorem* duty on the Inland Revenue affidavit to lead to the new grant is sufficient.

Methods of accounting for additional Probate Duty.

In cases of deaths occurring after 1st August 1894 the Probate duty is superseded by the Estate duty imposed by the Finance Act, 1894.

Estate Duty supersedes Probate Duty.

If property passes by the death after the Finance Act 1894, of a life-tenant under a will, and Probate duty has been paid thereon in connection with the testator's death, it is exempt from the payment of Estate duty in connection with the life-tenant's death. (Section 21 (1) of the Finance Act, 1894.) A testator *A* who died in 1890 devised and bequeathed his real and personal estate to *B* for life, and on *B*'s death, which occurred in 1904 (*i.e.* after the Finance Act, 1894), to *C* in fee and absolutely. On *B*'s death the investments representing *A*'s personal estate (Probate duty having been paid thereon) are exempt from Estate duty, but the real estate, on which Probate duty was not payable on *A*'s death, is not exempt from Estate duty. In addition, Legacy duty may also be payable on the personal estate, and Succession duty on the real estate (see under those duties below).

Payment of Probate Duty exempts from Estate Duty.

THE ACCOUNT STAMP DUTY.

Scale.—Same as that of Probate Duty.

The Customs and Inland Revenue Act, 1881 (44 and 45 Vict. c. 12, sec. 38) also created a duty, called the Account Stamp Duty, on certain kinds of personal property.

This duty was imposed partly to prevent evasions of Probate duty, to which it is in many respects analogous, and like Probate duty, it was displaced in cases of deaths occurring on or after 2nd August 1894 by the more comprehensive Estate duty.

Object of the duty.

The 1881 Act was slightly extended by the Customs and Inland Revenue Act of 1889, and the personal property liable to Account Stamp duty under the two Acts consisted of

Property liable.

1. *Donationes mortis causâ.*
   Inter vivos gifts made within twelve months of the donor's death.
   Gifts, whenever made, in which the donor had reserved any interest.
2. Property placed by the deceased in the joint names of himself and another so that a benefit survives to that other on his death. For example, if a husband or a father voluntarily transfers or purchases stock in the names of himself and his wife or child, it is a presumption of law that a provision is intended to be made in their favour subject to his receiving the income during his life, and on his death the beneficial gift of the whole fund would be liable to Account Stamp duty. Or if a husband and wife purchase an annuity on their joint lives with continuance to survivor, and the husband dies first, Account Stamp duty is payable on the value of the wife's annuity at that time proportionate to the sum contributed by the husband towards the purchase of the annuity.
3. Property passing under a voluntary settlement in which the settlor has reserved a life-interest, and property passing under a marriage settlement to "volunteers," *i.e.* to collateral relatives and to other persons outside the marriage consideration.
4. Any money received under a policy of assurance effected by any person dying on or after 1st June 1889, on his life, where the policy is wholly kept up by him for the benefit of a donee, or a part of such money in proportion to the premiums paid by him, where the policy is partially kept up by him for such benefit.

Property on which this Duty is paid exempt from Estate Duty.

If property passes by a death after the Finance Act, 1894, which has previously paid Account Stamp duty in connection with the death of a prior life-tenant under the same title, it is exempt from the payment of Estate duty in connection with the subsequent death. (Section 21 (1) of the Finance Act, 1894.) Thus by a voluntary settlement *A*, who died in 1890, settled real and personal property upon trust for himself for life and, after his death, to *B* for life, and after *B*'s death, which happened in 1904, to *C* in fee and absolutely. On *A*'s death,

Account Stamp duty would be payable upon the personal property (and possibly Succession duty), and Succession duty would be payable on *B*'s succession to the real property, while on *B*'s death Estate duty would be payable on the real property, on which no Account Stamp duty had been paid, but no Estate duty would be payable on the investments representing the personal property. On both descriptions of property, however, Succession duty may also be payable in connection with *B*'s death, according to the rules hereafter explained when dealing with that duty.

Incidence of the duty.

The charge of Account Stamp duty is considered to affect personal property locally situated in the United Kingdom passing under voluntary dispositions taking effect as British settlements, whether made by persons domiciled in the United Kingdom or by foreigners.

Liability.

Accounts are to be delivered by every person who as beneficiary, trustee, or otherwise, acquires possession or assumes the management of any personal or moveable property liable to the duty, upon retaining the same to his own use or distributing or disposing thereof.

THE LEGACY DUTY.

The Scale.

The rates of duty depend upon the consanguinity of the legatees to the testator or intestate, and are as follows :—

Lineal issue or lineal ancestor, ...	£1	per cent.
Brothers and sisters and their descendants, .	£3	„
Brothers and sisters of the father or mother and their descendants, ...	£5	„
Brothers and sisters of a grandfather or grandmother and their descendants, ...	£6	„
Any other person, ...	£10	„

The husband or wife of the testator is not chargeable with Legacy duty, and a legatee whose husband or wife is of nearer relationship to the testator is chargeable with duty at the rate appropriate to such relationship.

The principal Legacy Duty Acts at present in force are those of 1796 and 1815, 36 George III. c. 52, and 55 George III. c. 184.

Definition of a Legacy.

A legacy must be a gift by some will or testamentary instrument of a person domiciled in the United Kingdom and be payable or satisfied out of his personal estate or out of the personal estate over which he has a general power of appointment.

Formerly leaseholds, real estate directed to be sold, whether actually sold or not, moneys, rent-charges, annuities and jointures payable by will out of real estate were also legacies.

Some exceptions.

Under the Succession Duty Act, 1853, however, leaseholds were declared to be real property for the purposes of Succession duty and taken out of the scope of the Legacy Duty Acts, although they are still personal property for Probate duty. And under the Customs and Inland Revenue Act, 1888 [51 and 52 Vict. c. 8, sec. 21 (2)], the proceeds of sale of real estate, moneys, rent-charges, annuities and jointures charged on real estate were made liable to the duties under the Succession Duty Act, 1853, and to the additional duties imposed by the 1888 Act on property which had not paid Probate duty or Account Stamp duty under the Customs and Inland Revenue Act of 1881 (44 and 45 Vict. c. 12).

Incidence of the duty.

The incidence of Legacy duty depends on the domicile of the testator, and no legacy duty is payable upon the bequests of a testator who died domiciled abroad, although they may be satisfied out of his British personal estate.

When payable.

Legacy duty becomes payable upon retainer for the benefit of the legatee, or at the time of payment, delivery or other satisfaction or discharge, and not when the right to the legacy accrues.

On what amount payable.

The duty is chargeable upon the amount or value of the property as it stands, with its accretions of income [*Attorney-General* v. *Cavendish*; Wight., 82 (*a*)] at the time when the duty is computed, after deducting all proper payments actually made, and incumbrances then actually existing, and not upon the property as it stood at the time of the death of the testator.

Liability.

The Executors are primarily accountable for the duty, but if the legacies are paid to the legatees without deduction of duty, the legatees are also liable, as also are purchasers from legatees.

When one per cent. legacy duty not payable.

The £1 per cent. duty imposed upon lineals is not payable upon property on which Probate duty or Account Stamp duty has been paid under the 1881 Act, or on which Estate duty has been paid under the Finance Act, 1894.

Where a legacy is given to different persons in succession and the successive beneficiaries are all liable to the same rate of duty, the duty is payable at once on the capital as if on a legacy given to one person. The duty is deducted from the capital in the first instance, and no separate calculations by way of annuity are made for persons having only life or other

limited interests in the legacy. It may therefore happen by virtue of this enactment that a reversion will not be liable to Legacy duty on the death of the present tenant for life, although the reversioner is of collateral consanguinity to the testator, because the duty has been previously paid on the capital in connection with the death of the testator. Otherwise, if a legacy is settled upon different persons in succession not liable to the same rate of duty, then duty is payable upon the value of their respective interests, as and when they take effect. In particular, a legatee who is entitled to a life-interest only pays duty on the value of an annuity for his life calculated according to the 4 per cent. tables annexed to the Succession Duty Act, 1853, and the duty may be paid by four yearly instalments or in one sum under discount.

Duty payable on the capital when successive legatees are liable to the same rate of duty.

Legacy duty is payable upon every devolution. A legacy therefore falling into a deceased reversioner's estate by the death of a life-tenant may be liable to duty under the will of the original testator, and under the will or intestacy of the deceased reversioner, and this holds whatever may be the number of deceased reversioners. It is thus seen that for the purpose of ascertaining the duty payable it is not sufficient merely to know on whose death the legacy will fall in, and the original title under which it became payable, but the complete chain of the title giving all the subsidiary dispositions and devolutions must be known.

Duty payable on every devolution.

Where, however, by the same death a claim arises for Legacy duty and also a claim for Succession duty, the Legacy duty only is payable, and the claim for Succession duty is superseded.

Legacy v. Succession Duty.

THE SUCCESSION DUTY.

THE SCALE.

Consanguinity of the Successor to the Predecessor.	Prior to 1st July 1888.	On and after 1st July 1888.
Lineal issue or lineal ancestor, .	£1 per cent.	£1½ per cent.
Brothers and sisters and their descendants, ...	£3 „	£4½ „
Brothers and sisters of the father or mother and their descendants,	£5 „	£6½ „
Brothers and sisters of a grandfather or grandmother and their descendants, ...	£6 „	£7½ „
Any other person, ...	£10 „	£11½ „

The husband or wife of the predecessor is not chargeable with

Succession duty, and a successor whose husband or wife is of nearer relationship to the predecessor is chargeable with duty at the rate appropriate to such relationship.

Scope of the duty.

This duty, which was imposed by the Succession Duty Act, 1853, is supplementary to the Legacy duty, and brings into tax all successions to property, real as well as personal, whether the title be under settlement or will, by descent, intestacy, or survivorship. The intention of the Act was to impose a duty corresponding to the Legacy duty on all successions to personal property under settlements, and to real and leasehold property under settlements, wills, or intestacies.

Resembles Legacy Duty.

The Succession duty resembles the Legacy duty in its main features, and like the latter depends upon the consanguinity of the successor to the person from whom the succession is derived and the value of the succession.

Object of the higher rates.

Prior to 1st July 1888 the rates for Succession duty were exactly the same as for Legacy duty, but on and after that date they were increased respectively to 1½, 4½, 6½, 7½, and 11½ per cent. These higher rates were imposed with the object of making the duties on real estate more equal to those on personal estate, which alone was liable to Probate duty, or its equivalent Account Stamp duty, and therefore the Act imposing these duties included therein any moneys arising from the sale of real estate or payable thereout.

When not chargeable.

These higher rates, however, are not chargeable if Probate or Account Stamp duty had been paid on the property under the 1881 Act, nor are they payable if Estate duty has been paid. In such cases the lower rates only are payable.

In view of the comprehensive exceptions last mentioned these higher rates do not frequently occur now, except in cases of commutation of Succession duty, when, by reason of the settlement being put an end to, no Estate duty is payable.

When Succession Duty payable, and on what amount. (1) Personal estate.

In the case of personal property, Succession duty is to be paid when the successor shall become entitled in possession to his succession, and upon its value at the date of the death which gave rise to the claim. And if the personal property is settled upon different persons by way of succession, and they are all liable to the same rate of duty, the duty is payable at once on the capital.

Prior to the Finance Act of 1894, Succession duty on real estate and leaseholds was payable only on the successor's life-interest in the net annual value, according to the Annuity Tables annexed to the Act of 1853, whether the successor took the property absolutely or for life only. The duty thereon

could be paid by eight half-yearly instalments, the first due one year from the death, and the only difference between a successor who took for life and one who took absolutely was that if the former died before all the eight half-yearly instalments were due, those only which had accrued before his death were payable.

(2) Real estate. Payment by instalments.

Section 22 of the Customs and Inland Revenue Act of 1888 (51 Vict. c. 8) provided an alternative method of payment by instalments, but it is unnecessary for the purpose of this paper to set out these provisions in detail.

The Finance Act, 1894, however, provided that the value for Succession duty of property of which the successor was competent to dispose should be the principal value of such property and not the value of his life-interest therein.

Altered by Finance Act, 1894.

In order to render a succession liable to duty, the subject-matter of the disposition or devolution must be situate within this country; in the case of immoveable property (real estate and leaseholds) actually situate here, and in the case of moveable property either actually or constructively situate here, so that the question of domicile has some bearing on the liability to duty, although not so wide or important as in the case of Legacy duty. For example, if the deceased dies domiciled abroad and leaves personal property in this country, it will not be liable *at his death* to Succession duty any more than it would be to Legacy duty; but if he has created a British trust, whereby some person *B* becomes entitled to succeed beneficially at the death of another person *A* to personal property actually situate here, or constructively here, because vested in British trustees and subject to British jurisdiction, Succession duty would be payable on the death of *A*. And if a British settlement is made by a person who subsequently acquires a foreign domicile, Succession duty would be payable on the property passing by a death, even if the investments consisted of foreign securities.

Incidence of the duty.

Foreign testator.

The persons accountable for Succession duty are the trustees, successor or his alienee, or any person in whom the property or the management of it is vested.

Liability.

Although the Succession duty resembles the Legacy duty in so many respects, there is a difference in one respect which is important when considering what deduction should be made for future duties in valuing a reversion. In the case of Legacy duty it was pointed out that several duties may be payable by the death of a life-tenant owing to the property passing into the estates of reversioners who had predeceased the life-tenant. But this

Points of difference from Legacy Duty.

feature of the Legacy Duty Acts is expressly prevented from applying to transmitted successions in personal estate, the provision being that in successions to personal estate transmitted by death, it may be, three or four times before possession is acquired, one Succession duty only is to be charged at the highest rate, which, supposing every person in the chain of transmission to be liable to duty, would be payable by any one of them.

Example of a composite settlement.

Thus *A* settled a sum of money on himself for life, and on his death to *B*, a stranger in blood absolutely. *B* resettled his reversion in the fund upon himself for life and on his death to his nephew *C* absolutely. *B* died in *A*'s lifetime, and on *A*'s death *C* became absolutely entitled to the fund, but he does not pay two duties, one at £10 per cent. for *B* as derived from *A*, and the other at 3 per cent. for himself as derived from his uncle *B*, but he pays one duty only, viz. £10 per cent., that being the higher of the two rates.

Example of the joint operation of a will and settlement.

What Legacy or Succession duties are payable if the fund, when it falls into possession, becomes the property of a person who is entitled under the operation of a will and a settlement? In the example just given, suppose that *B*, instead of resettling the property by deed, bequeathed it by will to his nephew *C*. The Succession Duty Act, 1853, provides that no Succession duty shall be payable upon any property which is chargeable with duty under the Legacy Duty Acts in respect of the same acquisition of the same property. Therefore, in the case supposed, the duty payable would be the Legacy duty at 3 per cent., and there would be no Succession duty payable at all.

Similarly, if *A* had bequeathed the fund upon the trusts supposed, and *B* had resettled his reversionary legacy, the duty payable on *A*'s death (*A* having survived *B*) would be the Legacy duty at 10 per cent., and there would be no Succession duty.

So that in these circumstances the Legacy duty prevails, whether its rate is higher or lower than that of the Succession duty.

If a successor, before he becomes entitled in possession, sells his interest, and his alienee is alive when it falls in, the alienee pays duty at the same rate as the original successor would have done if he had not alienated.

If, however, the property were real estate or leaseholds, and the sale took place before the Finance Act, 1894, the value of the succession would be the value of the alienee's life-interest in the net annual value according to the Succession Duty Act Tables, and not the value of the vendor's life-interest therein.

Thus, if real property is settled upon *A* for life, with remainder to *B* in fee, and *B* sells his reversion to *C*, then on *A*'s death *C*

succeeds as alienee of *B* and pays duty at the same rate as *B* would have paid if he had not alienated. And if the sale took place before the Finance Act, 1894, *C* would pay on the value of his life-interest according to the Succession Duty Act Tables, and not on the value of *B*'s life-interest. In fact, *B* may then be dead.

Effect of new succession.

Also in the case of real estate and leaseholds, if the reversion is not sold but resettled, and a new succession is created, the person entitled when the property falls in is the successor, and he derives his interest not from the original settlor, but from the last settlor absolutely entitled to the property, and the instrument under which the property passes to such successor for the purpose of Succession duty, is the settlement made by such last settlor.

Thus under a deed of 1860 real property was settled upon *A* for life with remainder to *B* in fee, and *B* by deed of 1870 resettled it upon himself for life, with remainder to *C* in fee, and *B* dies in *A*'s lifetime. On *A*'s death *C* succeeds to the real property under the 1870 deed, and derives his succession from *B* as predecessor.

The difference between these two cases is that between a simple alienation and a resettlement under which a new succession is created.

How duty payable if succession accelerated.

While discussing transferred interests it should be mentioned that if an advance is made to a successor, or he comes into possession of his property before the death of the life-tenant, because the latter surrenders his life-interest, the Succession duty is payable at the same time and in the same manner as such duty would have been payable if no such acceleration had taken place.

Thus *A* by will devises real estate to *B* for life, and on his death to *C* for life, with remainders over. *B* surrenders his life-estate to *C* in 1900 and dies in 1906. No Succession duty is payable by *C* until *B*'s death in 1906, when it is payable on the then value of *C*'s life-interest in the net annual value.

Or *A* by deed settles personal estate upon *B* for life, and on his death to *C* absolutely. *B* surrenders his life-estate to *C* in 1900 and dies in 1906. No Succession duty is payable until *B*'s death in 1906, when it is payable on the capital of the fund then remaining together with any sums spent thereout by *C* since 1900.

In such a case it would be advisable to commute at the date of the surrender the future duty payable when the life-tenant dies.

THE ESTATE DUTY.

THE SCALE.

Where the Principal Value of the Estate				Estate Duty is payable at the Rate per cent. of
	£		£	
Exceeds	100	and does not exceed	500	One pound
"	500	"	1,000	Two pounds
"	1,000	"	10,000	Three pounds
"	10,000	"	25,000	Four pounds
"	25,000	"	50,000	Four pounds ten shillings
"	50,000	"	75,000	Five pounds
"	75,000	"	100,000	Five pounds ten shillings
"	100,000	"	150,000	Six pounds
"	150,000	"	250,000	Six pounds ten shillings
"	250,000	"	500,000	Seven pounds
"	500,000	"	1,000,000	Seven pounds ten shillings
"	1,000,000			Eight pounds

The rate of Settlement Estate duty where the property is settled is one per cent.

Its comprehensiveness.

The Estate Duty is the last and most important, because the most comprehensive, of all the Death Duties. It was imposed by the Finance Act, 1894 (57 and 58 Vict. c. 30), in connection with deaths occurring after 1st August 1894. It supersedes the Probate and Account Stamp duties. It is a stamp duty upon property, real or personal, settled or not settled, which passes on death; and is chargeable upon the principal value of the property without regard to its destination.

Property liable.

Incidence.

The duty is imposed upon all property which actually changes ownership on death, whether it was the deceased's own property or not, and even although he had no interest in it.

The liability to duty extends also to the following classes of property:—

Special classes of property caught by the duty.

(*a*) Property which by the deceased's death is relieved of an annuity;

(*b*) Gifts made by him within a year of his death;

(*c*) Gifts made by him at any time provided that he reserved some benefit to himself therein;

(*d*) Property placed by him in the joint names of himself and any other person, so that a benefit accrues to that other person on his death;

(*e*) Annuities and Assurances, or other provisions to arise at

his death, made by himself alone, or by him in concert with other persons; and

(*f*) Property over which he had an absolute power of appointment, whether he exercised the power or not.

The intention of the Act evidently was, in the words of Mr. Justice Grantham, "to sweep everything into the net of the Chancellor of the Exchequer that could be swept into it."

There are, nevertheless, exceptions to this sweeping enactment, some arising from the necessity of the case and others provided for by the Act itself. Naturally the foreign property of a foreigner could not be made liable to duty.

Foreign property.

Property out of the United Kingdom is made liable to Estate duty only if, under the law in force before the passing of the Finance Act, 1894, Legacy or Succession duty would be payable in respect thereof, or would be so payable but for the relationship of the person to whom it passes.

In effect, moveable property situated abroad is liable if subject to a British trust. Immoveable property is not liable, but if it is directed to be sold and a British trust of the proceeds is created, it can then become subject to the British Death duties.

Property not liable.

The Act of 1894 (as slightly amended by the Finance Act of 1896) provides for the following exceptions from liability to Estate duty:—

(*a*) Property which passes by virtue of a *bonâ-fide* purchase.

(*b*) Property which reverts to the settlor, who had merely given a life-estate therein to the deceased.

(*c*) Property in which a disponer's limited interest is enlarged, by a death after the Act, into an absolute interest.

(*d*) Property of common seamen, marines, or soldiers who are slain or die in service.

(*e*) Small estates under £100 in value.

(*f*) An annuity not exceeding £25 provided by the deceased, or by him in concert with others.

(*g*) Indian Government pensions for widows and children of Government officers.

(*h*) Church patronage until sold.

(*i*) Property settled before the Act by one spouse upon the other, and which, on the death of such other spouse, reverts to the settlor for life only.

Persons liable.

The executors are liable for the duty upon the personal estate of which the deceased was competent to dispose. In the case of other property passing the trustees, beneficiaries, or any other person in whom the property or the management thereof is vested are liable, and in this case the duty is made a charge on the property passing.

Property dealt with before 1894.

Where an interest in expectancy was sold or mortgaged before the Act of 1894 for full consideration in money or money's worth, the purchaser or mortgagee is to pay no other duty than would have been payable if the Act had not been passed.

Effect of the 1900 Act on release of a life-interest.

Until the Estate Duty Act of 1900 (63 & 64 Vict. c. 7), property in which the deceased tenant for life had released his life-interest, was exempt from Estate duty on the death of the tenant for life, but the Act of 1900 rendered such property liable to duty where the release was within a year of the death.

Two Estate Duties not payable on same death.

Payment of Estate duty on the reversion in settled property, as part of the estate of the deceased reversioner, exempts the settled property from Estate duty under the settlement on the subsequent death of the life-tenant thereunder (Section 5 (2) of the Finance Act, 1894).

Case where payment of Probate Duty excludes the payment of Estate Duty.

Payment of Probate duty on the reversion in settled property, as part of the estate of the deceased reversioner, where the disponer dies before the Act of 1894, similarly exempts the settled property from Estate duty under the settlement on the subsequent death of the life-tenant thereunder (Section 21 (1) of the Finance Act, 1894).

Estate duty is payable only once upon property passing under a settlement, until it passes to a person competent to dispose of it.

Settlement Estate Duty.

Provision, however, is made by the Finance Act, 1894, for property, tied up in a settlement, being allowed to pass free of Estate duty on the death of a subsequent life-tenant. Where, therefore, property is "settled," *i.e.* where "the enjoyment of estate under the same deed or will may be had by different persons succeeding to the estate in their order," a further Estate duty, called Settlement Estate duty, is to be levied upon the property on the death, on which Estate duty is levied, unless the immediate passing is to a person competent to dispose of it, and on whose death, therefore, a second Estate duty will be leviable, if the property then passes.

No Settlement Estate duty is payable in the following circumstances:—

When not payable.

(*a*) If the only life-tenant after the death of the deceased is the spouse of the deceased.

(*b*) Where the disposition by which the property was settled took effect before the Act of 1894.

(*c*) Where the property was settled by the will of the deceased,

and it and the other property, not settled but passing by the death, do not exceed £1000 in value.

If the stamp duty of 5s. per cent. has been paid on the settlement, it may be deducted from the Settlement Estate duty.

Settlement Estate duty is at the fixed rate of £1 per £100, and is payable only once during the continuance of the settlement.

The Estate duty is a graduated duty, the rate depending upon the value of the property passing on the death. The Act of 1894 provides that all property so passing, whether under the will or settlement of the deceased, or under the wills or settlements of others, or under joint tenancies, etc., is to be aggregated into one "estate," and the rate of duty is determined by the value of such one "estate."

Aggregation of all properties passing into one "estate" for rate.

The Act of 1894 provides that property in which the deceased never had an interest, and property which, under a disposition not made by him, passes on his death to a person who is neither his spouse nor a lineal relation, are not to be aggregated, but in each case such property is to form an "estate by itself," and its value is to determine the rate of duty to be levied in respect of it.

Exceptions.

Also where the deceased's own property, and the other property not settled, but passing on his death, do not, in all, exceed £1000 in net value, such combined property is to form an "estate by itself."

The law of aggregation was altered by the Act of 1900, and now the general rule is, as to persons dying after that Act (9th April 1900), that all property is to be fully aggregated, no matter by whom the disposition was made, or to whom the property passes.

Extension of aggregation by 1900 Act.

There is, however, this limitation to the general rule, that aggregation in the case of settled property, where the disponer died before the Act of 1894, and the property, if he had died after that Act, would have been chargeable with Estate duty on his death, is not to enhance the rate of Estate duty on that settled property, or on any other property, by more than one-half per cent. in excess of the rate which would have been chargeable if the settled property had been treated as an "estate by itself."

Limitation of aggregation in certain cases.

Also, property in which the deceased never had an interest still constitutes an "estate by itself," and if the combined net value of the deceased's own property and of the other property not settled does not exceed £1000, such combined properties are still not aggregated with any settled properties passing by his death.

Small estates exempt from aggregation.

In the case of Reversions sold or mortgaged between the Acts

of 1894 and 1900, the purchaser or mortgagee is not to be liable to any other duty than if the law of aggregation had not been altered.

Fixed duty on small estates.

Where the gross value of the deceased's own property, and of the other unsettled property does not, in all, exceed £300 or £500, a fixed composition duty of 30s. or 50s., as the case may be, can, if desired, be paid instead of the *ad valorem* Estate duty upon the net value.

Value for duty when property actually passes.

The value of property for the purpose of Estate duty is, in general, the price which, in the opinion of the Commissioners, it would fetch, if sold in the open market at the time of the deceased's death, but in the case of agricultural property, where no part of its value is due to the expectation of an increased income, the value is not to exceed twenty-five years' purchase of the net annual value arrived at by deducting, from the "Schedule A" assessment to income-tax, the average annual repairs and other necessary outgoings and expenses of management, the last not to exceed 5 per cent. of the assessment.

When an annuity ceases.

The value of the benefit by the termination of an annuity is to be the value of so much of the property subject to the annuity as was required to yield it. Therefore, if the whole income of the property was required to yield the annuity, the whole principal value of the property is to be charged with duty; but if a fraction only of the income was required, then the same fraction only of the principal value is to be charged.

Alternative methods of paying duty on Reversions.

Reversions are to be valued as at the death of the deceased owner, and are to be added to the rest of the estate passing by the death of the deceased for the purpose of ascertaining the rate on the rest of the estate. The duty on the Reversions may be paid either on their then value and at the same rate as the rest of the estate, or it may be postponed until the Reversions fall into possession. If the latter course is adopted, the duty is payable on their value in possession and at a rate ascertained by adding the value of the rest of the estate at the death of the reversioner to the value of the Reversions immediately after they have fallen into possession.

Deductions.

Deductions may be taken of funeral expenses, debts and incumbrances. Allowance, however, can only be made for debts and incumbrances created by the deceased (*a*) if they take effect out of his interest; (*b*) if they were incurred *bonâ fide* for full consideration in money or money's worth, wholly for his own use and benefit; and (*c*) if there is no right to reimbursement from any other estate or person, or if such reimbursement cannot be obtained.

In the case of settled property no allowance can be made of any mortgage or incumbrance created by the expectant successor subject to the deceased's life-interest against the principal value of the property chargeable with Estate duty (In re *Vernon* [1901] 1 Q.B. 297).

Mortgages on Reversions.

But an allowance is to be made of a mortgage created by the tenant for life and expectant successor, because it has been held by the House of Lords in such a case that what passes by the death of the life-tenant is the Equity of Redemption only (*Earl Cowley* v. *Commrs. of Inland Revenue*, 1899, A.C. 198).

If, however, the interest on the mortgage is not paid by the life-tenant during his life, *i.e.* if the mortgage is created by the life-tenant and remainderman with a guarantee to the life-tenant against loss of income, the mortgage cannot be deducted (*Attorney-General* v. *Lord Montagu* H.L., 1904, A.C. 316).

The following example of a "mixed fund" case may be useful.

Example of "mixed fund."

A, who died in 1890, devised and bequeathed his real and personal estate to Trustees upon trust to sell and pay his debts and funeral and testamentary expenses, invest the net proceeds, and pay the income thereof to *B* for life, and on *B*'s death, to divide the capital equally among *C*, *D*, and *E*. *B* died in 1900. On *B*'s death a claim for Estate duty arose upon the proportion of the investments, then representing the proceeds of the "mixed fund," attributable to the proceeds of the real estate. This proportion would be arrived at by ascertaining the values of the real and personal estate as they existed at the death of *A*, the testator, after deducting from each kind of property its due proportion of debts, etc. If any of the beneficiaries, *C*, *D*, and *E*, had predeceased *B*, and died possessed of their shares, the payment of Probate or Estate duty thereon, as part of their free estate, would exempt such shares from Estate duty in connection with *B*'s death, and a deduction of these shares would be made from the proportion of the investments previously arrived at.

To make this example illustrate a few other points, I will (1) assume that *B* was the wife of *A*, and *C*, *D*, and *E* his nephews. Here no claim for Legacy duty would arise until *B*'s death, when 3 per cent. Legacy duty would be payable on the entire capital, representing the mixed fund.

(2) Assume that *B* was *A*'s wife, but that *C*, *D*, and *E* were his children. In this case no Legacy duty is payable on *B*'s death, even upon the shares of the investments of the real estate which may be exempt from Estate duty on account of any of the beneficiaries predeceasing *B*.

(3) Assume that *B* was *A*'s son, and *C*, *D*, and *E* his grandchildren. In this case Succession duty at 1½ per cent. would

be payable on *A*'s death upon the capital of the net proceeds of his real estate as passing to *B* for life, and on his death to persons liable to the same rate of duty. There will still be, however, a claim for Estate duty in connection with *B*'s death on the proportion of the investments representing the real estate, but an allowance will be made from the Estate duty of the 1½ per cent. Succession duty paid in connection with *A*'s death.

Allowance of Succession Duty paid on the capital in connection with a previous death.

(4) Assume that *B* was *A*'s nephew, and that *C*, *D*, and *E* were the sons of *B*. In this case Succession duty at 4½ per cent.—the higher rate imposed by the 1888 Act—would be payable on *A*'s death upon the capital of the net proceeds of his real estate, and Legacy duty at 3 per cent. would be payable on the same event upon the capital of the net personal estate as passing to persons in succession all liable to the same rate of duty. Here, also, an allowance will be made from the Estate duty payable in connection with *B*'s death of the additional 1½ per cent. rate paid on *A*'s death.

This allowance against the Estate duty of the Legacy or Succession duty paid out of capital by lineals and of the additional Succession duty rate imposed by the 1888 Act is provided for by the Finance Act, 1896, Section 21.

Mr. C. R. V. Coutts, in his interesting paper on "Reversionary Securities as Investments" (*J. I. A.*, xl. p. 317), read before the Institute of Actuaries on the 30th April 1906, referred (pp. 325, 326) to a case where a reversion expectant on the death of the survivor of three lives had been dealt with before the Finance Act, 1894. In this case the Inland Revenue authorities claimed Estate duty upon the fund passing on the death of the first life-tenant to die after the Finance Act, 1894, on the ground that the exemption from Estate duty conferred by Section 21 (3) did not apply, because the reversion did not fall into possession by that death. In spite of this claim, however, the exemption from Estate duty and the substitution of the duties in existence prior to the Finance Act, 1894, provided by this subsection, apply, in effect, to the case mentioned by Mr. Coutts, so far as the purchaser of the reversion is concerned, because on the death of the last to die, *i.e.* when the reversion will fall into possession, a return of duty will be made equal in amount to the difference between the Estate duty paid on the first death and the old duties which would have been payable on the last death if the Finance Act, 1894, had not been passed, but without interest (*Att.-Gen.* v. *Walker*, In re *Reversionary Interest Society*, "*Times*," 14 *May* 1896).

Explanation of an apparent exception to the allowance of Succession Duty.

COMMUTATIONS OF DUTY.

Where settled property is being dealt with in such a way as to render it desirable to free it from all charges of duty, the future duties may, in the discretion of the Commissioners, be commuted.

When Commutations desirable.

Where property is being taken out of settlement, as by the release of the life-interest to the remainderman, by the assignment of the remainderman's interest to the life-tenant, or by the assignment of both their interests to a third party, an application for a commutation of the appropriate duty is entertained.

Where an interest in remainder only is being sold, or mortgaged, then if the duty would be a charge upon the property in the hands of a purchaser, or if it is made one of the conditions of the transaction that the quantum of property involved should be cleared from the prospective claim, an application to commute is entertained.

To know how the proceeds will be applied it is necessary to ascertain whether the moneys are to be held on the old trusts, or divided or how otherwise dealt with. If property is taken out of settlement, the nature of the arrangement between the tenant for life and remainderman must be known. Without this information it would not be possible to decide whether Estate duty or Succession duty or both were payable.

Necessary to know how the property will be dealt with.

When a commutation is agreed to, a present value is set upon the future duty, regard being had to the contingencies affecting the liability to and rate and amount of such duty, and interest is reckoned at three per cent. This rate of interest is used under statutory authority in the case of the Estate duty. In the case of the other duties the rate, like the other factors of a commutation, is at the discretion of the Commissioners, but in practice, three per cent. is used in every case.

Method of Commuting.

When real estate is sold under a trust for sale, or under a power of sale, no commutation is in general necessary, because the liability to duty shifts usually from the property sold to the proceeds of sale and the investments thereof.

POLICIES OF ASSURANCE AND DEATH DUTIES.

The following somewhat detailed remarks on claims for duty arising on policies of assurance, with the examples given, may, I think, appropriately find a place in this paper.

As previously stated, all classes of property liable to Account Stamp duty under the Customs and Inland Revenue Acts of 1881 and 1889 were made liable to Estate duty by the Finance Act, 1894 [Section 2 (1) (*c*)].

Gifts and Settlements of Policies and the Finance Act, 1894.

The Act of 1889 enacts, *inter alia*, that "The charge under the said section (*i.e.* Section 38 of the Customs and Inland Revenue Act of 1881) shall extend to money received under a policy of assurance effected by any person dying on or after the first day of June 1889, on his life, where the policy is wholly kept up by him for the benefit of a donee, whether nominee or assignee, or a part of such money in proportion to the premiums paid by him where the policy is partially kept up by him for such benefit."

Examples.

Ex. 1. A man who possesses a policy on his own life, not fully paid up, absolutely assigns it by way of gift more than twelve months before his death, without reservation, and no longer pays the premiums. His death does not give rise to any claim for duty.

Ex. 2. If the policy were fully paid up, or if the donor continued to pay the premiums, Estate duty would be payable upon the policy moneys on his death. Succession duty would also be payable unless the donee were a spouse or a lineal relative of the donor.

Ex. 3. If, after the gift, the donor wholly kept up the policy, but had not paid all (or it may be any) of the earlier premiums, it appears from *Lord Advocate* v. *Fleming or Robertson* (1897), 1 A.C. 145, that the words "wholly" or "partially" "kept up" for the benefit of the donee, in the 1889 Act, relate solely to the period subsequent to the date of the gift. In this case, therefore, Estate duty is payable on the whole of the policy moneys on the death of the donor after the Finance Act, 1894. Succession duty may also be payable.

Ex. 4. If, after the gift, the premiums were paid only in part, or not at all, by the donor, but he entered into an arrangement with some other persons, either at the time of or prior to the gift, to maintain the policy out of property or income, not his own, Estate duty is considered to be payable upon the whole of the policy moneys, on the donor's death, under Section 2 (1) (*d*) of the Finance Act, 1894. Succession duty may also be payable.

Ex. 5. If, after the gift, the donor partially kept up the policy, and the policy was otherwise maintained out of property or income, not his own and not in pursuance of an arrangement entered into by him with some other persons, Estate duty is payable under Section 2 (1) (*c*) of the Finance Act, 1894, upon a part of the policy moneys, such taxable part to be ascertained

with reference to the proportion which the premiums paid by the deceased bear to the total of the premiums paid.

The above examples hold, if, instead of the policy being the subject of an absolute gift to a single donee, it were given to Trustees upon the trusts of a settlement.

Policies under the Married Women's Property Acts.

A man effects a policy under the Married Women's Property Act on his own life for the sole benefit of his wife, and pays all the premiums. In this case it is assumed that he reserves no benefit to himself, and in the event of his wife's death in his lifetime, the then saleable value of the policy would form part of the wife's estate, and a grant of representation thereto would be necessary to enable her personal representatives to make a title to the policy. In these circumstances, on the husband's death in his wife's lifetime, a claim for Estate duty would arise on the policy moneys, as an interest purchased or provided by the deceased. Inasmuch, however, as the deceased never had an interest in the policy moneys, they would not be aggregated with his own free estate or any other property passing by his death, but they would constitute an "estate by itself" for the purpose of arriving at the rate of duty payable.

This principle of non-aggregation is of considerable benefit in small estates, *e.g.* consider that the deceased had effected such a policy for £400, and that the rest of his estate was £900. The duty payable would be 1 per cent. on the £400 and 2 per cent. on the £900, making £22 together, whereas if these two classes of property were liable to aggregation, the duty would be 3 per cent. on £1300, or £39, thus making a difference of £17 in the duty payable.

Policies on the life of a foreigner.

In cases where a person *dying domiciled abroad* has effected a policy of assurance on his life with a Company carrying on business in the United Kingdom, Section 19 of the Revenue Act, 1889 (52 and 53 Vict. c. 42), provides that the production of a British grant of representation shall not be necessary to establish the right to receive the money payable in respect of such policy. This section, however, does not exempt the moneys from liability to Estate duty under the Finance Act, 1894. Under Section 9 (1) of the Finance Act, 1894, the Crown has a charge on the moneys to the extent of its claim. The persons accountable for the duty are the persons to whom the moneys are payable by the Company. With regard to the liability of the Company, it would seem that if before the moneys were parted with by the Company, the Crown had notice of its intention to pay over, an injunction might be obtained on the part of the Crown to restrain the Company from paying over so much of the policy

moneys as represented the duty, until the Crown's claim was satisfied.

GENERAL REMARKS.

Full title necessary to ascertain duties payable.

It will be clear from the above short account of the various death duties that full particulars of the title under which an owner is entitled to a reversion must be known before any estimate of the duties payable on its falling into possession can be made. And not only so, but the various degrees of consanguinity between the different owners in the chain of title must also be known to enable a correct estimate to be given.

Not always supplied to actuary.

These particulars, I think, are not generally supplied by solicitors when submitting a case to an actuary for a Death Duty valuation, although of course they sometimes appear in the statement of a simple case. Possibly a request by the actuary for these particulars might be considered by the solicitors as unnecessary. They would perhaps prefer a valuation in the present terms as "subject to the payment of any death duties that may be payable," although in my experience they have never made any further deduction in respect of these duties from the valuation given subject to that limitation.

Usual type of Reversions.

In the great majority of cases the reversions which come before actuaries for valuation will be of a simple character, not involving a complicated chain of title. For it must be remembered that by far the most usual class of reversioners are the children and grandchildren of the testators or settlors, and (1) in the case of personal estate coming to the lineal issue of testators who died on or after 1st June 1881, and (2) in the case of both real and personal estate coming to the lineal issue of testators or settlors who died after 1st August 1894 there is no further duty payable under the title by which the reversion falls in.

Cases where no allowance for duties necessary.

Therefore in valuing such reversions as these for the purpose of paying duty in the reversioners' estate no allowance for duty should be made, because Probate duty or Estate duty is payable on the gross value. But if the reversions are valued for the purpose of sale or purchase after the reversioners' deaths, then deductions must be taken of the Probate or Estate duty, as the case may be, payable in the reversioners' estates.

In this large proportion of cases, and also in those cases where collateral relatives are the beneficiaries, and Legacy duty or Succession duty was paid on the capital in connection with the testator's death, no allowance (except as previously pointed out in

case of sale or purchase after the reversioner's death) requires to be made for duty.

While admitting that the market values of the funds at the date of death of the deceased reversioner are only to be used as a guide to the estimated value of these funds when they will fall in by the life-tenant's death, the following remarks on the values of property for the various death duties may be useful.

Values for various duties.

In the cases of Probate duty, Account Stamp duty, and Legacy duty, the value for duty is the market value of the property at the date of accounting for the duty together with all accumulations of rents, interest, dividends, etc., received and accrued up to the date of the account. In the case, however, of Legacy duty payable in connection with the death of a life-tenant, the duty is payable upon the property as it existed at the date of such life-tenant's death, and interest at 3 per cent. is added to the duty from that date.

In the cases of Estate duty and Succession duty the value for duty is the value at the date of the death which gave rise to the claim, and interest at 3 per cent. is payable on all duty in arrear.

Stocks and shares quoted on the Stock Exchanges.

There is no difficulty in arriving at the market value of stocks and shares quoted on the London, Edinburgh, or local Stock Exchanges, but in other cases a stockbroker's valuation, or a certificate from the secretary or some other responsible officer of the Company, is required. Shares in a private Limited Company, on the transfer or sale of which there are restrictions in the Articles of Association, are, for the purpose of Estate duty, to be valued at what, in the opinion of the Commissioners of Inland Revenue, they would fetch in the open market on the terms of the purchaser holding subject to the restrictions.

Market values for Estate Duty.

For Estate duty real estate is also to be taken at its market value at the date of the death giving rise to the claim. In this case, and also in the case of household effects, the best criterion of the value at the death is the result of a sale taking place shortly after the death, unless circumstances have arisen since the death to enhance or depreciate the value of the property.

Important exception.

To the general rule that, for Estate duty purposes, the market value is the value for duty, there is one very important exception. Purely agricultural property, where no part of the principal value is due to the expectation of an increased income from such property, the principal value shall not exceed twenty-five times the annual value as assessed under Schedule A of the Income Tax Acts, after making certain annual deductions, including Repairs, Insurance against

fire, and a sum not exceeding 5 per cent. of the gross annual value for expenses of management. Any agricultural property having a prospective building or mining value would not therefore come under this exception.

Full description required.

In every case a full description of the property is required, and an estimate is put upon it either by the accounting parties themselves or by their professional adviser, and this valuation is considered by the Estate Duty Office, and accepted, or questioned, or held over to await the result of a contemplated sale or otherwise as circumstances may warrant.

Market values of Policies.

With regard to the value of policies belonging to the deceased on the lives of other persons, the value for Estate duty is their market or saleable value, which is, of course, not always the same thing as their surrender value. As is well known, some Assurance Companies are more liberal in their surrender values than others, and no doubt in these more generous Companies the saleable and surrender values do not materially differ, except perhaps in the case of large with-profit policies.

And their surrender values.

It is apparently the fact that some surrender values in the early years of a policy are more than an outside purchaser could afford to give for these policies, even if he wished to make no more than 4 per cent. on his outlay.

It has also to be borne in mind that the owner of a small policy on the life of another would generally be well advised to accept the surrender value rather than resort to a sale with its attendant expenses and with the uncertainty of obtaining a better price than the Company will allow him.

Mining valuations.

Mining valuations are also of interest to an actuary. The general principle consists of an estimate by the mining expert, based on observations and the experience of the past, of the amount of coal or other minerals under the land, and of the time which will be taken in obtaining them. It is usually considered that the minerals will not be obtained at the same uniform rate of output throughout the period, but that a larger annual amount will be obtained during the earlier parts of the period than during the later parts.

The valuation of a mine, therefore, resolves itself into the valuation of immediate and deferred temporary annuities. As a rule, two rates of interest are used, the higher representing the remunerative rate desired on the investment, and the lower representing the sinking fund or accumulative rate at which the sinking fund may be re-invested in good securities to replace the capital at the end of the period.

As regards the immediate annuities, there is no difficulty in valuing these according to the well-known formula

$$a_{\overline{n}|}=\frac{s'_{\overline{n}|}}{1+is'_{\overline{n}|}}$$

where i is the remunerative rate and $s'_{\overline{n}|}$ is the amount of 1 per annum at the accumulative rate (j say).

Formula for a deferred temporary annuity at two rates of interest.

As regards the value of a deferred temporary annuity at two rates of interest, I do not know that a formula has ever been given for it, but it can be at once obtained with the aid of Mr. Lidstone's elegant expression for the present value of 1 at the end of t years on the conditions that throughout a period of n years ($n>t$) it shall pay interest at rate i and replace the capital at the end of the period of n years by a sinking fund invested at rate j. The present value of 1 under these conditions is

$$\frac{(1+j)^n}{1+is'_{\overline{n}|}}\times r'^t. \qquad (J.\ I.\ A.\ \text{xxxiii. p. 412.})$$

It follows that the value of a deferred temporary annuity of 1 at the two rates of interest to commence at the end of t years (*i.e.* first payment at end of $t+1$ years) and to continue for $n-t$ years is

$$\frac{(1+j)^n}{1+is'_{\overline{n}|}}(a'_{\overline{n}|}-a'_{\overline{t}|})$$

where the functions with dashes are based on the accumulative rate of interest j.

Incorrect formulas.

Although this is, I think, the correct formula, it does not appear to be used by mining engineers in their valuations. One of two incorrect formulas is generally used. Sometimes the value is taken as the difference between two immediate annuities, one for the whole period and the other for the period of deferment. This method, while correct if only one rate of interest is used throughout, is, of course, not correct when two rates of interest are used. The other incorrect method sometimes adopted is to find the value of the temporary annuity as if it were immediate, and to discount it during the period of deferment at the higher or remunerative rate.

It is not necessary to point out to actuaries that these methods are fallacious and do not adhere to the statement of the problem. In view, therefore, of the errors made in these calculations, it would seem advisable, when the mining experts have made their report on the prospects of the mine, that the necessary calculations should be submitted to an actuary.

Frequently annuities are payable under partnership deeds to the widow and children of a deceased partner for periods depending not only on life, marriage, etc., but on numerous other contingencies, such as the profits reaching a certain amount, one of the sons becoming a partner, etc., and it is desired to estimate the market value of these interests for the purpose of Estate duty. The chief factor in arriving at the value is, of course, the nature and stability of the business, because if it came to a standstill, all the charges thereon would be valueless. Even with a well-established and successful business a high rate of interest should, I think, be used, and as in such estimates it would not be easy to get satisfactory evidence of health, habits, and surroundings, certain margins would also have to be allowed. It may be argued that, inasmuch as these interests are rarely, if ever, sold in the open market, therefore they possess no "market" value, but that argument cannot, I think, be maintained; and probably the chief reason why they do not come into the market is because they are so much more valuable to their original possessors than they would be to a purchaser; in fact, like an ordinary interest for life or widowhood, their values change immediately they have been sold.

Benefits under partnership deeds.

Another case in which the question of the market value gives rise to some difficulty is that of shares in a private Limited Company, on the transfer or sale of which there are restrictions in the Articles of Association. In a case that came before the Courts [*Attorney-General for Ireland* v. *Jameson* (1905), 2 I.R. 218 (C.A.)] the deceased died possessed of certain fully paid £100 shares in a private Limited Company. The Articles of Association contained special provisions relating to the registration of the holders and to the alienation of the shares. The principal restrictive provision was that no share should be transferred to a person not already a member of the Company so long as any member was willing to purchase the same at the "fair value." The "fair value" was fixed, subject to a power of alteration which was never exercised, at £100 per share. The shares were paying a 20 per cent. dividend, and it was not disputed by the executors that their intrinsic value was much greater than the "fair value" as fixed by the Articles of Association. But it was contended that, as, owing to the restrictive nature of the Articles of Association, the shares could not be sold in the open market at all, and could only be sold subject to the right of the members to take them for £100 each, £100 was their only certain value.

The "Jameson" case.

When the case came before the Courts for decision, the Court of King's Bench, Ireland, were divided. A majority (Ke[illegible] and

Boyd, JJ.) held that the principal value of the shares ought to be ascertained with regard to the Articles of Association, but that such principal value might, in the opinion of an expert, exceed the "fair value" of £100, by reason that an existing member might be willing to buy at a higher price. Chief Baron Palles, however, was of opinion that the price was what would be fetched in the open market upon the terms that the purchaser should be registered as a member in respect of the shares.

The Commissioners of Inland Revenue appealed, and the Court of Appeal in Ireland ordered (19th Dec. 1904) that "The "principal value of the said shares is to be estimated at the price "which, in the opinion of the . . . Commissioners of Inland "Revenue, they would fetch, if sold in the open market on the "terms that the purchaser should be entitled to be registered as "the holder of the shares, and should take and hold them subject "to the said Articles of Association, including the Articles relating "to the alienation and transfer of the shares of the said Company."

The Court of Appeal were of opinion that the proper course was for the Commissioners to estimate the price under Section 7 of the Finance Act, 1894. The Commissioners were enabled to obtain expert evidence to guide them in applying the principle which the Court laid down, and the estimate of the Commissioners could be appealed against by the executors if they were dissatisfied. In consequence of this decision the executors made an offer which was accepted by the Commissioners.

Its effect on valuations of property not readily saleable.

This case has been detailed at some length, although perhaps it may not concern actuaries so much as interests involving life contingencies, because it seems to me that the principle involved is of wide application. For instance, when the question of the value of such interests of a widow and children under a partnership deed as were previously referred to, comes up for consideration, it will not, I think, be enough to put a nominal value on them, because they are not the usual investments of an Assurance Company, and because they would only be purchased by a speculator, and therefore the experience of the past is no guide to their value. These considerations would have to be taken into account in arriving at the value, but the main question is, "What would "an ordinarily careful man of business, whether within or outside "the family of the deceased, be reasonably expected to give to "stand, so far as these benefits are concerned, in the shoes of the "widow and children?" Surely this question can be answered with almost as much accuracy as that of the value of a reversion which, although complicated, is of a kind purchased by Assurance Companies, and in which therefore we have the experience of actual sales in the recent past to guide us. Possibly, however, this

construction may be an extension of the *Jameson* decision which is not justified, and I shall be glad to hear the views of the Faculty on the point.

Section 7 (5) of the Finance Act, 1894, enacts that "The "principal value of any property shall be estimated to be the "price which, *in the opinion of the Commissioners*, such property "would fetch if sold in the open market at the time of the death "of the deceased." So that if the Commissioners, who are able to obtain, and do obtain, expert advice in necessary cases, are of opinion that an interest of a kind which does not find a sale in the ordinary course in the open market, is of a certain value, it would not be enough, in support of a nominal value, to say, "We "should not think of buying it, and I do not think any other "Life or Reversionary-Interest Company would buy it," if the interest appeared, in view of all the facts, to be of considerable benefit to the present owners.

Conclusion.

In conclusion, I am conscious of the fragmentary character of these notes, but the subject of the Death Duties cannot be adequately treated in a paper of this size. At least I hope I have conveyed some general idea of their scope, which may be sufficient for ordinary purposes. Actuaries, I am told, generally specialise in some particular branch of their profession, and they cannot be expected to be experts in every subject which presents itself in some aspects of their work. To those, therefore, who do not desire to specialise in the subject of the Death Duties, or whose business occupations will not allow them to do so, this paper is offered with the hope that it may be useful to them and as a slight acknowledgment of the debt I owe to the Faculty of Actuaries.

SYNOPSIS OF THE DEATH DUTY ACTS

Duty	Acts	Duties Superseded, etc.	Property Liable	Persons Liable	Effect of Domicile or Locus on Liability
1. Probate Duty.	The Customs and Inland Revenue Act, 1881 (44 & 45 Vict. c. 12).	(*a*) Covers the 1 per cent. Legacy and Succession Duty payable by lineals. (*b*) Is superseded by the Estate Duty.	Personal property of the deceased, who died before 2nd August 1894, and personal property of which he disposed by will under a general power.	The executors and administrators.	The property must be British personalty.
2. Account Stamp Duty.	The Customs and Inland Revenue Acts of 1881 and 1889 (44 & 45 Vict. c. 12, and 52 & 53 Vict. c. 7).	(*a*) Covers the 1 per cent. and 1½ per cent. Succession Duty payable by lineals. (*b*) Is superseded by the Estate Duty.	Gifts of personal property and personal property voluntarily settled by the deceased who died before 2nd August 1894.	The trustees and beneficiaries.	The personalty must be locally situated in the United Kingdom or be subject of a British trust.
3. Legacy Duty.	The Legacy Duty Acts of 1796 and 1815 (36 Geo. III. c. 52, and 55 Geo. III. c. 184).	(*a*) The 1 per cent. Legacy Duty is not payable where Probate Duty or Estate Duty has been paid. (*b*) Legacies payable out of real property and proceeds of real property made liable to Succession Duty instead of Legacy Duty after 1st July 1888.	Personal property including proceeds of sale of real property given by will.	The executors, and if the legacies paid without deduction of duty, the legatees or their alienees.	The testator must have a British domicile.
4. Succession Duty.	The Succession Duty Act, 1853 (16 & 17 Vict. c. 51), and the Customs and Inland Revenue Act, 1888.	(*a*) Where Legacy and Succession Duties appear to be payable upon the same property by the same death, Legacy Duty only is in fact payable. (*b*) The 1 per cent. and 1½ per cent. Succession Duty payable by lineals, and the additional rate of 1½ per cent. imposed on Successions in 1888 not payable where Probate Duty or Account Stamp Duty or Estate Duty has been paid.	Real and personal property passing under a settlement, and real and leasehold property passing under a will.	The trustees, and successors and their alienees. The duty is a charge on the successions.	The property must be situated in the United Kingdom or be subject of a British trust.
5. Estate Duty.	The Finance Acts, 1894 (57 & 58 Vict. c. 30), 1896 (59 & 60 Vict. c. 28), 1898 (61 & 62 Vict. c. 10), 1900 (63 & 64 Vict. c. 7).	(*a*) Supersedes Probate Duty. (*b*) Supersedes Account Stamp Duty. (*c*) The 1 per cent. and 1½ per cent. Legacy and Succession Duties payable by lineals, and the additional rates of 1½ per cent. imposed on Successions in 1888 not payable in cases where Estate Duty is payable. (*d*) When the deceased's net free estate is under £1000, no other duty is payable.	Real and personal property passing or being deemed to pass on a death after 1st August 1894.	The executors in respect of property of which the deceased is competent to dispose. In the case of other property passing, the beneficiaries, trustees, or any other persons in whom the property or the management thereof is vested. In the latter case the duty is a charge on the property.	The free personalty of the deceased must be British. For other property the same rules as for Succession Duty apply.

DISCUSSION.

Mr MARR.—Some of the most interesting problems which come before the Actuary arise in connection with valuations of reversions and other limited interests and apportionments of settled funds as between the life-tenant and remaindermen. The principles of such calculations are now well established, but in all cases with which I have had to deal I have experienced great difficulty in deciding how to treat the death duties, and, personally, I owe a debt of gratitude to Mr. Denmead for his paper. We are to be congratulated that he, a member of our Faculty and a barrister at law, with special experience in questions of death duties, has given us such a contribution to our *Transactions*. Never was the idea of such a paper more present in my mind than on one occasion when engaged in an apportionment case. I inquired of the solicitors in the case what duties would be payable, and received the answer indicated by Mr. Denmead on p. 182: "We have no idea; make your calculations subject to the payment of any duties." The fund, however, was a large one, and the matter could not be so summarily dismissed.

In questions of apportionment it seems important not only to take into account the various duties which will become payable, but also to consider to whose share—the life-tenant's or the remainderman's—they should be allocated. When legacy or succession duty is payable in respect of the reversioner's interest, it should, I think, be taken into account in such a way as not to affect the share of the fund due to the life-tenant. Estate duty may, however, be on a different footing. Let it be regarded as a deferred tax payable at death in lieu of an additional income tax payable throughout life. We are told in the Report of the Select Committee on Income Tax issued last week that Sir Henry Primrose calculates that if the State had levied a special additional income tax, graduated as the death duties are, in order to raise the revenue actually produced by the death duties in the last ten years, such additional taxes would have ranged from 6d. per £ per annum on estates yielding an income of from £40 to £400 a year, to 1s. per £ per annum on estates yielding from £4000 to £6000 a year, and 1s. 3¾d. per £ per annum on estates yielding £40,000 and upwards a year. Take the case of settled property: In terms of the 1894 Act, estate duty is payable only once upon property passing under a settlement until it passes to a person competent to dispose of it. Assuming the fund to be settled by deed dated more than one year previous to the date of the settlor's death, the only payment of duty will be due on the death of the life-tenant, if she is not the wife of the settlor, and I think we must regard a portion of the duty then payable as a deferred tax due by the life-tenant in lieu of a higher rate of income tax than is charged on the life-interest, and, but for which, such higher rate would have been charged. Of course, on the other hand, the mere fact of apportioning a fund between life-tenant and reversioner is to throw into the life-tenant's separate estate a sum of money on which estate duty may ultimately be payable at his death. With regard to commutation of duty, Mr. Denmead remarks that interest is to be taken at 3 per cent. I take it that the various contingencies and sums payable should be discounted at 3 per cent. irrespective of the rate of interest actually yielded. It seems

strange that such a low rate should be specified when the "market values" are generally discounted on a 4 per cent. or 5 per cent. basis. Again, the Tables annexed to the Succession Duty Act do not differ much from the Government Annuitants' Experience, 1829, with 4 per cent. interest. Does the Crown recognise quotations based on those Tables at 3 per cent.? In any case it would surely be worth while to revise them in view of the Tables now available, embodying fuller and more recent statistics. On p. 181 Mr. Denmead deals with the question of payment of a claim by a Life Office without production of a British grant of representation. Is Mr. Denmead of opinion that in such cases, if notice of intention to pay were given to the Crown and no immediate action to restrain was taken—say within a specified number of days—the Office would be freed from further responsibility in regard to payment of duties? The same question sometimes arises in settling claims on policies effected under the Married Women's Property Act, 1870. Dr. A. E. Sprague, in his paper "Titles to Life Assurance Policies" (*T. F. A.*, ii, 360), states that evidence should be furnished of payment of all Government duties before the claim is paid. The interpretation of Section 7 (5) of the 1894 Act, on which Mr. Denmead invites our views, is of special interest to us in Scotland just now. A case came before the Court ten days ago (*Lord Advocate* v. *Marr*, etc.) in which additional duty of £500 is claimed because an estate valued for probate at £11,452—mainly comprised of a herd of cattle—realised at a sale four months later £17,722, 17s. The sale price, to judge from the reports of the case in the daily papers, was influenced by special conditions, and it will be interesting to learn the Court's decision.[1] Somewhat similar is the question: What value is to be placed on a policy of assurance for probate purposes? Mr. Denmead remarks (p. 184) that the value is the market or saleable value "which of course is not always the same thing as the surrender value." The surrender values of Offices do certainly differ, and no doubt the prices realised at auction sales will be relatively affected. One frequently hears that surrender values quoted for probate purposes are objected to as being too small by from 10 to 20 per cent. It seems to me that if the Crown is of opinion that such increased values represent the market or saleable values, its estimate is too high. I doubt whether, if say one hundred consecutive policies in any Office were put up for sale, the price realised for the lot would exceed or even equal their total surrender value, if it were possible, in fixing the sale prices, to leave out of account the Office quotations.

As regards the Crown's right to reopen the question of value, I must confess to a feeling that when an estate has been valued and the duty paid, the transaction should be considered at an end. The Crown should at the time take due precaution to satisfy itself as to the reliability of the valuation, and should not afterwards raise questions if the estate realise a higher price at a subsequent sale. You might as well expect one who has made a profit out of the purchase and subsequent sale of stocks or other property to hand over part or the whole of his profit to the original vendor.

Dr. A. E. Sprague.—I have often heard expressions of dissatisfaction with the Estate Duty Officials, and it is undoubtedly the fact that they sometimes attempt to obtain more than they are legally entitled to. Some people say that this is because the general public try to evade Estate duty by means of underestimates, and that consequently the Inland Revenue have, in defence, to try to balance this by making overcharges. I do not agree with this, but believe that, in the exercise of their duty and in perfect good faith, they look at things a little too exclusively from their own point

[1] Lord Johnston has given judgment against the Crown. Whether the Crown will appeal is not yet known.—Ed.

of view, viz., the collection of revenue, and that this leads them sometimes to construe a statute more favourably to the Revenue than is intended, and to overestimate the value of the estates they deal with. For example, a few years ago there was so much discontent with the values they placed on reversions, that the Institute of Actuaries sent a deputation to interview them. Since then (and, I believe, as a consequence of that deputation) they have appointed a qualified actuary in the person of Mr. Denmead, and matters are now much improved. As another example I may refer to a case which came before me last year, where an item in an estate consisted of a policy on the life of a third person, and the Inland Revenue wanted to add 25 per cent. to the value as given up, until I pointed out to them that the value was over 80 per cent. of the face value, and that the proposed addition would make their "market value" exceed the face value of the policy, including all bonus additions to date. With regard to foreign grants of representation, my Office paid, a few years ago, a claim on production of an American Probate. Shortly afterwards the Inland Revenue told us we had no right to pay without a British grant, and we had to refer them to the Customs and Inland Revenue Act cited in the paper, under which we had paid. No doubt their oversight of that Act was a clerical error on their part; but they subsequently contended that we should not have paid the claim without first accounting to them for the duty. The effect of this contention would have been to make us their agents for the collection of Estate duty, and there is apparently no authority for this. I am, therefore, glad to learn from Mr. Denmead that the Estate Duty Officials now recognise that an Assurance Company is right in paying over the full sum assured to the holder of the grant; though, of course, they may, if they know about it, interdict the Company from paying until their claim for Estate duty is settled. Turning now to the section on gifts and settlements of policies, I do not understand how it can be laid down that a paid-up policy which, more than twelve months before death, has been assigned absolutely by way of gift, still remains liable to Estate duty on the donor's death. I do not see the distinction between this and the gift of a sum of money, for the latter might be used to buy the policy, in which case there would be no claim for duty. There is one point not referred to in the paper, as to which I should like to have Mr. Denmead's opinion. Life Offices occasionally receive Probates or Letters of Administration in which the gross amount of the estate is set down at a sum less than the policy money. Is it our duty in such cases to make inquiry or to inform the Revenue Officials of what is going on? Mr. Barrand discusses the question in his paper (*J. I. A.*, xxxiii, 237), and seems to indicate that no definite decision could be got from the Somerset House officials. From various legal opinions that have come before me, I gather that the Company is bound to pay over the whole of the policy money to the person named in such a grant, but perhaps Mr. Denmead may give us his view. In conclusion, I should like to join in thanking him for his interesting and valuable paper.

Mr. CHATHAM.—I desire to associate myself with the other speakers in thanking Mr. Denmead for coming here to-night and giving us such an excellent paper. He has taken us through the mazes of his subject with the skill of a master hand, and when I arrived towards the end of the paper I thought that at last I knew all about the death duties; but in the last paragraph I found I had only reached a "clearing," and that there was still much to be done. The question then occurred to me whether it would not have been better if Mr. Denmead had confined himself to the Estate duty, leaving out the duties which are practically obsolete, or at least relegating them to a back instead of a front seat; but no doubt Mr.

Denmead had a good reason for the method he had adopted. At the same time I must admit my reading of the paper confirmed me in the impression I formed long ago, and have always acted upon, that the determination of the death duties is not a question for the Actuary. Notwithstanding what has been said, I think it would be dangerous for an Actuary to take up a complicated subject of this kind where his knowledge must necessarily be imperfect. For myself, although I am no longer concerned in the matter, I hold it is a question for the Lawyer rather than the Actuary. We are more especially interested in, and very much indebted to Mr. Denmead for his remarks regarding policies of Assurance. In regard to Section 19 of the Revenue Act, 1889, which deals with the case of persons dying domiciled abroad, it should be noted that the Company is not *compelled* to pay over the policy moneys to the persons entitled, without requiring production of a British grant of representation. All the Act says is that the production of such a grant shall not be necessary. It does not say what is necessary; it is negative, not affirmative, and, according to the best advice we can get, a Company paying under that section is still liable to be called in question by creditors in this country. One part of the paper as to which I find myself in substantial agreement with Mr. Denmead is that where the question of market values is dealt with. I know that that question has exercised the minds of other Actuaries, and that they have had grave doubts upon the subject. To my mind there is no one better fitted than the Actuary to come to a decision upon a question of such importance; especially if, in the course of his duties, he has to consider the various investments coming before an Assurance Company. That I hold to fall within the Actuary's province, and in all my experience I have never known of a single case in which the market value I arrived at was not accepted by the parties, even although it differed substantially from the figures which they themselves had in view. A short time ago a case came before me of a division of shares in a private limited Company between a life-tenant and a reversioner, where the Company could exercise a right of pre-emption. The Company referred to dealt in articles manufactured under a patent. It was doing very well at the time, but of course might not, under other circumstances, do so well. The parties were at first surprised at the smallness of the number of shares that I allotted to the reversioner, but on my explaining fully the position of matters they expressed themselves as being quite satisfied. Mr. Denmead says at the close of his paper that he owed a debt to the Faculty, but I think we shall all unite in acknowledging that he has discharged that debt in a very handsome manner indeed.

Mr. Wm. Stuart Fraser, W.S.—Speaking as a member of the legal profession, I am sure Mr. Denmead's paper cannot fail to prove of great value to those who have to do with the subject of the death duties. I do not, however, propose to discuss in detail the many points involved, especially as my own practical experience relates to the older duties rather than to the provisions of the more recent and complicated Statutes. Dr. A. E Sprague has related what seems to me a curious experience with regard to the amounts sometimes set forth in Probates or Letters of Administration. I should not care to say off-hand what is the precise effect of such documents under English Law. So far as Scotland is concerned, however, and speaking very much from recollection, my impression is that Confirmation is a title merely to the Estate contained in the Inventory, which I need hardly remind you is a detailed statement and is usually attached to the Confirmation. If, in such a case, it were found that a policy of assurance had been omitted, I scarcely think the Office could be advised to make payment, and similarly, in the case where a manifestly insufficient value

had been placed upon a policy, I should hesitate to advise that the Office was bound to pay.

Mr. H. GLANVILL, Registrar of Estate Duties, Inland Revenue, Edinburgh.—We of the Estate Duty Office were on one occasion described by a taxpayer as "experts with predatory instincts in favour of Revenue." I can, however, assert with confidence that in the exercise of our official duties we are only guided by one principle, viz., that of holding a just balance between the taxpayer on the one hand, and the Crown on the other. I can also assure the Faculty generally that any actuarial valuation which is produced to us is always considered with the greatest respect, and usually, I think, is adopted as overriding any preconceived notions we may have formed upon the matter in question. The subject of the present paper is one on which it is exceedingly unsafe to dogmatise or to speak "without book." All I should like to say is that I and my colleagues will at all times be happy to place our official experience at the service of any member of either the actuarial or the legal profession in regard to any difficult matter which it is thought we might help to elucidate. It is, I may add, an everyday occurrence with us to formulate presumptive claims for duties expectant on death for the guidance of intending purchasers of reversions.

Mr. PERCIVAL WAUGH, Estate Duty Office.—In regard to the question of the surrender value as against the market value of a policy, I may refer to the opinion of the late Lord President Inglis on which we in the Estate Duty Office rely to some extent. In the case of *Pringle's Trs.* v. *Hamilton*, (10 Macph. 621), his Lordship said it was a matter of notoriety that the surrender value and the market value were not the same thing, the former being less. The surrender value is merely the consideration offered by the Company for taking over the policy irrespective of the health of the assured. As regards the incidence of Estate duty, the question as between life-tenant and reversioner really does not arise if the whole property is taxed on the death of the owner. As I understand it, the whole of the duty comes out of the capital of the fund, and the effect as regards the life-tenant is an abatement of income proportionate to the amount of the duty. I may quote in point the English case of In re *Parker Jervis*; *Salt* v. *Locker* [1898] 2 Ch. 643. In that case there was an annuity to a widow, and the question of Estate duty arose. The Court held that you could not assume the annuity to be on an average life, as the life might be good or bad. It was necessary to ascertain what proportion of the whole estate was required to produce the annuity; and, a rateable part of the duty being payable on the ascertained proportion, the annuitant was to pay interest during her life on that part. The difficulty raised by Dr. Sprague in connection with certain Probates is one that I think can hardly arise under Scotch Confirmations. As Mr. Fraser has already stated, the Inventory here contains every item of the moveable estate passing, and therefore it ought to be evident to the Company whether or not it can pay over to the executors. In England it seems possible, by deduction of debts, to show a smaller estate covered by the Probate than the amount of the policy.

Mr. PAULIN.—I observe that on one point Mr. Denmead pays the Faculty the compliment of asking an expression of its views on a question that has perplexed him, and I should think many others. He refers (p. 186) to the *Jameson* case as a typical instance of the difficulty of valuing for Estate or Legacy duty the shares of a private limited Company on the transfer or sale of which there are restrictions in the Articles of Association. It is a question between a traditional "fair value" and the "intrinsic" value. The shares in question were par value £100,

paying a 20 per cent. dividend, and the restrictive provision regarding them was that no share could be transferred to a person not already a member of the Company so long as any member was willing to purchase the same at the "fair value." After the case had been litigated at some length in the Irish Courts, and the Court of Appeal had indicated that they were against the idea of a traditional "fair value," the executors came forward and made an offer which was accepted by the Commissioners. It would be very interesting if Mr. Denmead felt himself at liberty to tell us what that offer was. Such a restriction as is here described is not uncommon in the articles of private limited Companies in Scotland. A right of pre-emption is given to the other shareholders, limited, however, in some cases by the right of the Trustees of a deceased shareholder to transfer his shares to heirs by will or heirs at law. It is most difficult to assess the value of shares which have such a restriction. The deceased shareholder may have been the mainspring or moving spirit of the Company, and his death would certainly depreciate the value of the shares. A not uncommon practice in Scotland is to refer the valuation of the shares to the Auditor of the Company. Another method occasionally adopted is to refer the matter to a man of skill, who is acquainted with the class of property to be valued, and who would take into account the handicap on the value created by the clause of restriction. The assessable value of the shares would depend largely on the history of the Company and the nature of the business. The whole subject of Estate and Legacy duties is well within the province of the Actuary, who has to deal in various ways with reversionary interests. On the question of payment of the claim under a policy on a person dying domiciled abroad without the production of a British grant of representation, I entirely agree with Mr. Chatham that the Statute does not oblige the Company to pay, and, in fact, Mr. Denmead speaks of the liability of the Company if it pays the money before the Crown's claim is satisfied. I should like to see any dubiety on this point thoroughly cleared up.

Mr. J. J. Waugh, B.L.—As one who has been in personal contact with the officials of the Estate Duty Office, I may be permitted to say that my experience on the whole confirms Mr. Glanvill's assurance that the Department tries to ascertain what the law awards to it and does not in practice ask any more. I wish Mr. Chatham's belief that application to a Solicitor would invariably lead to a successful definition of the duties payable in a particular case were well founded. Unfortunately it frequently happens that we cannot say with certainty what claims do affect the estate, and on such occasions we are glad to apply to the Estate Duty Office and learn what claims they have noted. When an estate comes first within their cognisance it is their practice to note down what claims will arise at future dates, and perhaps Mr. Denmead may be able to say how far such information is communicated to an inquirer. I hope any future Estate Duty Acts will contain, as the 1894 Act does, a clause protecting purchasers of reversions from any increased duties which could not be within their view when the purchase was made. It is quite possible, however, that already the screw has been turned, if I may use the phrase, as far as is profitable. Certainly the duties are at present felt somewhat burdensome, and in order to escape them, people frequently divide property which would otherwise be liable at their death; and this practice is commoner with the very rich than in the case of estates of moderate amount, in other words, with those estates that would yield the largest sums to the Revenue. As regards reversions, it might be supposed that the Government would tax every benefit as it arises, but that is subject to the important exceptions mentioned in the paper, the chief of which perhaps is that when a legacy is

left to two or more persons in succession who are liable to duty at the same rate, the capital is taxed only once. Practically all benefits arising on death are now subject to taxation at some time, and the explanation is that the whole "code" or statutory law of Estate duties is very largely the work of the Inland Revenue Officials. It is an easy matter to get a remedial Statute through Parliament to meet any attempted evasion, and the result is that the death duties are not to be got round by the ingenuity of any conveyancer.

Mr. RONALDSON.—As regards the alternative methods of paying duty when Reversions form part of the deceased's estate (p. 176), I may point out that in some cases it would be advisable to pay at once on the value of the Reversions rather than postpone payment of the duty until the Reversions fall in, since the difference between the value in possession and the present value might increase the total value of the estate so as to bring it under a higher rate of duty. I take it, however, as Mr. Denmead indicates, that the duty on the rest of the estate would be on the scale applicable to the value of that portion with the addition of the *present value*, and not with the addition of the value in possession. Mr. Denmead points out (p. 178) that where an interest in expectancy is mortgaged for full consideration before 1894 (Section 21 (3) of the Finance Act, 1894), the mortgagee pays no other duty than would have been payable if the Act had not been passed. But if the mortgagee sells, in what position is the purchaser? Would he pay the old duties or the new? If he had to pay the new, the price would be smaller, and if the price did not cover the sums due under the mortgage and expenses, this provision in favour of the mortgagee would not in such a case be any protection to him.

THE PRESIDENT.—I must dissociate myself from Mr. Chatham's view regarding the Actuary's and the Lawyer's work. We all know that the Actuary has a very wide outlook and that the Lawyer is indebted to him, as he is to the Lawyer. Things go much better when they assist each other in these complicated matters. I have much pleasure in proposing a very hearty vote of thanks to Mr. Denmead for the great labour and pains he has taken in preparing what will be a valuable addition to our *Transactions*, and for his trouble in coming from London to submit his paper in person. It is important to Actuaries and to Life Offices to know that there is in Somerset House an Official such as Mr. Denmead who can talk our language. I should like to say that my own observation and my own experience go far to show that Inland Revenue Officials have an honest desire to act fairly and justly to all parties. If we take them the right way and deal with these matters in a businesslike spirit, they are quite willing to consider everything reasonable. We are glad to learn that those Government Officials are appreciating more and more the work of the Actuary, who is always glad to assist and is prepared to explain the principles he follows and the conclusions at which he arrives. I have already occupied too much of your time, and I hope you will adopt with acclamation the vote of thanks I have proposed.

Mr. DENMEAD having thanked the President and the members of the Faculty for the kind reception accorded to him and his paper, replied to some of the points raised in the discussion as follows:—Mr. Marr referred to a case of apportionment between a life-tenant and a reversioner, and asked if, when the future duties were ascertained, they should also be apportioned. I think that, inasmuch as those future duties must come out of the reversioner's interest, they should be provided for entirely out of the share ascribed to him. Mr. Marr also suggested that when once an account has been assessed it should not be reopened when a subsequent sale shows that the property in the account was undervalued. In practice,

accounts are not reopened after the lapse of any considerable time from the death on which the duty became payable. But in the absence of any infallible method of estimating the value of property at the time of rendering the account, I do not think that Mr. Marr's suggestion is a good one in the interest either of the public or of the revenue, because presumably the adoption of his suggestion would prevent a return of duty if the property sold for less than its estimated value. It seems to me advisable to retain the present system of making the assessment of duty subject to rectification should better evidence of the true value of the property be forthcoming. As Mr. Marr says with reference to commutations, the Tables attached to the Succession Duty Act, 1853, are antiquated and 3 per cent. is a low rate to use. The rate is, however, fixed by Statute in the case of Estate duty at 3 per cent., but it is not necessary to use these tables, although in practice they are used where possible. The amount of the commuted duty is not quite the same thing as the "market value" of the future duty, and although I cannot defend the 3 per cent. rate, I do not think any great injustice is done by the use of the old tables. Of course these tables and this rate are quite unsuitable for "market values" of taxable interests, but in the case of these interests there is no necessity to use any specified tables of mortality or rate of interest. Referring to Mr. Marr's question whether a Company would be relieved from responsibility for duty if it paid a claim without a grant under Section 19 of the 1889 Act after giving notice to the Revenue, I should say that such a notice, not being provided for by Statute, could have no legal effect, but would undoubtedly have the effect of enabling the Company to ascertain as early as possible what liability, if any, attached to it. Dr. Sprague thought no claim for Estate duty should arise on the death of the donor in respect of the gift of a fully paid-up policy to a donee more than twelve months before the donor's death. His view that it should be regarded purely as an *inter vivos* transaction is arguable, but the claim is made under Section 2 (1) (*d*) of the Finance Act, 1894, which provides that property deemed to pass shall include "any annuity or other interest purchased or provided by the deceased, either by himself alone or in concert or by arrangement with any other person, to the extent of the beneficial interest accruing or arising by survivorship or otherwise on the death of the deceased." The "interest" referred to in this section appears to include such a policy (see *Attorney-General* v. *Murray and another* [1904] 1 K.B. 165), and when the donor dies, a "beneficial interest" provided by him does accrue to the donee. Dr. Sprague also raised the interesting question as to how far, if at all, an Assurance Company was liable for duty on paying a claim when the Probate showed gross assets of less value than the amount assured. It is impossible to dogmatise on this difficult question, but I would point out that the only case in which a personal representative can obtain a grant of representation without full disclosure of the assets is where he is able to swear that any property not included is property of which he does not know the amount (see *In the goods of Beech*, "Times," 9th August 1904, and *In the goods of Baron Grimthorpe, Beal* v. *Baron Grimthorpe*, "Times," 6th June and 1st and 8th August 1905), and a policy would not as a rule come within this category. Generally, therefore, in the case supposed it would follow that the grant had been obtained by means of a false affidavit. If a payment were made under these circumstances, and the Revenue suffered loss, it does not seem unreasonable to suppose that the Company might be held responsible. Otherwise there is no protection afforded to the Revenue by the certificate on the grant. In any case there are certain moral duties imposed upon all citizens, and the least a Company could do in such a case

would be to call the attention of the Revenue to the fact. Referring to Mr. Chatham's remarks, I should like to say that my reason for giving some account of the Probate and Account Stamp duties, which have been practically superseded by the Estate duty, was chiefly for the purpose of comparison, and because of their bearing upon the Estate duty in the matter of exemption, although it must be borne in mind that these duties (or at any rate the Probate duty) are still payable in appropriate cases, and must be taken into consideration when dealing with reversionary interests. With reference to Mr. Paulin's instance of a Company under whose Articles of Association the price of shares was fixed by the auditors, I do not doubt that such price would be accepted in a valuation for Probate purposes, because it would show what competent persons, who were valuing for other than Probate purposes, considered to be the market value. I may mention that the compromise offered by the executors in the *Jameson* case was the value originally suggested by the Commissioners. Presumably, if such a compromise had not been arrived at, the Commissioners would have consulted some well-known stockbrokers, and acted on their advice. Mr. J. J. Waugh asks if information as to the future duties payable in a particular case would be given by the Estate Duty Office to an interested inquirer. My impression is that the fullest possible information would be given in such a case, and this is borne out by Mr. Glanvill's remarks. Referring to Mr. Ronaldson's remarks, and taking a concrete case, a man may die possessed of a reversion worth £2000, and other property in possession worth £7000 (total £9000). If Estate duty is paid upon the whole immediately after his death, 3 per cent. duty would be paid upon the total of £9000, and no further Estate duty would be payable when the reversion fell into possession. If, however, duty is not paid upon the reversion until it falls into possession when it is worth (say) £4000, the 3 per cent. would be paid upon the £7000 immediately after the death of the deceased as being the rate appropriate to the then value (£9000) of his estate. And when the reversion fell into possession, duty would be payable upon £4000, but at 4 per cent., because then the £4000 would have to be added to the £7000, making a total of £11,000, on which 4 per cent. is the appropriate rate. In such a case as this, which is not unusual, two different rates of duty are payable upon different parts of the free estate of the deceased. As regards Mr. Ronaldson's other point, whether the purchaser from a mortgagee of property, mortgaged for full consideration before 1894, would pay the old duties or the new, it seems to me quite clear that the old duties and not the new would be payable.

CORRESPONDENCE

TABLE for obtaining m_x from q_x

To the Editor of the Transactions of the Faculty of Actuaries.

SIR,—The accompanying table * has been made up to enable m_x to be obtained by inspection from q_x for all values of q_x not exceeding ·06940, or in other words, where the difference between m_x and q_x is less than ·00250. The table is of use in getting out the central death, marriage or secession rates in making up combined marriage and mortality, secession and mortality tables, etc. For example, for all values of q_x from ·00316 to ·00546 inclusive, the addition to obtain m_x is ·00001; thus, if $q_x = ·00435$, $m_x = ·00436$.

The table covers the bulk of the values required in any case, and to carry it to higher values of q_x would make it of unwieldy length. The values of m_x beyond the limits shown are most easily obtained from a table of reciprocals, as the difference between the reciprocals of q_x and m_x is ·5.

I may add that the table was constructed by finding values of q_x corresponding to the values of $m_x - q_x$, or Δ, = ·000005, ·000015, etc., the formula being $q_x = \frac{1}{2}\{(\Delta^2 + 8\Delta)^{\frac{1}{2}} - \Delta\}$. This gave the limiting values of q_x for the values of Δ = ·00000, ·00001, etc., and the correctness of these limiting values was afterwards checked inversely by the ordinary formula for m_x.

I am, &c.,

ALEX. FRASER.

19 ST. ANDREW SQUARE,
EDINBURGH, 11*th September* 1906.

* See p. 200. The table of course holds good for any two quantities x and y so related that $y = \frac{x}{1+\frac{1}{2}x}$, or $x = \frac{y}{1-\frac{1}{2}y}$ where y is given. *E.g.* if y be the rate of interest on the average of the funds of an Office for any year, say 3·848 per cent., the addition for the effective rate—Mr. G. F. Hardy's formula—is shown by the table to be ·075.—ED.

TABLE *showing addition* (Δ) *to* q_x *to obtain* m_x.

q_x = or <	$\Delta \times 10^5$	q_x = or <	$\Delta \times 10^5$	q_x = or <	$\Delta \times 10^5$	q_x = or <	$\Delta \times 10^5$	q_x = or <	$\Delta \times 10^5$
·00315	0	·03152	50	·04433	100	·05411	150	·06232	200
·00546	1	·03183	51	·04455	101	·05429	151	·06248	201
·00705	2	·03214	52	·04476	102	·05446	152	·06263	202
·00834	3	·03244	53	·04498	103	·05464	153	·06278	203
·00946	4	·03274	54	·04519	104	·05482	154	·06293	204
·01046	5	·03304	55	·04541	105	·05499	155	·06309	205
·01136	6	·03333	56	·04562	106	·05516	156	·06324	206
·01221	7	·03362	57	·04583	107	·05534	157	·06339	207
·01299	8	·03391	58	·04604	108	·05551	158	·06354	208
·01373	9	·03420	59	·04625	109	·05568	159	·06369	209
·01443	10	·03448	60	·04646	110	·05586	160	·06384	210
·01510	11	·03476	61	·04666	111	·05603	161	·06398	211
·01574	12	·03504	62	·04687	112	·05620	162	·06413	212
·01636	13	·03532	63	·04708	113	·05637	163	·06428	213
·01695	14	·03559	64	·04728	114	·05654	164	·06443	214
·01752	15	·03586	65	·04748	115	·05671	165	·06458	215
·01808	16	·03613	66	·04769	116	·05687	166	·06472	216
·01862	17	·03640	67	·04789	117	·05704	167	·06487	217
·01914	18	·03667	68	·04809	118	·05721	168	·06502	218
·01965	19	·03693	69	·04829	119	·05738	169	·06516	219
·02014	20	·03719	70	·04849	120	·05754	170	·06531	220
·02062	21	·03745	71	·04869	121	·05771	171	·06545	221
·02110	22	·03771	72	·04888	122	·05788	172	·06560	222
·02156	23	·03797	73	·04908	123	·05804	173	·06574	223
·02201	24	·03822	74	·04928	124	·05821	174	·06589	224
·02245	25	·03848	75	·04947	125	·05837	175	·06603	225
·02288	26	·03873	76	·04967	126	·05853	176	·06618	226
·02331	27	·03898	77	·04986	127	·05870	177	·06632	227
·02373	28	·03923	78	·05005	128	·05886	178	·06646	228
·02414	29	·03947	79	·05024	129	·05902	179	·06661	229
·02454	30	·03972	80	·05043	130	·05918	180	·06675	230
·02494	31	·03996	81	·05063	131	·05934	181	·06689	231
·02533	32	·04020	82	·05081	132	·05950	182	·06703	232
·02571	33	·04045	83	·05100	133	·05966	183	·06717	233
·02609	34	·04068	84	·05119	134	·05982	184	·06732	234
·02646	35	·04092	85	·05138	135	·05998	185	·06746	235
·02683	36	·04116	86	·05157	136	·06014	186	·06760	236
·02719	37	·04139	87	·05175	137	·06030	187	06774	237
·02755	38	·04163	88	·05194	138	·06046	188	·06788	238
·02791	39	·04186	89	·05212	139	·06062	189	·06802	239
·02825	40	·04209	90	·05231	140	·06078	190	·06816	240
·02860	41	·04232	91	·05249	141	·06093	191	·06830	241
·02894	42	·04255	92	·05267	142	·06109	192	·06843	242
·02927	43	·04277	93	·05285	143	·06124	193	·06857	243
·02961	44	·04300	94	·05304	144	·06140	194	·06871	244
·02993	45	·04322	95	·05322	145	·06156	195	·06885	245
·03026	46	·04345	96	·05340	146	·06171	196	·06899	246
·03058	47	·04367	97	·05358	147	·06186	197	·06912	247
·03090	48	·04389	98	·05376	148	·06202	198	·06926	248
·03121	49	·04411	99	·05393	149	·06217	199	·06940	249

THE FACULTY OF ACTUARIES IN SCOTLAND

TRANSACTIONS

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OF THE

FACULTY OF ACTUARIES

No. 32

Paid-up Policies and Surrender Values. By Alfred Ernest Sprague, M.A., D.Sc., F.F.A., F.I.A.

Abstract of Discussion thereon

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1907

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TRANSACTIONS

OF THE

FACULTY OF ACTUARIES

VOL. III.—PART VIII.

No. 32

Paid-up Policies and Surrender Values. By Alfred Ernest Sprague, M.A., D.Sc., F.F.A., F.I.A.

Abstract of Discussion thereon

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OF THE

FACULTY OF ACTUARIES

Paid-up Policies and Surrender Values. By ALFRED ERNEST SPRAGUE, M.A., D.Sc., F.F.A., F.I.A., *Assistant Secretary to the Edinburgh Life Assurance Company.*

[Read before the Faculty, 14 January 1907.]

THE principles upon which surrender values should be calculated are discussed at length in papers by Mr. Macfadyen (*J. I. A.*, xvii, 381); Mr. Crisford (*J. I. A.*, xxi, 301); and Mr. Fulford (*J. I. A.*, xxxv, 199), but none of the methods proposed appear entirely satisfactory in practice, and I have therefore attempted to devise a scheme which shall not only give results in reasonable accordance with strict theory, but also prove simple and easily worked in its practical application.

Mr. Macfadyen held the view that the surrender of a policy should be regarded as the purchase of a reversion, and that the price should be the net value of the policy by the Carlisle table, with interest at the rate of 5 per cent or 6 per cent per annum. In an earlier paper (*J. I. A.*, xv, 297) he showed that under ordinary circumstances the paid-up policy to be allowed for a

(Notation: Throughout this paper P, P′, and P″ respectively denote Office annual premiums for assurances "without profits," "with profits," and "with discounted bonuses"; and π and π' respectively denote net annual premiums for uniform assurances and for assurances increasing at a fixed rate per annum.)

limited-payment or endowment assurance policy might safely be arrived at by reducing the original sum assured in the ratio of the number of premiums actually paid, to the full number originally payable. This is now generally known as a "proportionate" paid-up policy, and will hereafter be referred to under that name. He did not, however, suggest any method of calculating the amount of the paid-up policy in the case of ordinary whole-life assurances, and no scheme can be considered complete which omits so important a matter.

Mr. Crisford based his surrender values on the reserve held by the Office, making deductions therefrom to allow for the possibility of loss to the Office through withdrawal of healthy lives, and for the loss of future contributions to expenses. These deductions were: (1) 5 per cent of the reserve to allow for possible adverse selection: (2) $2\frac{1}{2}$ per cent of the value of the future Office premiums, to provide for future expenses: so that his formula was (for a whole-life policy for £1)

$$\text{Surrender Value} = \cdot 95 \times {}_nV_x - \cdot 025\, P_x a_{x+n}.$$

He arrived at the paid-up policy by reducing the sum assured in the ratio of the reserve to the value of the future premiums plus the reserve, the formula being (leaving bonuses out of account)

$$\text{Paid-up Policy} = \frac{{}_nV_x}{{}_nV_x + P_x a_{x+n}}.$$

Mr. Fulford also held that the surrender value should be equal to the reserve less deductions for (1) expenses, and (2) adverse selection; but he arrived at the amount of his deductions in a different manner from Mr. Crisford. Owing to the fact that "select" mortality tables have come into general use, we are now able to calculate the reserve that should be made for a life that is still "select"; and Mr. Fulford naturally adopted this value in place of making an arbitrary deduction of five per cent from the Office reserve. He also modified the allowance for 'expenses' in view of the changed conditions under which the business is now carried on, arriving finally at the formula (for a whole-life policy for £1),

$$\text{Surrender Value} = {}_n(hV)_x - \left\{\frac{\cdot 021}{a_{[x]}} + \cdot 025\, P_x\right\} a_{[x+n]}$$

from which the paid-up policy was deduced by dividing it by a loaded single premium.

Both Mr. Crisford's and Mr. Fulford's methods are complicated, and therefore involve considerable trouble in their practical application; and both lead to the inconsistency that the surrender value is altered by the mere conversion of the policy into a paid-up policy.

Mr. Chatham has given a method (applicable to endowment assurances and limited-payment policies) which avoids this inconsistency, and has the great advantages of simplicity and ease of application: namely, to take the O^M $4\frac{1}{2}$ per cent net value of the "proportionate" paid-up policy and of the existing bonuses. (See pp. 12, 13, and 39 of the current volume of the *Transactions*.)

It has been suggested that, after making proper allowances for expenses and selection, deductions should also be made from the reserve, as compensation for (1) loss of interest said to be caused by the fact that surrender values are practically payable on demand, and that this necessitates the investment of a portion of the funds in readily convertible securities; (2) alleged breach of contract caused by surrender—the policyholder being the only party thereto who has power to terminate the policy at will; and (3)—in a proprietary Office—loss of future profits which might be earned on the policy if it were continued.

Now an examination of the published accounts of the Offices shows that surrenders form only a small fraction (about 6 per cent) of their total outgo, and that their total funds are increasing. This increase seems likely to continue for many years, and it follows that in ordinary circumstances a well-managed Office may safely invest the bulk of its funds in securities which need not necessarily be readily convertible, inasmuch as its income is more than sufficient to meet all its outgoings, and there is, therefore, no need to realise any portion of its investments. Of course it is prudent to keep a portion of the funds invested in readily convertible securities, as a provision against any sudden and unforeseen demand; but the probability of such a demand arising seems to be small, and to depend more upon possible fluctuations in mortality than upon the amount of surrenders. Again, it should be borne in mind that the policyholder has entered into no contract to continue payment of the premiums, and that, in most cases, the Office has expressly bound itself to pay a surrender value in the event of discontinuance. It is therefore clear that a surrender is not in any sense a breach of contract, and that no claim for compensation can be founded on that ground. The third point is of very small moment, as the shareholders are, as a rule, entitled to only a small proportion of the profits, and in practice the distinction between the mutual and proprietary systems is very fine. I therefore do not think that any of these three considerations should affect the amount of the surrender value or paid-up policy.

There is great difference of opinion as to the effect of sur-

renders upon the rate of mortality; and I doubt whether it is possible to produce conclusive evidence on the point. On the one hand we have the figures deduced by Mr. Chatham (*J. I. A.*, xxix, 81) from the Institute experience, which proved that the rate of withdrawal in the Scottish portion of the experience was less than in the total, but the rate of mortality was higher. This seems to show that withdrawals have a favourable, rather than an unfavourable effect on the rate of mortality; but, on the other hand, Mr. G. F. Hardy has proved (*J. I. A.*, xxiii, 1) that those policyholders of the "British Empire" Office who surrendered their bonuses for cash were better lives than those who did not. With such contradictory evidence on the point it seems to me only prudent to assume that on the average the option of withdrawing is exercised in the same direction as most other options, namely, adversely to the Office; and to calculate surrender values of life assurance policies on the assumption that the lives withdrawing are still "select." It will be seen later that this does not apply to the surrender of annuities. In this connection I take the opportunity of pointing out a fact which students are apt to overlook, namely, that the great majority of assured lives are "select"; and that the difference between the "select" and the "ultimate" rates of mortality arises from the inclusion among the latter, of a comparatively few lives who have fallen into bad health. An examination of the $O^{[M]}$ table shows that out of 94,974 "ultimate" lives aged 20, only 1278 or 1·35 per cent are "damaged"; and 93,696 or 98·65 per cent are still "select." The proportion of "damaged" lives is 2·11 per cent at age 40, and 6·58 per cent at age 60, and it gradually increases with age; but even at age 75 the "damaged" lives are still in a small minority, as nearly 80 per cent remain "select." Taking all the "ultimate" lives from ages 20 to 75 inclusive, it will be found that 96 per cent are still "select" and only 4 per cent are "damaged." It is therefore clear that the majority of policyholders are "select" lives, and remain so for the greater part of their lifetime; and that the increase in the rate of mortality as "selection" wears off, is not due to a general or widespread deterioration in the lives, but to the breakdown in health of a small minority. That is to say, lives as a rule are "select," the "damaged" lives being exceptions; and the effect of withdrawals upon the rate of mortality will depend upon whether the withdrawing lives contain a greater or smaller proportion of these exceptions than the lives who remain. Now an examination of the mortality of these exceptions, *i.e.* of the "damaged" lives,

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TABLE A.—*Office Premiums for "non-participating" Whole-life Assurances of* 100.

Age. x	Average of published office rates. (1)	100 P_x. (2)	Difference. (2)−(1)
10		1·309	
15	1·450	1·438	− ·012
20	1·621	1·600	− ·021
25	1·808	1·806	− ·002
30	2·046	2·067	+ ·021
35	2·358	2·402	+ ·044
40	2·767	2·835	+ ·068
45	3·379	3·397	+ ·018
50	4·008	4·136	+ ·128
55	4·979	5·113	+ ·134
60	6·217	6·417	+ ·200
65	8·088	8·173	+ ·085
70	10·842	10·548	− ·294
75		13·769	
80		18·124	
85		23·956	
90		31·694	
95		41·723	
100		54·459	
		Total,	+ ·369

The following table shows the paid-up policies deduced from these rates by the formula $1000\left(1-\frac{P_{x+1}}{P_{x+n}}\right)$; compared with: (1) the full theoretical amounts by the formula $1000\left(1-\frac{\pi_x}{\pi_{x+n}}\right)$ on an O^M 3 per cent basis; and (2) the total premiums paid.

TABLE B.—*Paid-up Policies. Single and Joint Lives. Original Policy for* 1000 *without Profits.*

Duration.	Single Lives.			Two Joint Lives, Equal Ages.		Three Joint Lives, Equal Ages.	
	OM 3% net.	Office.	Total Premiums paid.	OM 3% net.	Office.	OM 3% net.	Office.
	Ages at entry, 10.						
5	129	74	65	123	61	120	54
10	247	167	131	240	141	236	129
15	355	262	196	349	228	345	211
20	451	356	262	446	317	443	299
30	611	530	393	608	494	606	477
40	736	678	524	738	652	739	641
50	833	802	655	838	777	840	772
60	901	874	785	907	866	909	863
70	945	927	916	949	922	950	920
80	970	958	1047	972	955	971	952
90	984	976	1178	982	972	979	968
	Ages at entry, 20.						
5	143	94	80	143	82	142	78
10	270	208	160	270	188	270	180
15	383	318	240	383	294	383	284
20	483	423	320	484	398	484	389
30	650	604	480	655	586	658	580
40	778	745	640	786	735	791	733
50	868	845	800	877	841	881	840
60	926	910	960	933	908	935	907
70	960	948	1120	963	946	961	944
80	979	970	1280	976	966	972	962
	Ages at entry, 30.						
5	154	114	103	155	107	155	105
10	291	250	207	292	239	294	235
15	412	374	310	417	362	419	360
20	520	486	413	527	476	531	475
30	695	669	620	707	665	714	666
40	819	798	827	832	799	837	800
50	899	883	1034	908	883	911	883
60	946	933	1240	949	932	948	930
70	971	961	1447	967	957	962	953

Duration.	Single Lives.			Two Joint Lives, Equal Ages.		Three Joint Lives, Equal Ages.	
	O^M 3% net.	Office.	Total Premiums paid.	O^M 3% net.	Office.	O^M 3% net.	Office.
Ages at entry, 40.							
5	171	136	142	175	134	177	134
10	323	290	284	332	289	336	290
15	456	426	425	469	427	476	429
20	570	542	567	586	546	594	549
30	745	722	851	762	727	770	730
40	858	838	1134	870	842	873	842
50	923	907	1418	937	908	926	906
60	959	946	1701	953	942	943	936
Ages at entry, 50.							
5	196	157	207	205	159	210	161
10	365	328	414	381	334	389	337
15	506	473	620	527	480	536	484
20	623	591	827	644	599	653	603
30	790	762	1241	805	767	809	768
40	887	864	1654	891	864	888	861
50	939	921	2068	930	915	919	906
Ages at entry, 60.							
5	223	177	321	235	181	241	182
10	406	362	642	425	369	433	371
15	554	511	963	573	518	579	518
20	669	623	1283	685	633	688	632
30	821	788	1925	825	786	817	780
40	905	876	2567	887	866	867	852
Ages at entry, 70.							
5	248	193	527	258	194	259	193
10	442	387	1055	453	387	450	384
15	590	536	1582	595	533	585	527
20	699	649	2110	695	643	677	632
30	839	796	3164	804	776	765	751
Ages at entry, 75.							
5	258	198	688	263	197	258	194
10	455	393	1377	454	389	441	381
15	600	541	2065	589	532	565	518
20	702	652	2754	679	635	641	613
25	776	733	3442	735	707	683	675

JOINT-LIFE ASSURANCES WITHOUT PROFITS.

Following the same lines as above, the formula for the paid-up policy is $1-\frac{P_{x+1:x+1}}{P_{x+n:x+n}}$; and the formula deduced, after a series of trials, for the Office premiums necessary to illustrate it, is $P_{x:x}=1{\cdot}1\pi_{[x]:[x]}+{\cdot}002$, which gives a close approximation to the average Office premium.

TABLE C.—*Office Premiums for "non-participating" Joint-life Assurances of* 100.

Age. x	Average of published office rates. (1)	100 $P_{x:x}$. (2)	Difference. (2)-(1)
20	2·613	2·567	- ·046
25	2·822	2·854	+ ·032
30	3·275	3·227	- ·048
35	3·733	3·712	- ·021
40	4·346	4·353	+ ·007
45	5·133	5·187	+ ·054
50	6·229	6·329	+ ·100
55	7·817	7·848	+ ·031
60	10·050	9·901	- ·149
		Total,	- ·040

There are no published tables of Office premiums for three joint lives, and I have therefore assumed that a similar loading to the above would be fairly representative, and have calculated premiums and paid-up policies on that basis.

Table B shows these paid-up policies and the corresponding O^M 3 per cent net amounts.

Owing to the property of "Uniform Seniority" possessed by the $O^{[M]}$ and $O^{[NM]}$ tables, it is possible to substitute for any number (n, say) of joint lives of different ages (who have been insured for the same number of years less than 10, or who have all been insured more than 10 years) the same number of lives whose ages are equal, by means of the equation

$$c^x+c^y+c^z+\ldots\ (n \text{ terms in all})=nc^w$$

$$\text{or } \mu_x+\mu_y+\mu_z+\ldots\ (n \text{ terms in all})=n\mu_w,$$

and it is therefore only necessary to consider the case where the ages are equal. It should be noticed that the value of c is the same in both these tables, so that the substituted lives are of the same age in both cases.

An examination of the table shows that the "single-life" and the "joint-life" paid-up policies correspond very closely, and that for most of the ordinary ages at entry and durations the

"Office" amounts are nearly identical. There is, at the younger ages, a tendency in the amounts to decrease as the number of lives is increased; but, on the other hand, it may be said that the formula does not quite do justice to the joint-life policyholders. The premium is larger than in single-life cases, and therefore the deduction of the whole of the first year's premium presses the more heavily upon them. In view of this, and more especially of the immense practical advantage gained by reducing the number of tables to a minimum and thus simplifying the work, I propose to use the same table of paid-up policies for single and for joint lives, irrespective of the number of lives involved. In each joint-life case the equivalent age w can be determined at the outset, and used throughout the existence of the policy as the age with which to enter the table of paid-up policies.

LAST-SURVIVOR ASSURANCES WITHOUT PROFITS.

The principle of uniform seniority does not, in the case of a last-survivor assurance, enable us to make such a substitution as that mentioned above; and it is therefore necessary to consider the case where the ages differ, as well as where they are equal.

The formula which I found gave the closest approximation to the average office premiums is:—

$$P_{\overline{x:y}} = 1{\cdot}075\pi_{[x]:[y]} + {\cdot}00175.$$

TABLE D.—*Office Premiums for "non-participating" Last-Survivor Assurances of* 100.

	Ages. x	Ages. y	Average of published office rates. (1)	100 $P_{\overline{x:y}}$. (2)	Difference. (2)–(1)
Equal Ages.	20	20	1·038	1·004	– ·034
	25	25	1·153	1·155	+ ·002
	30	30	1·336	1·343	+ ·007
	35	35	1·566	1·582	+ ·016
	40	40	1·869	1·886	+ ·017
	45	45	2·266	2·278	+ ·012
	50	50	2·782	2·790	+ ·008
	55	55	3·459	3·468	+ ·009
	60	60	4·376	4·374	– ·002
Difference of Age 30 Years.	20	50	1·451	1·407	– ·044
	25	55	1·642	1·632	– ·010
	30	60	1·915	1·907	– ·008
				Total,	– ·027

I have assumed that a similar loading to the above would give fairly representative premiums for assurances on the last survivor of three lives, and have calculated premiums and paid-up policies

on that basis. The following table shows these paid-up policies and the corresponding O[M] 3 per cent net amounts:—

TABLE E.—*Paid-up Policies.—Last Survivor.—While all the lives survive. Original Policy for* 1000 *without Profits.*

Duration.	Two Lives. Equal Ages. O[M] 3% net.	Two Lives. Equal Ages. Office.	Two Lives. Difference of Age 30 Years. O[M] 3% net.	Two Lives. Difference of Age 30 Years. Office.	Three Lives, Equal Ages. O[M] 3% net.	Three Lives, Equal Ages. Office.
Age of youngest life at entry, 10.						
5	148	91	149	99	150	96
10	278	200	281	215	283	210
15	391	305	395	324	397	316
20	489	402	494	421	495	414
30	648	574	652	590	653	584
40	766	712	770	733	770	720
50	853	816	855	823	855	821
60	914	889	915	893	915	892
70	952	937			953	939
80	975	965	Older life	dead.*	976	966
90	988	980			989	981
Age of youngest life at entry, 20.						
5	156	107	159	113	158	110
10	292	232	296	241	296	237
15	410	348	414	358	414	355
20	512	453	516	462	516	459
30	676	630	679	637	679	635
40	797	764	799	768	798	767
50	880	858	882	859	881	860
60	934	919			934	920
70	966	954	Older life	dead.*	966	956
80	984	975			984	976
Age of youngest life at entry, 30.						
5	166	124	168	126	168	126
10	310	265	312	269	313	268
15	435	392	437	395	437	395
20	542	503	545	506	545	506
30	713	683	714	684	713	685
40	831	809	832	808	831	811
50	907	891			907	892
60	951	939	Older life	dead.*	951	940
70	977	966			978	967

* As the limiting age in the O[M] table is 105 years.

Duration.	Two Lives. Equal Ages. O^M 3% net.	Two Lives. Equal Ages. Office.	Two Lives. Differences of Age 30 Years. O^M 3% net.	Two Lives. Differences of Age 30 Years. Office.	Three Lives, Equal Ages. O^M 3% net.	Three Lives, Equal Ages. Office.
	Age of youngest life at entry, 40.					
5	181	141	181	141	181	142
10	336	299	338	299	337	300
15	470	436	472	435	470	437
20	583	553	585	551	583	554
30	755	731	756	728	754	732
40	865	846			864	847
50	929	914	Older life	dead	930	915
60	966	952			968	953
	Age of youngest life at entry, 50.					
5	201	161			201	160
10	372	335			371	335
15	514	480			512	480
20	630	599			628	600
30	796	770			795	772
40	893	871			893	873
50	949	928			952	930
	Age of youngest life at entry, 60.					
5	226	182			225	180
10	411	370			409	369
15	559	521			558	521
20	676	639			674	640
30	830	798			831	800
40	919	887			923	890
	Age of youngest life at entry, 70.					
5	252	197			251	198
10	449	395			449	398
15	600	547			600	550
20	712	661			713	665
30	863	811			870	816
	Age of youngest life at entry, 75.					
5	264	203			264	205
10	466	403			467	406
15	615	554			617	558
20	722	667			726	671
25	816	751			826	757

It will be noticed that the amounts depend almost entirely upon the age of the youngest life, a difference of even 30 years in the ages, or the introduction of a third life, making no important difference in the amounts. Further, a comparison with the table of single-life paid-up policies shows that although the last are always the smaller, yet the difference is comparatively small except at the younger ages at entry when the duration is short. It may, however, be said that as the last-survivor premiums are less than those for single lives, the formula for the paid-up policy, perhaps, tends to give rather too large amounts as compared with single-life policies. Partly for this reason, but chiefly for the sake of simplicity, I propose to use the same table of paid-up policies for single-life and last-survivor policies, irrespective of the number of lives involved, and of the difference (if any) in their ages; taking, in last survivor cases, the age at entry of the youngest life, as the age with which to enter the table of paid-up policies.

The above amounts are calculated on the assumption that the lives are all still surviving at the time the paid-up policy is quoted; and if one or more of them be dead, a slightly different procedure must be followed.

First take the case of a policy on two lives, x and y; one of whom, say x, is dead. Since the assurance then depends solely on the survivor, y, the premium (per unit of the sum assured) for a new policy of the same amount would be P_{y+n}, and the formula for the paid-up policy would become $1-\frac{P_{\overline{x+1:y+1}}}{P_{y+n}}$. Now by an inspection of the Office tables of premiums we can at a glance find a number r such that $P_{y-r}=P_{\overline{x:y}}$ (unless x and y are both very young), and by entering the table of single-life paid-up policies with age $y-r$ and duration $n+r$ we get a very close approximation indeed to the exact amount by the formula; the amounts being, in fact, practically identical. For $P_{\overline{x+1:y+1}}$ will be practically equal to P_{y-r+1}, therefore, approximately, $1-\frac{P_{\overline{x+1:y+1}}}{P_{y+n}}=1-\frac{P_{y-r+1}}{P_{y-r+n+r}}$, which is the paid-up policy given by the single-life table where the age at entry is $y-r$, and the duration is $n+r$. The number r remains unaltered during the remainder of the policy's existence, and therefore only requires to be determined once, when it can be recorded for future reference.

Secondly, take the case of a policy on three lives, x, y, and z, one of whom, say x, is dead. In these circumstances the premium (per unit of the sum assured) for a new policy of the same amount

is $P_{\overline{y+n:s+n}}$ and the paid-up policy is therefore $1-\frac{P_{\overline{x+1:y+1:s+1}}}{P_{\overline{y+n:s+n}}}$.

Now, an inspection of the Office tables of premiums enables us to find a number s such that $P_{\overline{y-s:z-s}}=P_{\overline{x:y:z}}$ (unless the lives are very young), and if the table of single-life paid-up policies be entered with age at entry $y-s$ (where y is less than z), and duration $n+s$, the result will be the paid-up policy.

For $P_{\overline{x+1:y+1:s+1}}$ will be practically equal to $P_{\overline{y-s+1:z-s+1}}$, so that (approximately) $1-\frac{P_{\overline{x+1:y+1:s+1}}}{P_{\overline{y+n:s+n}}}=1-\frac{P_{\overline{y-s+1:z-s+1}}}{P_{\overline{y-s+n+s:z-s+n+s}}}$, which is the paid-up policy allowed for a policy effected on the last survivor of two lives, $y-s$ and $z-s$, which has been $n+s$ years in force; and as has been shown above, this is very nearly the same as the paid-up policy allowed for a single-life policy effected at age $y-s$, and of duration $n+s$. The number s will remain unaltered so long as both y and z survive.

Thirdly, take the case of a policy on the last survivor of three lives, x, y, and z, two of whom, say x and y, are dead, in which case the formula for the paid-up policy becomes $1-\frac{P_{\overline{x+1:y+1:s+1}}}{P_{s+n}}$. By inspection of the Office tables of premiums we can find a number t such that $P_{s-t}=P_{\overline{x:y:z}}$ (unless the lives are very young), and if the table of single-life paid-up policies be entered with age at entry $z-t$, and duration $n+t$, the result will be the paid-up policy. For (approximately) $1-\frac{P_{\overline{x+1:y+1:s+1}}}{P_{s+n}}=1-\frac{P_{s-t+1}}{P_{s-t+n+t}}$, which is the paid-up policy allowed for a single-life policy effected at age $z-t$, which has been $n+t$ years in force. The number t remains unaltered during the remainder of the policy's existence.

It is therefore possible to construct from the Office premiums for whole-life assurances without profits, a single table of paid-up policies, which can be used safely and conveniently in practice for every whole-life policy by uniform annual premiums, irrespective of the number of lives involved.

If the Office has changed its rates of premium between the issue of the policy and its conversion into a paid-up policy, the question arises: What difference should this make in the value to be allowed? If the rates have been raised in the meantime, the policyholder has all along been paying less than a new entrant at the same age would be charged; and therefore as the Office has received less from him, it would appear reasonable that he should receive a smaller paid-up policy than would be given by the formula according to the present scale of premiums. On the other

hand, it may be said that as he has secured the same reversion as an entrant of the same age under the new tables, but pays a smaller premium for it, his contract is more valuable; and that this fact should be recognised by granting him a somewhat larger paid-up policy. Similar considerations apply, *mutatis mutandis*, if the rates of the Office have been lowered. That is to say, purely theoretical reasoning by the retrospective and prospective methods leads to two diametrically opposite conclusions. In these circumstances I think the best course is to construct a table of paid-up policies from the premiums at present in use, and to apply it to all policies whether the premiums payable thereunder are the same as, or different from, those under the present scale.

LIMITED-PAYMENT POLICIES AND ENDOWMENT ASSURANCES.

In dealing with these classes of policies we have to recognise the fact that owing to the pressure of competition and the force of public opinion many Offices have adopted the arbitrary rule of allowing a "proportionate" paid-up policy corresponding to the number of premiums paid, with or without the addition of existing bonuses; and I have therefore examined the prospectuses and other publications of a number of Offices in order to ascertain the general practice on this point. There are 59 British Offices each of which has funds of over half-a-million pounds sterling, and I confined the investigation to these. Fifty-one of them guarantee a "proportionate" paid-up policy in some form or another as shown in the following table:—

Offices guaranteeing "proportionate" paid-up Policies.

From Outset.	After 1 Year's Premium paid.	After 2 Years' Premiums paid.	After 3 Years' Premiums paid.	After ¼th of total Premiums paid.	Bonus added to proportionate paid-up Policy.	Totals.
1	0	5	4	0	None.	10
2	0	0	0	0	Future profits.	2
0	1	1	3	0	Proportion of existing Bonus.	5
0	1	2	1	0	Do. and future profits.	4
0	1	13	10	2	Whole of existing Bonus.	26
1	0	0	3	0	Do. and future profits.	4
4	3	21	21	2		51

The great majority of these guarantees apply both to limited-payment policies and to endowment assurances; but six of the Offices appear to limit the privilege to the former class, and one seems to limit it to the latter. The table cannot be regarded as an absolutely accurate exhibit of the practice of the 51 Offices, for some of them introduce slight modifications which have been disregarded, as otherwise the groups would have been too numerous. For example, one Office guarantees a paid-up policy after two years' premiums have been paid under an endowment assurance; but requires at least three years' premiums in the case of a limited payment policy; and another Office which has the same condition regarding the former class, guarantees a proportionate paid-up policy under a limited-payment policy after only one year. Of the remaining eight Offices, one states that the paid-up policy is "about" the same proportion of the sum assured that the number of premiums paid bears to the full number originally payable; another guarantees a paid-up policy equal to all the premiums paid plus any vested bonuses—while six make no definite statement on the subject.

This shows that the majority of Offices adopt some form or other of a "proportionate" paid-up policy, and I think they are right in doing so. The public can understand the system, and it appeals to them as being in accordance with "common-sense," and therefore reasonable and fair; and it will be seen from Tables K—S that as a rule it gives values which are within those provided by the reserves of the Offices—the reserve for the proportionate paid-up policy being generally less than the net value of the original policy. At ages above 50 the system gives rather too much in the case of limited-payment policies, and also in the case of endowment assurances for a short term of years, though the discrepancy here is less.

If reference be made to Mr. Macfadyen's paper above mentioned (*J. I. A.*, xv, 297) it will be seen that he did not extend his investigation to a sufficiently high age at entry to exhibit fully this defect in the method; and it may be that had he done so, he would have come to a somewhat modified conclusion. Some Offices attempt to remedy the defect by limiting their guarantee of a proportionate paid-up policy to cases where the age at entry does not exceed a fixed age, say 50 or 60, but they do not state how they deal with cases falling outside this limit. In view of the comparative infrequency of such cases, and of the great advantage of having one rule instead of a number, I am of opinion that an Office may justifiably guarantee a pro-

portionate paid-up policy in the case of every policy effected by limited payments, provided at least two years' premiums have been paid thereon, and irrespective of the class of assurance. I would therefore extend the rule to double endowments, pure endowments, and deferred annuities, with or without return of premium in the event of death, it being understood that the reduced benefit must be of the same nature as the original. It appears to me that any disadvantage of this course will be more than compensated by the popularity and simplicity with which it is attended.

INCOME POLICIES.

Many Offices have schemes under which the sum assured is payable in instalments over a period of years, instead of in one sum on the policy becoming a claim. That is to say, the benefit is an annuity-certain instead of a sum of cash. The term of this annuity is usually 20 years, 5 per cent of the nominal sum assured being paid in advance each year, so that each £1 assured is equivalent to $\cdot 05a_{\overline{20}|}$. Assuming the Office to value on a 3 per cent basis, this is equal to ·76619 ($\cdot 05 \times 15\cdot 3238$), and such policies are therefore virtually the same as ordinary policies for that fraction of the nominal sum assured, so that the paid-up policies could be calculated by the same rules. It is, however, usually inconvenient to grant paid-up policies on this system, as the annual instalment would be very small; and the trouble of keeping so small an annuity on the books would outweigh any advantages afforded thereby. Accordingly the paid-up policy should generally be made payable in one sum on the policy becoming a claim; and its amount got by applying the rules for the conversion of ordinary policies of similar class (whole-life, limited-payment, or endowment assurance as the case may be), and multiplying the result by ·76619.

DEBENTURE POLICIES.

Under this scheme the Office agrees to retain the sum assured in its hands for a specified term of years, usually 20, allowing interest in the meantime at the rate of, say, 4 per cent.

Assuming a 3 per cent valuation basis as before, the policy money may be regarded as deposited with the Office at that rate, so that the contract is virtually equivalent to an ordinary assurance, plus a 20-year annuity-certain of 1 per cent of the sum assured. Accordingly each unit of the sum assured may be

treated as equivalent to $1+\cdot 01a_{\overline{20}|}$ (3 per cent); or $1+\cdot 01a_{\overline{20}|}^{(2)}$, if the interest be payable half-yearly. This last expression is equal to 1·14958 (say 1·15), so that a "4 per cent Debenture" policy may be treated as if it were an ordinary policy for an amount 15 per cent larger than the nominal sum assured.

CHILDREN'S DEFERRED ASSURANCES.

Under this scheme the assurance does not commence until the attainment of a specified age, or the expiry of a fixed number of years after entry; and the premiums are consequently much less than those for immediate assurances. If death occurs before the specified age (usually 21), the premiums are generally returned; but policies are occasionally issued at even lower rates with "non-returnable" premiums. The benefit is therefore equivalent to a deferred assurance combined with a pure endowment for a smaller amount. After the assurance has come into force, the paid-up policy may be found by the formula $1-\frac{P}{{}^{s}P}$, where P is the premium actually payable, and ${}^{s}P$ is the premium that would be charged in the case of a new entrant at the attained age, for an immediate assurance similar to that secured by the existing policy, namely whole-life, limited-payment, or endowment assurance, as the case may be. Having regard to the profit obtainable by the Office under the endowment portion of the contract, during the period for which the assurance was deferred, I do not think it is necessary to omit the first premium, as in the case of immediate assurances. At any time before the assurance has come into force, the paid-up policy may be found by the "proportionate" method. Thus, if the assurance be deferred for r years; if the paid-up policy that would be given by the above formula at the end of that term be denoted by ${}_{r}(FP)$; and if n premiums ($n<r$) have been paid; then the amount of the paid-up policy at the end of n years is ${}_{r}(FP)\times\frac{n}{r}$. The nature of the paid-up benefit must of course be the same as that of the original contract, namely, a deferred assurance, with or without return of premiums, as the case may be.

REVERSIONARY BONUSES.

In the case of a whole-life policy with participation in profits the paid-up policy might be found by the formula $1-\frac{P'_{x+1}}{P'_{x+n}}$, with

suitable modifications for joint-life and last-survivor policies. This will give a greater or less paid-up policy than the formula $1-\frac{P_{x+1}}{P_{x+n}}$

according as $\frac{P'_{x+1}}{P'_{x+n}}$ is $<$ or $>$ $\frac{P_{x+1}}{P_{x+n}}$,

or as $\frac{P'_{x+1}}{P_{x+1}}$ is $<$ or $>$ $\frac{P'_{x+n}}{P_{x+n}}$,

that is to say, according as the ratio of the "participating" to the "non-participating" premium increases or diminishes with the age at entry.

The following table shows the averages of the published rates of premium for assurances with profits, and their ratio to the corresponding rates without profits; and it will be seen that, on the whole, this ratio tends to diminish as the age at entry increases, though there are several irregularities caused by the somewhat irregular progression of the average office premiums.

TABLE F.—*Average Office Premiums for Whole-life Assurances of* 100, *with and without Profits.*

x	Average Premiums		$\frac{P'_x}{P_x}$
	With Profits P'_x	Without Profits P_x	
15	1·721	1·450	1·187
20	1·967	1·621	1·213
25	2·183	1·808	1·207
30	2·467	2·046	1·206
35	2·817	2·358	1·195
40	3·258	2·767	1·177
45	3·817	3·379	1·130
50	4·567	4·008	1·139
55	5·621	4·979	1·129
60	7·042	6·217	1·133
65	8·996	8·088	1·112
70	11·321	10·842	1·044

As is explained hereafter (p. 238), the cost of a compound bonus of 1 per cent per annum is approximately represented by a difference of 1 per cent in the rate of interest used in valuing the initial benefit. Accordingly the net annual premium $\pi'_{[x]}$ for an assurance with the right to such a bonus is approximately equal to

$$\frac{A_{[x]}\ (\text{at rate } i-\cdot 01)}{a_{[x]}\ (\text{at rate } i)}.$$

Using the same basis as before ($O^{[NM]}$ $3\frac{1}{2}$ per cent) I have constructed by this formula the following table:—

TABLE G.—*Net "Select" Premiums for Assurance of* 1.

x	With Compound Bonus of 1 per cent $\pi'_{[x]}$	Without Profits $\pi_{[x]}$	$\frac{\pi'_{[x]}}{\pi_{[x]}}$
10	·01542	·01099	1·403 } Approx.
15	·01685	·01216	1·397 } Approx.
20	·01864	·01364	1·367
25	·02077	·01551	1·339
30	·02345	·01788	1 312
35	·02682	·02093	1·281
40	·03108	·02486	1·250
45	·03652	·02997	1·219
50	·04359	·03669	1·188
55	·05282	·04557	1·159
60	·06497	·05743	1·131
65	·08123	·07339	1·107
70	·10312	·09498	1·086
75	·13274	·12426	1·068
80	·17244	·16385	1·052
85	·22566	·21687	1·041
90	·29622	·28722	1·031
95	·38754	·37839	1·024
100	·50374	·49417	1·019

I have also constructed (on an O^M $3\frac{1}{2}$ per cent basis) the following table, which gives the premiums for an assurance with right to a simple bonus of 1 per cent per annum, and their ratio to the corresponding non-profit premiums :—

TABLE H.—*Net "Aggregate" Premiums for Assurance of* 1.

x	With Simple Bonus of 1 per cent π'_x	Without Profits π_x	$\frac{\pi'_x}{\pi_x}$
10	·01231	·00893	1·378
15	·01400	·01032	1·357
20	·01604	·01204	1·332
25	·01849	·01416	1·306
30	·02146	·01677	1·280
35	·02503	·01997	1·253
40	·02945	·02400	1·227
45	·03503	·02917	1·201
50	·04225	·03597	1·175
55	·05174	·04504	1·149
60	·06445	·05733	1·124
65	·08169	·07416	1·102
70	·10541	·09749	1·081
75	·13836	·13007	1·064
80	·18441	·17580	1·049
85	·24853	·23963	1·037
90	·33596	·32682	1·028
95	·44905	·43973	1·021
100	·62214	·61262	1·016

These tables show that both for a simple and a compound bonus the ratio $\frac{\pi'_{[x]}}{\pi_{[x]}}$ steadily diminishes as x increases (when the mortality table is well graduated), and the same thing holds for the Office premiums if they are loaded with the same constant and percentage in each case. For, on this hypothesis,

$$\frac{P'}{P}=\frac{\pi'(1+k)+c}{\pi(1+k)+c}=\frac{\pi'+h}{\pi+h} \text{ where } h=\frac{c}{1+k},$$

and the form of this expression shows that since $\frac{\pi'}{\pi}$ diminishes with the age but is always >1, $\frac{P'}{P}$ will be slightly $<\frac{\pi'}{\pi}$ but >1, and will diminish in a similar manner though not quite so rapidly. Accordingly it appears that if an Office charges premiums based on a well-graduated table, and properly loaded, the paid-up policy given by the formula $1-\frac{P'_{x+1}}{P'_{x+n}}$ will in all cases be less than that given by the formula $1-\frac{P_{x+1}}{P_{x+n}}$. If the rates are based on the $H^{[M]}$ table, irregularities will appear at the early ages at entry, owing to the irregular mortality shown by that table between ages 20 and 30; but for the greater part of the table the same relation holds. As a general rule, therefore, it will be found that the proposed method gives smaller results when applied to "participating" than when applied to "non-participating" policies; which was perhaps only to be expected, as it implies that in the former case the paid-up policy will share in future profits and therefore be increased by the addition of bonuses from time to time. There are, however, several objections to allowing a paid-up policy to share in the future profits; among which may be mentioned the facts that the assured ceases to contribute towards the "bonus-loading," that the allocation of periodical bonuses to a large number of small paid-up policies causes a great deal of labour—possibly out of proportion to the benefits of such a course, and that when a policyholder cannot continue to pay his premiums it is generally to the advantage of his family that the paid-up policy should be for as large an amount as possible from the outset. I therefore propose that in all cases (unless a request to the contrary be made), the paid-up policy should not share in future profits; and that its amount should, for "participating" as well as "non-participating" policies, be ascertained by the formula $1-\frac{P_{x+1}}{P_{x+n}}$, with the addition, in "participating" cases, of any vested bonus addi-

tions; and I would follow a similar course in all cases where a "proportionate" paid-up policy is granted. The only difference in the amounts of paid-up policy allowed for "participating" and "non-participating" assurances would therefore be that in the former case the amount of any existing bonus would be added.

In the case of an Office dividing its profits on the compound reversionary bonus plan, some modification is required if inconsistencies are to be avoided; since the proposed paid-up policy is without profits, and a compound bonus is virtually a paid-up assurance with profits. If they were simply added together and the total were not to share in future profits, it will be seen from what follows (p. 256) that the policyholder would get a smaller surrender value if he surrendered the whole benefit outright, than if he first surrendered the bonus for its cash value, and immediately thereafter surrendered the policy. To meet this difficulty I propose that in such a case the amount of the bonus should be increased in the ratio of the surrender value of a "participating" policy to the surrender value of a similar "non-participating" policy, and that this increased amount be added to the paid-up policy given by the formula $1-\frac{P_{x+1}}{P_{x+n}}$. The difficulty could be also met, in whole-life cases, by making the paid-up policy $=\left(1-\frac{P'_{x+1}}{P'_{x+n}}\right)\times$ sum assured + existing bonus, and allowing it to share in future profits; but this would not apply to limited payment and endowment assurance policies where a "proportionate" paid-up policy is guaranteed; and it seems to me better to adopt a rule which can be applied to all classes of assurance. It will be seen that this proposed rule assumes that the existing bonuses are in each case the equivalent of the difference between the "participating" premiums actually paid and those that would have been paid for a "non-participating" policy; and though this will not always be scientifically accurate, it is probably sufficiently near the truth to form a good practical rule.

DISCOUNTED BONUS SYSTEM.

Under this plan the Office anticipates a certain bonus, and applies its present value in reduction of the premium. If the bonus actually realised should exceed the amount anticipated, the excess is applied either in increasing the sum assured, or in further reduction of the premium; but if it falls short thereof

the policyholder has to make good the deficiency, either by suffering a reduction of the sum assured or paying an increased premium. Now an examination of the figures given by Mr. G. F. Hardy (*J. I. A.*, xxxi, 265, 268) shows that though the reduction of premium $P'-P''$ increases with the age at entry, it does not increase nearly so rapidly as the premium P'.

From this it follows that $\frac{P'_{x+n}-P''_{x+n}}{P'_x-P''_x}$ is less than $\frac{P'_{x+n}}{P'_x}$, and therefore $\frac{P''_{x+n}}{P''_x}$ is greater than $\frac{P'_{x+n}}{P'_x}$ and $1-\frac{P''_x}{P''_{x+n}}$ is greater than $1-\frac{P'_x}{P'_{x+n}}$. That is to say, the paid-up policy on this system is larger than that on the ordinary "participating" plan.

I examined a set of Office tables of premiums based on the $H^{[M]}$ (select) tables, and consistently loaded, and found that the ratio $\frac{P''}{P}$ increased with the age at entry (though there were a few irregularities at the younger ages due to the irregular mortality above mentioned), but that this increase was very slow.

It follows that, according to those tables, $1-\frac{P''_x}{P''_{x+n}}$ is generally greater than $1-\frac{P_x}{P_{x+n}}$, but that the excess is very small. According to a set of tables published by a second Office, the former expression was very slightly less than the latter; and a third set gave slight deviations in both directions. The general conclusion to be drawn from these facts is that $1-\frac{P_x}{P_{x+n}}$ is a very close approximation to $1-\frac{P''_x}{P''_{x+n}}$, and this was perhaps to be expected, since the "discounted bonus" premiums published by an Office are generally very close to its "non-participating" premiums.

I therefore suggest that the *amount* of a paid-up policy allowed for an assurance under this plan, might be the same as that allowed for a "non-participating" assurance. The reasoning implies, however, that this paid-up policy is to be granted on the "discounted bonus" plan, and is therefore liable to be increased or diminished if the future rate of bonus exceeds or falls short of the anticipated rate. In order to avoid the trouble of numerous small changes in the amounts assured by such contracts, it would probably be better to issue the paid-up policy on the "non-participating" plan, and this raises the question: Upon what terms

can a paid-up policy on the discounted bonus system be converted into one without profits? The rate of bonus anticipated under the system is generally a substantial proportion of the rate declared by the Office, *e.g.* four-fifths, five-sixths, or even more—and having regard to the tendency of the rate of interest to fall, and the rate of expenditure to rise, it may be said that the anticipated rate represents approximately the rate which, on a cautious estimate, the Office is likely to maintain in the future. If this be correct, it follows that, on converting a paid-up policy on this plan into a "non-participating" assurance, its amount should be reduced; as the former benefit depends upon estimates which may not be realised, while the latter is fixed and guaranteed. As will be explained hereafter, under the heading of surrender values, it seems to me that the Office might, in such a calculation, make allowance for one-half to two-thirds of the estimated future bonus. Thus, if the estimated rate is 30s., the policyholder might be credited with a rate of, say, 20s., leaving a difference of 10s. per annum to be deducted from the paid-up policy on its conversion into a fixed benefit. This leads to the conclusion that the amount of the paid-up policy to be granted on the "non-participation" plan might be

$$\left(1-\frac{P_{x+1}}{P_{x+n}}\right)\times\frac{A_{[x+n]}(O^{[M]}\,4\tfrac{1}{2}\%)}{A_{[x+n]}(O^{[M]}\,4\%)}$$

where the difference of one-half per cent in the rate of interest represents the deduction to be made in respect of the 10s. per annum of bonus. Of course the exact difference in the rates of interest should depend upon the circumstances of the particular Office; the above formula being given merely as an illustration. It will be noticed that this method, though arrived at by a different process, is in principle the same as that suggested by Mr. G. F. Hardy, namely, to reduce the surrender value by increasing the rate of interest used in the calculation thereof.

If, during the currency of the original contract, the profits have exceeded the anticipated rate, the sum assured will have been increased by bonuses, or the premiums will have been further diminished. The amount of such bonuses, or the reversionary equivalent of such further reduction of future premiums, should be added to the paid-up policy given by the above formula. On the other hand, if it should happen that the profits fall short of the anticipated rate, and the sum assured has to be reduced, or the premiums increased, the paid-up policy would have to be diminished to a corresponding extent.

MINIMUM PAID-UP POLICIES.

As it would cause a great deal of trouble to have a large number of paid-up policies for trifling amounts placed on the books of an Office, I think it is a good rule to fix upon a minimum sum below which no paid-up policy will be granted. Some Offices fix this limit at £50, but I am inclined to think this is rather too high, and would suggest £25 or even £20. If it were smaller than this, it is very doubtful whether the results would repay the extra trouble involved. There would be a likelihood of such policies being in some cases overlooked by the holders owing to their insignificance, or of the relatives thinking the amount too small to be worth the trouble and expense of taking out a grant of representation, so that a number of them might remain on the books of the Office long after the deaths of the assured.

PAID-UP POLICY COMPARED WITH PREMIUMS PAID.

One Office guarantees a paid-up policy without profits, for an amount equal to the premiums paid, plus any existing bonus additions, provided that the assurance has been in force for at least five years, that at least £20 has been paid in premiums, and that the age of the assured does not exceed 60 years. No statement is made regarding its practice in cases where this age limit is exceeded. This system was investigated by Dr. T. B. Sprague in 1857 (*J. I. A.*, vii, 58), and he showed that, according to the tables then in use, it gave too liberal values when the age at entry exceeded 50 years; and it will be seen from Table B that when the age at entry exceeds 40, it gives larger values for whole-life assurances than those now proposed by me, while for younger ages it gives less, the difference being in many cases of substantial amount. Therefore, although the method has the advantage of simplicity, I do not think it can be recommended for general adoption.

SURRENDER VALUES.

NON-PARTICIPATING PAID-UP POLICIES.

It appears to me that the surrender of a policy is virtually the purchase by the Office of a thoroughly well-secured absolute

reversion free from liability to estate duty or expenses of realisation, the amount of the reversion being the amount of the paid-up policy that the Office would allow in lieu of the original assurance. On this footing the rate of interest used in the calculation should be an investment rate approximating to the market rate for such transactions, which is probably about $4\frac{1}{2}\%$, or possibly a little less. My own view is that $4\frac{1}{2}\%$ is a suitable rate to employ; but as some may consider the values brought out on this basis to be rather too low, I have calculated corresponding values on a 4% basis in order to exhibit the effect of using a lower rate of interest. (See Tables J—S.)

For the reasons given above, and because the mortality among life-tenants is known to be light (see the paper by Mr. Neil Campbell, *T. F. A.*, vol. i. p. 79), I think the Office should assume that the assured are still "select" lives at the date of surrender, and that their mortality will not exceed that shown by the $O^{[M]}$ table. That is to say, I propose that the surrender value should be equal to the paid-up policy as found by the above rules, multiplied by the appropriate net Single Premium according to the $O^{[M]}$ tables with $4\frac{1}{2}\%$ interest; and I append tables of $A_{[x]}$ and $A_{[x]:\overline{r}|}$ on this basis which have been calculated by Mr. J. A. Rankin (Appendix I.). It will be noticed that this rule gives the same cash value for a paid-up non-participating policy as for an equal amount of simple bonus, and this appears to me to be correct since the two benefits are identical. Some authorities have held the view that a higher cash value should be allowed in the latter case, on the ground that the policyholder is only drawing out his share of the accrued profits and not terminating his connection with the Office: but I have never been able to satisfy myself of the validity of this plea.

The following Tables, then, show the Surrender Values (excluding bonus) of Whole-life, Limited Payment and Endowment Assurance Policies for 1000 each, deduced from the above-mentioned Paid-up Policies on the bases of $O^{[M]}$ "select" mortality and interest at $4\frac{1}{2}$ per cent and 4 per cent per annum; compared with the O^{M} 3 per cent Net Values of the original assurances, and (in the case of Limited-Payment and Endowment Assurance Policies) with the O^{M} 3 per cent net values of the corresponding "proportionate" paid-up policies.

SURRENDER VALUES OF POLICIES FOR 1000.

TABLE J.—*Whole Life—Whole-Life Premiums.*

Duration.	Om 3% Reserve.	Surrender Value 4½%	Surrender Value 4%	Total Premiums paid.	Om 3% Reserve.	Surrender Value 4½%	Surrender Value 4%	Total Premiums paid.
	Age at entry 10.				**Age at entry 20.**			
5	35·8	13·6	15·8	65	49·2	21·3	24·4	80
10	76·5	34·0	39·1	131	103·0	53·0	60·2	160
15	121·9	59·4	68·0	196	161·1	91·8	103·2	240
20	171·6	90·8	103·1	262	224·0	138·8	154·4	320
30	283·4	173·9	193·5	393	364·9	256·1	278·6	480
40	413·5	287·4	312·7	524	519·8	400·9	426·6	640
50	556·5	431·5	459·2	655	670·5	555·5	579·8	800
60	695·7	574·5	599·7	785	795·7	696·3	716·0	960
70	811·3	709·3	729·4	916	882·1	805·5	819·5	1120
80	891·2	814·0	826·1	1047	934·6	880·0	889·3	1280
90	939·6	885·5	894·8	1178				
	Age at entry 30.				**Age at entry 40.**			
5	64·9	32·9	37·0	103	87·7	50·8	55·9	142
10	135·0	82·0	91·3	207	181·6	122·9	133·8	284
15	210·8	139·6	153·6	310	280·3	204·2	219·7	415
20	292·0	206·0	224·2	413	381·1	291·6	310·3	567
30	464·6	360·0	383·1	620	575·4	474·6	495·4	851
40	632·7	524·6	547·6	827	736·7	641·2	659·3	1134
50	772·2	675·7	694·7	1034	848·1	770·7	784·1	1418
60	868·6	792·8	806·6	1240	915·7	858·3	867·3	1701
70	927·1	871·9	881·0	1447				
	Age at entry 50.				**Age at entry 60.**			
5	120·6	75·3	81·0	207	161·0	105·9	111·5	321
10	243·8	176 5	187·8	414	313·9	238·0	248·4	642
15	365·5	282·9	298·0	620	453·3	364·7	377·6	963
20	481·2	388·5	405·5	827	574·6	476·7	490·2	1283
30	678·3	583·1	599·5	1241	754·6	669·6	681·2	1925
40	814·4	734·1	746·9	1654	863·8	794·8	803·1	2567
50	897·0	835·6	844·3	2068				
	Age at entry 70.				**Age at entry 75.**			
5	203·1	137·7	142·6	527	221·9	151·5	155·8	688
10	379·9	296·1	304·5	1055	406·0	318·6	325·7	1377
15	526·6	434·6	444·2	1582	551·1	459·7	467·7	2065
20	642·3	551·5	561·0	2110	659·1	574·9	582·7	2754
25	728·3	647·2	656·0	2637	750·8	665·0	672·0	3442
30	801·4	722·2	729·7	3164				

TABLE K.—*Whole Life—10 Payments.*

Duration.	Age at entry 20.				Age at entry 30.			
	O^M 3% net value of		Surrender Value		O^M 3% net value of		Surrender Value	
	Original Policy.	Proportionate Paid-up Policy.	4½%	4%	Original Policy.	Proportionate Paid-up Policy.	4½%	4%
1*	33	32	21	24	40	39	26	30
2	67	65	42	49	82	79	54	61
3	102	99	65	74	124	121	82	93
4	138	134	88	102	168	165	112	127
5	176	172	113	130	214	210	144	162
6	214	210	139	159	261	257	178	199
7	254	251	166	190	309	306	212	238
8	295	292	194	221	359	357	249	278
9	337	336	220	255	411	410	287	320
	Age at entry 40.				Age at entry 50.			
1*	48	47	34	37	57	57	43	47
2	98	97	69	77	115	116	89	97
3	149	148	106	118	176	178	137	148
4	202	200	145	160	238	241	187	202
5	257	256	187	205	303	307	240	258
6	314	313	230	252	370	375	295	316
7	372	372	275	301	440	445	352	377
8	433	433	322	352	513	517	411	440
9	496	496	372	405	589	591	473	505
	Age at entry 60.				Age at entry 70.			
1*	64	68	55	58	66	78	67	70
2	129	138	112	119	133	158	136	142
3	197	210	172	182	203	240	207	216
4	268	284	234	247	275	324	281	291
5	341	361	299	315	352	409	357	369
6	418	439	366	385	434	496	435	449
7	498	520	435	456	523	585	514	531
8	583	602	506	531	621	674	596	615
9	675	686	581	608	731	766	680	700

* It is not proposed to allow a paid-up policy or surrender value until two years' premiums are paid, and these values are merely inserted for the sake of completeness.

TABLE L.—*Whole Life—20 Payments.*

Duration.	Age at entry 20.				Age at entry 30.			
	O^M 3% net value of		Surrender Value		O^M 3% net value of		Surrender Value	
	Original Policy.	Proportionate Paid-up Policy.	4½%	4%	Original Policy.	Proportionate Paid-up Policy.	4½	4%
2	36	32	21	24	44	40	27	30
4	74	67	44	51	89	83	56	63
6	114	105	70	80	137	129	89	100
8	156	146	97	111	188	179	125	139
10	201	190	128	145	241	232	164	183
12	247	238	161	181	298	289	207	229
14	297	289	197	222	358	351	254	281
16	349	343	237	266	421	417	306	336
18	405	402	280	313	489	487	363	396
	Age at entry 40.				Age at entry 50.			
2	52	48	35	38	62	58	45	48
4	107	100	73	80	126	121	94	101
6	164	156	115	126	193	188	147	158
8	224	216	161	176	262	259	206	220
10	288	281	212	231	335	334	269	286
12	355	349	267	289	410	414	337	357
14	425	422	328	353	490	498	410	433
16	501	500	393	421	575	586	488	513
18	581	582	463	495	668	677	570	598
	Age at entry 60.							
2	73	69	56	60				
4	147	142	117	124				
6	220	220	183	192				
8	295	301	254	266				
10	371	386	329	343				
12	450	470	408	425				
14	532	567	492	510				
16	623	662	579	599				
18	728	759	671	691				

TABLE M.—*Whole Life*—30 *Payments.*

Duration.	Age at entry 20.				Age at entry 30.			
	O^M 3% net value of		Surrender Value		O^M 3% net value of		Surrender Value	
	Original Policy.	Proportionate Paid-up Policy.	4½%	4%	Original Policy.	Proportionate Paid-up Policy.	4½%	4%
3	40	33	22	25	48	40	27	31
6	83	70	46	53	100	86	59	66
9	128	112	75	85	155	137	96	107
12	177	159	107	121	213	193	138	153
15	230	210	144	162	276	256	187	205
18	286	268	187	209	343	325	242	264
21	347	331	236	261	415	400	304	330
24	412	401	290	321	492	483	374	404
27	484	478	353	387	576	572	452	484
	Age at entry 40.				Age at entry 50.			
3	59	49	35	39	75	59	46	49
6	122	100	77	84	152	125	98	105
9	188	165	124	135	231	197	158	168
12	257	233	178	193	311	276	225	238
15	330	307	240	258	391	361	299	315
18	407	388	308	330	472	451	380	398
21	487	475	385	409	553	548	468	488
24	573	569	469	495	639	648	562	583
27	666	668	560	587	736	752	661	683

TABLE N.—*Endowment Assurances*—10 *Years.*

Duration.	Age at entry 20.				Age at entry 30.			
	O^M 3% net value of		Surrender Value		O^M 3% net value of		Surrender Value	
	Original Policy.	Proportionate Paid-up Policy.	4½%	4%	Original Policy.	Proportionate Paid-up Policy.	4½%	4%
1*	86	77	68	71	85	77	68	71
2	174	159	142	147	173	159	142	147
3	266	245	221	229	264	245	222	229
4	360	336	308	317	358	336	308	317
5	458	432	402	412	456	432	402	412
6	559	534	504	513	557	534	504	513
7	664	641	614	623	662	641	614	623
8	772	754	733	740	770	754	733	740
9	884	874	861	865	883	874	861	865
	Age at entry 40.				Age at entry 50.			
1*	84	78	68	71	83	78	69	72
2	171	159	142	148	168	160	143	148
3	261	246	222	230	257	247	223	231
4	355	337	309	318	349	338	310	319
5	452	433	403	412	445	435	404	413
6	553	534	504	514	546	535	505	515
7	658	641	614	623	651	642	614	623
8	767	754	733	740	761	754	733	740
9	881	874	861	865	877	874	861	865
	Age at entry 60.				Age at entry 70.			
1*	79	80	70	73	73	83	73	76
2	161	163	146	151	148	168	151	156
3	247	250	226	233	226	256	233	239
4	336	341	313	321	308	348	320	328
5	430	437	406	416	396	443	412	421
6	529	538	507	516	491	543	511	520
7	634	643	616	624	595	647	618	627
8	747	755	733	740	711	757	735	741
9	868	874	861	865	845	874	861	865

* It is not proposed to allow a paid-up policy or surrender value until two years' premiums are paid, and these values are merely inserted for the sake of completeness.

TABLE O.—*Endowment Assurances—20 Years.*

Duration.	Age at entry 20.				Age at entry 30.			
	OM 3% net value of		Surrender Value		OM 3% net value of		Surrender Value	
	Original Policy.	Proportionate Paid-up Policy.	4½%	4%	Original Policy.	Proportionate Paid-up Policy.	4½%	4%
2	74	60	48	52	73	61	48	52
4	152	127	103	110	151	129	104	111
6	235	202	167	177	233	204	168	179
8	324	284	240	254	321	286	242	256
10	418	376	326	342	414	378	328	343
12	519	477	425	441	515	478	426	443
14	627	588	540	555	622	590	540	556
16	742	712	672	685	738	713	672	685
18	866	848	824	832	864	848	824	832
	Age at entry 40.				Age at entry 50.			
2	72	63	50	53	73	66	53	56
4	149	131	106	114	150	137	112	119
6	231	207	171	182	231	214	179	189
8	317	290	246	259	316	298	254	267
10	410	381	331	346	405	390	340	354
12	509	481	429	445	501	489	437	452
14	616	592	543	558	605	597	548	563
16	732	714	673	686	720	718	676	689
18	859	849	825	833	849	850	825	833
	Age at entry 60.							
2	77	71	59	62				
4	155	147	124	130				
6	234	228	194	203				
8	316	315	272	283				
10	400	406	358	371				
12	488	504	453	467				
14	585	609	560	573				
16	694	724	682	694				
18	827	851	827	834				

TABLE P.—*Endowment Assurances*—30 *Years.*

Duration.	Age at entry 20.				Age at entry 30.			
	O^M 3% net value of		Surrender Value		O^M 3% net value of		Surrender Value	
	Original Policy.	Proportionate Paid-up Policy.	4½%	4%	Original Policy.	Proportionate Paid-up Policy.	4½%	4%
3	64	49	35	39	65	51	37	41
6	134	105	78	86	135	109	81	89
9	210	170	129	142	210	175	134	146
12	292	245	192	208	292	251	198	213
15	382	331	269	288	381	337	275	294
18	481	430	363	383	479	435	369	389
21	590	543	478	498	587	547	482	502
24	712	674	618	635	707	676	620	638
27	847	825	789	801	843	825	790	801
	Age at entry 40.				Age at entry 50.			
3	68	54	40	44	78	61	47	51
6	140	116	88	96	158	128	102	109
9	217	185	144	156	241	203	164	175
12	300	262	211	225	325	285	235	249
15	388	350	289	306	411	374	316	332
18	483	447	382	401	500	472	408	425
21	586	558	492	511	593	578	514	531
24	702	683	626	643	696	696	639	655
27	836	827	792	803	822	832	795	806

TABLE Q.—*Endowment Assurances*—40 *Years.*

	Age at entry 20.				Age at entry 30.			
4	58	41	28	32	62	45	31	35
8	122	90	63	71	130	98	70	77
12	193	149	107	119	203	159	117	129
16	270	218	162	178	282	231	176	191
20	356	299	231	251	369	314	248	267
24	452	395	319	341	464	411	336	357
28	561	508	430	453	568	522	445	467
32	684	642	572	594	685	652	582	602
36	827	803	757	772	823	806	761	775

Endowment Assurances—40 Years, continued.

Duration.	Age at entry 40.							
	O^M 3% net value of		Surrender Value		O^M 3% net value of		Surrender Value	
	Original Policy.	Proportionate Paid-up Policy.	4½%	4%	Original Policy.	Proportionate Paid-up Policy.	4½%	4%
4	73	51	37	41				
8	151	110	83	90				
12	234	179	138	149				
16	320	256	203	218				
20	409	344	280	298				
24	501	442	370	390				
28	596	550	476	496				
32	697	672	603	622				
36	819	814	767	780				

TABLE R.—*Endowment Assurances—50 Years.*

	Age at entry 20.				Age at entry 30.			
5	57	37	25	28	67	43	29	33
10	120	82	56	63	140	94	67	74
15	189	137	96	107	219	156	115	126
20	265	204	147	163	305	229	174	189
25	350	283	213	233	396	315	248	266
30	445	377	297	320	492	413	336	357
35	550	489	405	429	591	524	443	464
40	670	623	544	567	695	650	572	593
45	812	787	731	748	816	798	742	758

TABLE S.—*Endowment Assurances—60 Years.*

	Age at entry 20.							
6	61	36	24	27				
12	129	80	54	62				
18	204	136	95	109				
24	287	204	149	164				
30	379	287	218	237				
36	477	385	305	326				
42	581	498	412	435				
48	689	629	544	567				
54	811	783	719	737				

An examination of these tables shows that the proposed surrender values are less than the O^M 3 per cent net values in all whole-life cases; and that the same holds good for limited payment and endowment assurance policies except when the term is 10 years and the age at entry is 70. In the case of the limited-payment policies the excess of the proposed surrender value over the reserve is comparatively small, never exceeding 2½ per cent of the latter, and it disappears after six years. In the endowment assurances the excess is greater, and lasts throughout the whole duration of the policy, but does not exceed 5 per cent of the reserve. Of course, in both cases, if the age at entry were more than 70 years, the excess would be greater. It will also be noticed that in the case of the 50-year endowment assurances, where the age at entry is 30 years, the proposed surrender value is in some cases less than that proposed for a whole-life assurance effected at the same age. These are undoubted defects in the proposed methods; but they are inherent in the system of "proportionate" paid-up policies, and cannot be entirely avoided if that system be adopted. Moreover, the errors are of comparatively small magnitude and of very rare occurrence (as such assurances are seldom effected), and I think, therefore, that any objection to the proposed methods on this account, is far more than outweighed by the great gain in simplicity which the methods secure.

PARTICIPATING PAID-UP POLICIES.

In cases where the paid-up policy carries the right to future profits (for example, a limited-payment policy with profits on which all the premiums have been paid) a higher value should be given, as the reversion (being an increasing one) is more valuable; and in order to allow for this it is necessary to assume a future rate of bonus. Obviously this should be less than the rate which the Office expects to maintain, as the policy-holder, by surrendering, is exchanging a contingent benefit which may never be realised, for a definite and certain sum of cash. There must necessarily be considerable difference of opinion regarding the rate to be assumed for this purpose; but probably most authorities will agree that it should not be much less than one-half nor much more than three-quarters of the rate which the Office is likely to maintain. The exact proportion to be allowed must be largely a matter of opinion, influenced by considerations such as the strength and progress of the Office. Most Offices make an allowance of a somewhat similar nature to this, by

granting interim bonuses on policies which become claims between two valuation years; the interim bonus being generally from two-thirds to three-fourths of the bonus which the Office expects to declare at its next valuation. In the case of a surrender of the bonuses for the whole future lifetime of a policyholder, it seems to me that a smaller proportion of the estimated bonuses should be allowed than is granted in the case of interim bonus—say, from one-half to two-thirds. Thus, if an Office is likely to maintain a rate of bonus varying between 30s. and 40s., I think it might value its participating paid-up policies for surrender by treating them as assurances increasing at the rate of, say, 1 per cent per annum; and if the bonus is likely to range between 25s. and 30s., or 20s. and 25s., the rates of increase in the assurance might be taken as, say, $\frac{3}{4}$ per cent and $\frac{1}{2}$ per cent respectively. Having assumed a rate of future bonus, the formulas for valuing such policies as increasing assurances can be readily deduced; but they involve a good deal of work, being rather lengthy and complex. For the sake of simplicity I propose to substitute for the increasing assurance a uniform assurance valued at a lower rate of interest, the difference in the rate of interest being about the same as the rate of future bonus assumed. If the bonus be compounded annually this gives a very close approximation; and an exact value can be obtained by making a slightly larger difference in the rate of interest. Thus, if the assumed rate of bonus be b per unit per annum from the outset, the value of the increasing assurance is $v(1+b)q_1+v^2(1+b)^2q_2+v^3(1+b)^3q_3+\ldots$ where $q_1, q_2, q_3, \ldots$ are the probabilities of the assurance becoming payable in the 1st, 2nd, 3rd . . . years respectively. Putting $v(1+b)=\frac{1}{1+j}$ this expression becomes $\frac{q_1}{1+j}+\frac{q_2}{(1+j)^2}+\frac{q_3}{(1+j)^3}+\ldots$ which is the value of a uniform assurance when the rate of interest is j.

Now $1+j=\frac{1}{v(1+b)}=\frac{1+i}{1+b}$, therefore $j=\frac{i-b}{1+b}$, and as b will always be small, seldom if ever exceeding ·02, we have very approximately $j=i-b$.

Accordingly the value of such an increasing assurance, at rate of interest i, is very nearly equal to the value of a uniform assurance at rate of interest $i-b$, and is exactly equal to the value of a uniform assurance at rate of interest $\frac{i-b}{1+b}$. In practice, bonuses are frequently "simple"; and even when "compound," the compounding does not take place annually, but as a rule only

once every five years, and the interim bonus is usually at a lower rate, so that the above approximation would tend to place a slightly higher value on a participating policy than would be given by a more exact calculation. The difference, however, is small, as is shown by the following table; and as a substantial margin should be allowed in fixing the assumed rate of future bonus, I think it may safely be disregarded.

TABLE T.—*Values of Uniform and Increasing Whole-life Assurances of* 1.

Age.	Value by $O^{[M]}$ Table at 4½% interest of				Value by $O^{[M]}$ Table at 3½% interest of
	Uniform Assurance. (1)	Assurance with simple bonus of 1% per annum. (2)	Assurance with bonus of 1% per annum compounding annually. (3)	(3) - (2) (4)	Uniform Assurance. (5)
10	·16864	·21927	·23022	·01095	·22993
15	·18423	·23864	·24958	·01094	·24928
20	·20339	·26174	·27255	·01081	·27224
25	·22677	·28906	·29954	·01048	·29922
30	·25502	·32104	·33100	·00996	·33067
35	·28865	·35796	·36719	·00923	·36686
40	·32806	·39992	·40818	·00826	·40784
45	·37322	·44659	·45379	·00720	·45346
50	·42396	·49748	·50346	·00598	·50314
55	·47938	·55149	·55616	·00467	·55586
60	·53807	·60710	·61055	·00345	·61027
65	·59810	·66248	·66482	·00234	·66456
70	·65736	·71577	·71714	·00137	·71691
75	·71364	·76519	·76583	·00064	·76564
80	·76518	·80947			
85	·81074	·84787			
90	·84971	·88017			
95	·88179				
100	·90726				

It will be noticed that the difference between the values of an assurance with "compound" and with "simple" bonus is greatest at the youngest age, but that even then it is less than 5 per cent of the latter value; and that it steadily decreases throughout life until after age 75 it becomes almost negligible in comparison with the value of the assurance.

Although the above table only gives figures for whole-life assurances, the same principle holds good for endowment assurances; and in the latter case the values of "simple" and "compound" bonuses will be even closer to each other, as the term of

the assurance is shorter. On these grounds, therefore, I propose that the cash value of a "participating" paid-up policy should be equal to the amount of the policy multiplied by the "select" value of A according to the $O^{[M]}$ table at some such rate of interest as follows:—

If bonus likely to range between 20s. and 25s., say 4 per cent.
If bonus likely to range between 25s. and 30s., say $3\frac{3}{4}$ per cent.
If bonus likely to range between 30s. and 40s., say $3\frac{1}{2}$ per cent.

These figures are given more for the sake of illustrating the method than as indicating the exact rates that should be adopted in practice, and there are circumstances which might render their modification desirable. For example, a compound bonus might (other things being equal) be valued at a slightly lower rate of interest than a simple bonus, in order to make some allowance for the greater intrinsic value of the former.

CASH VALUES OF COMPOUND BONUS.

Seeing that a compound bonus and a participating paid-up policy of the same Office, and for the same amount, secure identical benefits, I think they should have the same cash values; and I therefore propose that the above rules be applied also to find the cash values of bonuses in Offices dividing their profits on the compound reversionary bonus plan.

DOUBLE ENDOWMENTS.

The surrender values of these may be got from the "proportionate" paid-up double endowments by a precisely similar method, namely, by multiplying them by the $O^{[M]}$ $4\frac{1}{2}$ per cent (or 4 per cent) value of $A_{[x]:\overline{n}|}+A_{[x]:\overline{n}|}^{\;\;1}$. It will be found, however, when the date of maturity is approaching, that this gives a cash value in excess of the sum that would be payable if the paid-up double endowment were to become a claim by death. Accordingly, if the rule were to be applied without modification in such cases, any holder of a double-endowment policy who fell into bad health might be able, by surrendering, to obtain a larger sum than his representatives would receive at his death; and there would be a tendency for this option to be exercised. The Office would thus lose part of the profit expected on the transactions of this kind which turn out in its favour; and as this profit is discounted in the calculation of the premium, it might find that this class of

business resulted in a loss. It would therefore be a wise precaution (in the absence of evidence of health) to limit the amount of cash surrender value to the amount of the assurance portion of the benefit, and to apply the balance, if any, of the value given by the rule, in the purchase of a paid-up endowment. The amount of this endowment should be calculated on the same basis as the surrender value, namely, $O^{[M]}$ $4\frac{1}{2}$ per cent—not by the use of an Office single premium. Although at first sight it may appear that this is equivalent to granting the endowment at less than cost price, it is easily seen that this view is erroneous; the fact being that a sufficient part of the paid-up double endowment is surrendered on an $O^{[M]}$ $4\frac{1}{2}$ per cent basis to yield the maximum cash value, and the balance is allowed to remain as a paid-up endowment for a still further reduced amount. For, suppose that the proportionate paid-up policy secures S at death, or 2S on survivance to age $x+n$, x being the present age, then its cash value by the rule would be $S(A_{[x]:\overline{n}|}+A_{[x]:\overset{1}{\overline{n}|}})$; and suppose that this value is greater than S, say S+R. Now, if the endowment assurance portion of the benefit be surrendered for its cash value $S\times A_{[x]:\overline{n}|}$, there remains a pure endowment of S whose cash value is $S\times A_{[x]:\overset{1}{\overline{n}|}}$, and a part of this pure endowment may also be surrendered so as to bring the total cash value up to S (but not more, unless proof of health be furnished). Therefore the part to be surrendered must have a cash value of $S-S\times A_{[x]:\overline{n}|}$, and by simple proportion its amount must be $\dfrac{S-S\times A_{[x]:\overline{n}|}}{A_{[x]:\overset{1}{\overline{n}|}}}$. This leaves us (having paid S in cash) with a pure endowment of $S-\dfrac{S-S\times A_{[x]:\overline{n}|}}{A_{[x]:\overset{1}{\overline{n}|}}}$, or $\dfrac{S(A_{[x]:\overline{n}|}+A_{[x]:\overset{1}{\overline{n}|}})-S}{A_{[x]:\overset{1}{\overline{n}|}}}$ which is equal to $\dfrac{R}{A_{[x]:\overset{1}{\overline{n}|}}}$, since by hypothesis the first term of the numerator is equal to S+R. That is to say, the balance, R, of the surrender value, after paying over S in cash, must be cast into reversion by the net $O^{[M]}$ $4\frac{1}{2}$ per cent "select" single premium.

If, however, the policyholder is able to prove to the satisfaction of the Office that the assured is still in good health, there is no objection to paying over the full surrender value in cash when it exceeds the sum that will become payable in the event of death; but it would be well in such a case to require a certificate of health by a competent medical man, his fee, of course, being paid by the policyholder.

BENEFITS CONTINGENT UPON SURVIVANCE.

Similar considerations apply to all classes of benefit where the sum payable in the event of survivance exceeds the sum payable in the event of death; and we may therefore lay down the general principle that unless proof of health be furnished, the cash surrender value should never exceed the latter amount. Applying this principle to endowments, and to deferred assurances and annuities during the period of deferment, we see that when the premiums are returnable in the event of death, the surrender value of these benefits should not exceed the total premiums paid; and therefore the rule adopted by some Offices, of allowing a cash value equal to all the premiums paid after the first year, with interest thereon, will give too large values when the duration of the policy has been long—unless proof of health be insisted upon in such cases. When the premiums on such benefits are not returnable in the event of death, no cash surrender value should be allowed unless proof of health be furnished; but instead thereof, a paid-up policy should be offered. The well-known necessity of requiring proof of health before purchasing a life-interest, or paying a surrender value for a life-annuity, is merely a particular case of this general principle, since the benefit disappears in the event of death, and therefore the cash value should be zero unless proof of health be furnished. Even when such proof is forthcoming, it is perhaps doubtful whether the Office should treat the life as "select" for the purpose of surrender value. If it be accepted as a general principle that policyholders who have any option allowed them will, on the average, exercise it in a manner advantageous to themselves and adverse to the Office, it follows that a withdrawing holder of a "non-returnable" endowment or deferred annuity should not receive the "select" value of his benefit, even if he submits to the most searching medical examination. I propose that the O[M] $4\frac{1}{2}$ per cent "ultimate" value of the paid-up benefit should be allowed in such a case—this procedure being analogous to that hereafter suggested for adoption in the case of life-annuities.

ENDOWMENTS, AND DEFERRED ASSURANCES AND ANNUITIES, WITH "RETURNABLE" PREMIUMS.

It appears to me that the best course to follow in dealing with endowments, and with deferred assurances and annuities during the period of deferment, on which the premiums are returnable, is to adopt some simple arbitrary rule for arriving at the surrender

value. As there is no mortality risk under such policies, and the initial commission thereon is usually small, it is clear that a considerable proportion of the premiums can safely be refunded on surrender. On the other hand, it appears only fair that such policies should contribute something to the general expenses of obtaining new business, and therefore that in their early years the surrender value should be less than the total premiums paid. In order to secure this, and to follow as closely as possible the principles proposed in the case of whole-life assurances, I propose to leave the first year's premium out of account and to return the remaining premiums on surrender with a small addition for interest; but to limit the surrender value to the total premiums paid. On these principles I have adjusted the following arbitrary scales:—

TABLE U.—*Surrender Values of Endowments; and of Deferred Assurances and Annuities before the period of deferment has expired—premiums "returnable."*

Yearly Premiums.		Half-yearly Premiums.		Quarterly Premiums.	
Number paid.	Number returnable on surrender.	Number paid.	Number returnable on surrender.	Number paid.	Number returnable on surrender.
				5	1·2
		3	1·2	6	2·3
				7	3·4
2	1·2	4	2·3	8	4·5
				9	5·6
		5	3·4	10	6·7
				11	7·8
3	2·3	6	4·5	12	8·9
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
10 and upwards	all	20	19·9	41	40·8
		21 and upwards	all	42	41·9
				43 and upwards	all

These scales are so constructed as to return a rather smaller proportion of the premiums paid in half-yearly than in yearly cases, and a still smaller proportion if the premiums be payable

quarterly, in order to allow for the additional expense caused by the more frequent collection of premiums, and the loss of interest caused by part of the premiums being deferred for a portion of a year. This may, however, be considered by some to be an unnecessary refinement; and the following rule, which is somewhat simpler, is therefore suggested as an alternative:—

If the duration be less than 11 years, allow all the premiums paid after the first year, increased by 10 per cent.

If the duration be 11 years or more, allow all the premiums paid.

CHILDREN'S DEFERRED ASSURANCES ON, OR AFTER, THE EXPIRY OF THE PERIOD OF DEFERMENT.

As stated above, these benefits are equivalent to an endowment combined with a deferred assurance; and some Offices allow the option, at the commencement of the assurance, of withdrawing in cash the full amount of the endowment that could have been secured according to their published rates, by the premium that has actually been paid. It seems to me that this gives the holder of such a policy an unfair advantage over the holder of an endowment; and it is obvious that no one would effect an endowment with such an Office if he were aware that for the same premium he could secure an endowment for the same amount, plus the option to continue the policy, after maturity, as an ordinary assurance at a low rate of premium and without medical examination. I therefore think that on surrender at this stage of the contract, some deduction (say 5 per cent) should be made from the amount of the endowment in question. After the expiry of some years the $O^{[M]}$ $4\frac{1}{2}$ per cent "select" value of the paid-up policy as found by the proposed method, will give a larger surrender value; and when this happens, the larger value should be allowed on surrender.

LIFE ANNUITIES.

The mortality among annuitants is on the whole lighter than that among assured lives, and this fact seems to show that the medical selection by an Office is less effective than the self-selection exercised by persons who buy annuities. Accordingly, when an annuitant proposes to surrender his annuity, and tenders medical evidence that he is in good health, this does not necessarily prove that he is as good a life as a "select" annuitant. The most stringent medical examination may be said to do no more than

prove that he is as good as a "select" assured life; and the Office might treat the matter on this footing, and calculate the surrender value in the same manner as the price of a life-interest, using the Office premium for a whole-life assurance. It is, however, simpler to value the annuity on a net basis; and I propose to allow the net value by the $O^{[am]}$ or $O^{[af]}$ Table, as the case may be, using the "ultimate" values at $4\frac{1}{2}$ per cent interest.

Of course an Office is not obliged to pay any surrender value for an annuity, and may refuse to allow the contract to be broken.

SINGLE-PREMIUM POLICIES.

The net $O^{[M]}$ $4\frac{1}{2}$ per cent "select" value would be too small during the early years of a policy effected by a single premium, inasmuch as that premium contains a commuted loading to cover expenses during the whole term of the policy, and it is clear that some of this should be refunded unless it has all been absorbed. Further, if the policy be "with profits," the premium will contain a bonus loading; and some portion of this ought to be returned if the policy be surrendered before any bonuses have been added to it. I have not been able to arrive at a more satisfactory way of dealing with these cases than the rule of allowing an arbitrary percentage of the single premium; say 85 per cent if the policy be "non-participating," and 90 per cent if it be "participating." One objection to this course is that it perhaps tends to press with undue weight upon endowment assurances, especially where the term of the assurance is short. It may be said that the Office runs practically the same mortality risk whether the policy be a whole-life or an endowment assurance, and that it should in consequence deduct from the single premium the same amount in both cases (instead of a larger amount in the latter) and hand over the balance to the policyholder. If the surrender value in a whole-life case without profits be, as suggested above, 85 per cent of the single premium, the Office retains 15 per cent thereof for its trouble and risk; and it might therefore be argued that in any other case the surrender value should be equal to the single premium actually paid less 15 per cent of the "whole-life without profits" premium for the same age at entry. A rule of this kind would be simple, and of easy application in practice; but it would not allow for the fact that the expenses of the Office depend largely on the amount of premium. Thus if the commission in such cases be a percentage of the premium, it is clear that the Office is put to greater expense in respect of an endowment assurance than a whole-life policy, and

that it should accordingly retain a larger amount out of the premium in the former case. Of course, after the lapse of a number of years the net O[M] $4\frac{1}{2}$ per cent "select" value (allowing, in the case of participating policies, for the right to future profits) will give a larger surrender value than the arbitrary percentage of the single premium; and when this happens, the larger value should be allowed on surrender.

GUARANTEED MINIMUM SURRENDER VALUES.

In view of the common practice of guaranteeing on surrender a return of not less than a fixed arbitrary percentage of the premiums paid, I examined the prospectuses and Board of Trade Returns of the 59 Offices mentioned above, in order to ascertain their practice on this point. This is so diverse that it cannot be fully represented within the limits of a short table, but the following statement shows the general features:—

Non-participating Policies.		Participating Policies.	
No. of Offices.	Guaranteed Surrender Value.	No. of Offices.	Guaranteed Surrender Value.
30	None guaranteed.	28	None guaranteed.
5	25 per cent of prems. paid.	7	30 to 35 per. cent of prems. } Plus the cash value of any vested bonus.
11	30 to 35 per cent do.	4	Do. excluding 1st year's prem. } Plus the cash value of any vested bonus.
10	30 to 35 per cent } Excluding the first year's prem.	1	40 per cent do. } Plus the cash value of any vested bonus.
3	40 per cent } Excluding the first year's prem.	8	30 to 35 per cent of prems. paid.
59		2	40 per cent do.
		1	50 per cent do.
		5	40 per cent do. } Excluding the first year's prem.
		3	50 per cent do } Excluding the first year's prem.
		59	

There are many minor variations, such as a limitation of the guarantee to cases where the age at entry does not exceed 55 or 60 years; the allowance of a larger percentage in the case of endowment assurances; and in one case the exclusion of the premiums for the first *two* years. The general result is that about half the Offices guarantee a minimum surrender value of (on the average) about one-third of the premiums paid under a non-participating policy, and rather more in the case of a policy with profits; and that the remaining half make no such guarantee. A study of my tables of paid-up policies and

surrender values shows that one-third of the premiums paid may exceed, not only the reserve value, but even the amount of the full theoretical paid-up policy, when the age at entry is advanced; and this occurs to a greater extent in joint-life cases, as the risk thereunder is from the outset greater than that under a single-life policy. Accordingly every Office which gives such a guarantee without qualification, is liable to suffer a considerable loss whenever an entrant of advanced age surrenders his policy. In one case which came under my notice, a gentleman aged over 70 effected a large assurance, and surrendered it a few years later, when the Office found that the guaranteed cash value exceeded the amount they could have afforded to give as a paid-up policy. This shows that those Offices which limit their guaranteed surrender value to cases where the age at entry does not exceed say 55 or 60, have good reason for so doing. Of course, every limitation or qualification of a general rule causes more trouble and increases the chance of errors occurring; but I do not think that this consideration is sufficient to warrant an Office in giving an unqualified guaranteed surrender value of one-third of the premiums paid. Further, I doubt whether such a guarantee, even with suitable limitations, is of such advantage to an Office as to outweigh the serious drawbacks attendant upon it.

The circumstances are very different from those attending a guaranteed "proportionate" paid-up policy, as that is a benefit which can be appreciated by the man in the street, and is likely to help to popularise the Offices which grant it, and thus add to their prosperity by assisting them to obtain a satisfactory flow of new business. But the prospect of getting back one-third of the premiums on surrender can hardly be said to be attractive, as it involves the "loss" of the remaining two-thirds; and therefore I do not approve of guaranteeing a fixed arbitrary percentage of the premiums paid. It seems to me better to adopt some simple but more scientific method, such as that proposed above, so adjusted as to give reasonably accurate results at all ages from the youngest to the oldest, and thus to have one uniform rule for calculating the surrender value of each class of assurance. The arbitrary method involves the necessity of having other rules for cases where the arbitrary value would be too small; and this again means in many, if not most cases, that two or more separate calculations have to be made by the different rules in order to ascertain which gives the largest value. Any one who has had much practical experience of the calculation

of surrender values in a large Office will admit that it takes a great deal of time and trouble, and that simplification of the existing methods is very desirable, provided it can be effected without undue loss of accuracy.

It is usual not to allow any surrender value or paid-up policy until after a certain minimum number of premiums have been paid; and the following table exhibits in a general way the practice of the above-mentioned 59 Offices in this respect:—

No. of Offices.	Surrender Value or Paid-up Policy allowed after payment of	No. of Offices.	"Proportionate" Paid-up Policy allowed after payment of
4	Not stated.	6	Not stated.
7	1 year's premium.	8	1 year's premium (4 from outset).
13	2 years' premiums.	20	2 years' premiums.
34	3 ,, ,,	22	3 ,, ,,
1	5 ,, ,, (or after one-fifth of the total premiums have been paid).	3	5 ,, ,,
59		59	

There are many minor variations which it would take too long to describe in detail. For example, several Offices allow a surrender value one year sooner in the case of limited payments or endowment assurances than in whole-life assurances. Others draw a similar distinction between "participating" and "non-participating" policies; and one Office makes a difference of two years, granting a surrender value after payment of one year's premium under a "participating" policy, but not until three years' premiums have been paid in the case of a "non-participating" policy. The second portion of the table, relating as it does to the grant of a "proportionate" paid-up policy, applies only to limited-payment and endowment assurance policies, and therefore reflects the above-mentioned distinction. It is clear from these figures that the majority of Offices consider the limit should be fixed at two or three years, and that there is a decided preponderance in favour of the latter limit for whole-life cases; but that in respect of limited payments and endowment assurances opinion is fairly evenly divided between the two. One or two Offices make a point of allowing a paid-up policy and surrender value even if only one premium has been paid, and this presumably includes half-yearly and quarterly premiums; but probably most of us will agree that

this is too liberal. My own opinion is that a limit of two years is justifiable for limited-payment and endowment assurance policies, as the premiums thereon usually leave a large margin for expenses after meeting the mortality risk of the first year or two. In whole-life cases the margin available for expenses is very much less, and this constitutes a good reason for treating them in a different manner; but I do not think that this should take the form of having different limits for the two classes. It seems to me preferable to have the one rule for all classes, namely, to grant a surrender value after two years' premiums have been paid; and to allow for the difference by giving a smaller proportion of the reserve in whole-life cases. This is achieved, according to the methods now proposed, by omitting the first year's premium in those cases, while in the case of limited-payment and endowment assurance policies all the premiums are taken into account in the calculation of the surrender value or paid-up policy. Whichever limit may be fixed upon, whether two or three years, it will occasionally happen that a surrender value should be allowed at an earlier date—especially in half-yearly or quarterly cases. This will occur whenever the surrender value that will be allowed at the end of the two years (or three, as the case may be) exceeds the amount of one premium. For example, suppose the Office guarantees, after three years, a surrender value equal to one-third of all the premiums paid, and assume that the premiums are payable quarterly, each instalment being P. At the end of three years, 12 premiums will have been paid, and the guaranteed surrender value will be 4P. Now, if the policyholder has only paid 9 premiums, he could come to the Office, tender payment of three more premiums at once, and then demand the surrender value. That is to say, the Office should in such a case allow a surrender value equal to the amount of one premium, after the policy has been only $2\frac{1}{4}$ years in force. Similarly, after $2\frac{1}{2}$ years the value should, on the same supposition, be equal to two premiums, and after $2\frac{3}{4}$ years it should be equal to three premiums.

INTERIM BONUS.

It is a question of some importance whether any allowance in respect of interim bonus should be made in the case of paid-up policies and surrender values. Further, as a bonus cannot be surrendered until it has vested, it becomes necessary to consider what conditions, if any, should attach to its vesting. Excluding two Offices whose constitutions provide for a tontine bonus, the

following table shows in a general way the practice of the above-mentioned Offices on these points:—

No. of Offices.	Interim Bonus allowed	No. of Offices.	Bonus vests
3	Nil.	18	Immediately.
1	If premiums accumulated at 4 per cent exceed the sum assured.	6	After 1 year.
		12	,, 2 years.
11	Either about one-half of last bonus or of indefinite amount.	3	,, 3 ,,
		5	,, 4 ,,
17	About two-thirds of last bonus.	12	,, 5 ,,
11	About three-fourths of last bonus.	1	,, 7 ,,
8	Nearly equal to last bonus.		
6	Make an annual declaration of bonus.	57	
57			

This shows that interim bonus in one shape or another is allowed by the great majority of Offices; and there seems to be a decided tendency to allow the periodical bonuses to vest immediately they are declared. The object of any restriction in the latter respect, is to secure a modified tontine arrangement for the benefit of policies which do not become early claims; and also to facilitate the start and early progress of a new Office by keeping down the amount of its claims during the early years of its existence. So long as the bonus distribution rules of the Office are applied impartially, and the assured clearly understand the nature of the contracts, I do not see any objection in principle to this; but it is undoubtedly the fact that in many cases the smallness of an interim, or the forfeiture of a non-vested bonus, causes keen disappointment to claimants, and this must tend to prejudice the Office. To my mind the ideal system would be one which avoids as far as possible any violent fluctuations in the rates of bonus added from time to time throughout the duration of a policy; and the system of quinquennial valuations has this great advantage that it helps largely to avoid the fluctuations which would otherwise be much more frequently experienced by all Offices except perhaps those of the largest size. By allowing bonuses to vest immediately they are declared, and giving from the outset an interim bonus approximating to the rate which the Office is likely to maintain, still greater uniformity would be obtained in the rate of increase of any participating policy; and I am inclined to think that this is desirable, and that the same interim bonus should be allowed on surrender or the grant of a paid-up policy. The result of this would be (1) a much greater uniformity in the rate of increase of

the sum assured, paid-up policy, and surrender value, which under the ordinary methods are all apt to increase slowly during a valuation period, with a bound upward immediately on the declaration of the next bonus; and (2) that each holder of a participating policy would receive a bonus for each premium paid, whether the policy became a claim, or were made paid-up, or surrendered. The drawbacks to this procedure are of course (1) the cost, as it would tend, by distributing the profits more uniformly among the participating policyholders, to reduce the rates of bonus declared at the periodical valuations; and (2) the fact that it would necessitate the allowance of a surrender value (equal to the cash value of the interim bonus) after only one premium had been paid. This last consideration perhaps indicates that if an Office adopts the rule of not allowing any surrender value until after, say, two years, its bonuses should not vest, and interim bonus should not be added, until after the lapse of that time.

"RATED-UP" LIVES.

When a life has, on account of some defect in personal or family history, been accepted at a rate of premium higher than the normal, the usual method of fixing that rate is to add an arbitrary number of years to the age and to charge the premium applicable to the age thus increased. The Office thus makes the assumption that, at the commencement of the assurance, the life is equivalent to that of a healthy person of the "rated-up" age; and though this is only an assumption, it is a very convenient one, and has been found to work well in practice. A number of Companies virtually make the further assumption that this equivalence will continue throughout the existence of the contract, for they make the same reserve for such policies as they do for policies on first-class lives of the "rated-up" age; and it appears to me that the proper course in such circumstances is to calculate the surrender value and paid-up policy on the same footing. On the other hand, many Companies, when making their periodical valuations, omit the extra premium from their calculations, and thus reserve the same amount in respect of a "rated-up" policy as in the case of a similar policy on a life accepted at ordinary rates; and in these circumstances it seems proper to allow simply the normal surrender value and paid-up policy. It would be outside the scope of this paper to discuss the respective advantages of these two methods, and I therefore confine myself to pointing out the fact that the former method gives the larger reserves, if other things be equal and the valuation tables be well graduated.

COMMUTATION OF PREMIUMS: NORMAL AND "RATED-UP" LIVES.

When a policyholder desires to commute future premiums wholly or in part, either by a cash payment or by surrender of bonus, it should be assumed that he is in good health, since the transaction is virtually the purchase of an annuity. The rate of interest should be not greater than the Office is likely to realise on its funds, and 3½ per cent seems suitable under present conditions, being the rate at which premiums are calculated by a number of the best authorities. When a "rated-up" life effects a policy by a single payment or by limited payments, he virtually commutes the premiums at the outset; and as the premium is calculated at the "rated-up" age, the commutation involved therein is also on that basis. I think that the same principle should be followed if the commutation take place, not at the date of entry, but subsequent thereto, as otherwise inconsistencies would result and a "rated-up" policyholder commuting his premiums shortly after entry would be unduly penalised as compared with one who did so at the outset.

CHANGES OF CLASS: "RATED-UP" LIVES.

Similar considerations apply when a policyholder desires to change the class of his assurance; and I therefore think it a good plan in all cases of "under average" lives to take the "rated-up" age as the basis of all the calculations relating to the policy, whether these relate to the paid-up policy; surrender value; cash value of bonus; commutation of premiums by cash or surrender of bonus; division of profits on the "contribution" plan; change of class; or reserve.

NON-FORFEITURE.

I have no sympathy with the view that a surrender value partakes of the nature of an *ex gratiâ* payment by the Office. It seems to me that the statements in the prospectuses and policies of most Offices form a binding contract that a surrender value will be allowed on certain conditions, and that when a value has been quoted correctly according to the Office's ordinary rules, the Office has no moral right to forfeit this value on the ground merely that the policyholder fails to apply for it within a definite time. Unless the value be applied to keep the policy in force, I consider that it should be held at the disposal of the policyholder, and should be paid over on application and proof of title, however long that may be delayed.

There are three well-known ways of applying the value to keep the policy in force :—

(1) By advancing unpaid premiums and keeping the policy in force for the full amount, subject to a debt which can be repaid at any time during the currency of the policy. This has the disadvantage of gradually decreasing the net surrender value; and in most cases, if the debt be not repaid, it sooner or later exhausts the whole value, and the policy perforce becomes void.

(2) By providing a paid-up temporary assurance for the full amount. This is objectionable, as it deprives the assured of the right to pay up arrears and have the policy reinstated on its original footing, irrespective of the state of his health.

(3) By providing a paid-up assurance of the same class as the original policy but for a reduced amount. This presses very hardly upon those who accidentally omit to pay a premium, and who desire to pay the arrears, but are not in good enough health to justify the Office in accepting payment.

From a short investigation into some hundreds of cases of unpaid premiums, I am satisfied that the majority of these arose through inadvertence or negligence, and not from any intention of dropping the policies; and that an arrangement whereby the assurance is kept in force for a limited time, say one year, is sufficient to protect the interests of almost every policyholder. At the end of that time, if the policyholder disregards all notices issued by the Office on the subject, I think that on the whole the best course is to employ the net surrender value in providing a paid-up assurance of reduced amount. This not only leaves the surrender value intact, but causes it to become gradually larger as the age increases; and a portion of the assurance is also preserved. Whatever course be adopted, notice should be sent to all persons who appear to be interested in the policy, stating the position of matters in plain language, so as to remove as far as possible all risk of misunderstanding.

LOANS.

It is usual to grant loans to the extent of 95 per cent of the surrender value, and this is a convenient rule in practice, though a smaller margin would generally suffice. The loan should be of

such an amount that the total loan with interest to the next renewal date will be not greater than the surrender value at that date; and as the rate of interest is often less than 5 per cent, and the next renewal date frequently falls less than a year from the date of granting the loan, it is clear that a margin of 5 per cent is in such circumstances more than sufficient. If the policyholder has, however, the right to extend the days of grace for payment of the premium and interest on the loan, by paying a small fee, the margin should be larger, so as to allow for the longer period during which interest will accrue, and for any decrease in the surrender value that may be caused by the additional mortality risk run by the Office during that period.

If a policyholder has only borrowed a portion of the available value, there will be a balance, or net surrender value, to which he is entitled on surrender of the policy. Sometimes the policyholder prefers to take a paid-up policy free of loan instead of the net surrender value, and the following is the obvious method of calculating this. Find the gross paid-up policy; that is, the amount that would be allowed if there were no loan: then find what proportion of this must be surrendered in order to produce sufficient cash to repay the loan with interest; and the remainder is the net paid-up policy that can be granted free of loan. This leads to the equation $\frac{\text{net paid-up policy}}{\text{gross do.}} = \frac{\text{net surrender value}}{\text{gross do.}}$, and as the gross surrender value is got from the gross paid-up policy by multiplying the latter by A, the $O^{[M]}$ $4\frac{1}{2}$ per cent net single "select" premium, it follows that the net paid-up policy is equal to the net surrender value divided by A. A remark similar to that on p. 241 regarding Double Endowments applies to this case also; as the reasoning shows that the transaction is not the grant of an assurance on an $O^{[M]}$ $4\frac{1}{2}$ per cent net basis, but a partial surrender on those terms.

PRACTICAL APPLICATION.

In the practical application of the proposed methods it is of great importance to arrange the work so that it can be performed with a minimum expenditure of time and labour, and the application of the Card system helps largely towards the attainment of this object. The old-fashioned plan of entering the calculations in books was more expensive, as it entailed the cost of binding; and more laborious, as it involved the necessity of indexing every calculation. Also it had the great drawback that only one clerk

could use the book at a time, and when any one else wished to refer to a calculation contained in it, he had either to wait or to interrupt the work, both of which courses result in the long run in a considerable waste of time. The calculations should therefore be made on loose sheets or cards, kept in numerical order according to policy number, and an example of such a card will be found in Appendix II. The particulars should be filled in from the principal records of the Office, care being taken, in the case of endowment assurances maturing at a stated age, that this age be placed above the age at entry. This enables the computer to subtract more readily the latter from the former, thus getting the full number of years' premiums originally payable—which is wanted for the purpose of calculating the "proportionate" paid-up policy. A complete table of $1-\frac{P_{x+1}}{P_{x+n}}$ should be constructed from the Office table of premiums for whole-life assurances without profits, and if these do not proceed with sufficient smoothness they should be adjusted so as to remove any marked irregularities, and to secure that for any given duration the paid-up policy shall increase with the age at entry. As the table of Office premiums rarely extends beyond age 80, if so far, it should be continued to the extreme limit likely to be met with in practice, so that the table of paid-up policies may be sufficiently extensive to cover every case that is likely to arise.

When the premium is half-yearly it will be found that $1-\frac{P_{x+1}}{P_{x+n}}$ is not always equal to $1-\frac{P^{(2)}_{x+1}}{P^{(2)}_{x+n}}$; but it seems quite unnecessary to take into account any such small differences: and when (as is now common) the half-yearly are got from the yearly premiums by the addition of a constant percentage thereto, the two values will coincide. Similar considerations apply to quarterly premiums; and I therefore propose to use the one table whether the premiums be payable yearly, half-yearly, or quarterly; interpolating by first differences when the duration (or the age at entry) is fractional.

A table of $\frac{n}{r}$ should also be constructed, for the purpose of finding the "proportionate" paid-up policies allowed in the case of limited-payment and endowment assurance policies; and the limits of this table should be wide enough to include every case likely to arise. It is useless to make an affectation of accuracy by carrying the calculation to a large number of decimal places, as the fundamental mortality tables are themselves only approxi-

mately accurate; and I think it is quite sufficient to quote paid-up policies to the nearest integral number of pounds, and thus avoid the trouble caused by the insertion of a large number of shillings and pence in the "sum assured" columns of the Office Registers. Further, I think it is sufficient to tabulate the values of $1-\frac{P_{x+1}}{P_{x+n}}$ and of $\frac{n}{r}$ to three and four decimal places respectively. To avoid any possible confusion between the two tables, and the insertion of a number of decimal points and zeros, I would suggest tabulating to the nearest integer the function $1000\left(1-\frac{P_{x+1}}{P_{x+n}}\right)$ in the former case—that is to say, to construct a table showing the amount of paid-up policy that will be allowed for a whole-life assurance of 1000. The reason for tabulating the function $\frac{n}{r}$ to four decimal places, is that if the table only extended to three decimal places, slight errors would sometimes be introduced if the original assurance were for a large amount; and although these would be of trifling consequence having regard to the amounts involved, they might cause complaints by those policy-holders who calculated the "proportionate" paid-up policy for themselves. A simple mental multiplication of the value in one or other of these tables will give the paid-up policy to be allowed for a "non-participating" assurance of any ordinary amount. In the case of a "participating" assurance, the amount of the bonus is added, and this gives the total paid-up policy that can be granted without right to future profits. If the bonus be "compound," it virtually forms a participating paid-up assurance, and as it is converted into a non-participating assurance on the grant of the paid-up policy, its amount should be increased, so as to allow for the surrender of the right to future profits thereon. As already explained (p. 224), this increase should be of such an amount that the cash value remains unaltered by the change; that is to say, the bonus should be multiplied by $\frac{A'}{A}$, where A′ and A respectively represent the surrender values allowed for 'participating' and for "non-participating" paid-up policies of an otherwise similar nature; and accordingly a table of this ratio should be constructed. It might be argued that the increase should be in the ratio $\frac{(SP)'}{(SP)}$ where (SP)′ and (SP) represent respectively the Office single premiums for such policies; but this will generally be greater than

$\frac{A'}{A}$ owing to the difference between selling and buying; the single premiums being the prices at which the Office sells assurances, and the surrender values being the prices at which it is willing to buy them back again from the public.

The surrender value is got by multiplying the paid-up policy by the net single premium for the benefit ($O^{[M]}$ $4\frac{1}{2}$ per cent "select" value), and this operation may be performed in the ordinary way, or by the use of logarithms or an arithmometer, by contracted multiplication, or by inspection of a slide rule. The first of these methods causes, as a rule, about twice as much work as is necessary, and therefore it should never be employed. For a single multiplication involving a few figures, the use of logarithms or an arithmometer does not result in much saving of labour; and I think the two last-mentioned methods are the best, as with a very little practice they are both remarkably quick and quite accurate enough for the purpose. It seems to me quite sufficient to carry the multiplication to one decimal place, and thus to calculate the surrender value to the nearest integral number of florins.

In the case of whole-life policies on single lives an alternative method would be to construct a table of surrender values by the formula $A^{O[M]\,4\frac{1}{2}\%}_{[x+n]} \times \left(1 - \frac{P_{x+1}}{P_{x+n}}\right)$. The surrender value would then be got by entering this table, and adding the cash value of the bonus, if any. This would save some trouble if the Office declared bonuses on the "compound" system and the surrender value alone were required; but if the bonus be "simple," I doubt whether the table would repay the trouble of constructing it. As most policies are "with profits," the calculation of the surrender value generally involves the necessity of one multiplication in order to get the cash value of the bonus; so that the only saving effected by the table would be that one of the factors would be reduced. On the other hand, this method would give the cash value of the bonus and the surrender value, while the former method would give the paid-up policy and surrender value, which is the combination more generally required.

In the case of limited-payment and endowment assurance policies it would be impracticable to construct complete tables of surrender values, as the introduction of a third variable, namely the term of the assurance, would make them too extensive. The values of $A_{[x]:\overline{r}|}$ should be tabulated according to the values of x, the age at valuation; and $x+r$, the age at maturity. This is more convenient than using the variables x and r, because

the age at maturity remains constant throughout the whole existence of the policy and can be noted in the primary records of the Office, so that x is the only quantity to be ascertained before entering the table. If r were taken as one of the variables, it would be necessary to find the original term of the assurance and to subtract therefrom the "duration" in order to get the term with which to enter the table. Accordingly a policy effected at age 27 say, and payable in 30 years or on previous death, should be classified not as a "30-year endowment assurance," but as a "Death or (57)" policy, the brackets being a convenient way of denoting that the assurance is payable not upon the 57th birthday, but upon the preceding anniversary of the policy.

In order to exhibit in a concise form the methods now proposed, I append a summary in the shape of a series of rules for calculating paid-up policies and surrender values.

Before quoting any value it should be checked by an independent calculation, not by an examination of the original work, as that is apt to lead the examiner to repeat any error therein; and the value should be compared with any previous quotation, and with the total amount of premiums paid. Care should be taken to see that the person making the inquiry is entitled to the information, as it sometimes happens that an unauthorised person inquires about a policy in which he has no interest, and the policyholder would have good cause for complaint if confidential information regarding his affairs were given to everybody who might ask for it.

The net value of a policy does not increase steadily with the duration, but as each premium is paid it immediately increases by the amount of the net premium. Thus at the end of n years, before the $(n+1)$th premium is paid, the net value is ${}_nV_x$, but immediately thereafter it is ${}_nV_x+\pi_x$. Now, unless a policy has been a considerable number of years in force, it will be found that ${}_nV_x+\pi_x$ is greater than ${}_{n+1}V_x$, so that the value decreases during the year, owing to the interest on the reserve being less than the cost of the current mortality risk. It would be theoretically correct to give effect to this principle in the calculation of surrender values; but it would cause a great deal of complication in the work, and would almost certainly raise questions with the policyholders, who would have great difficulty in understanding why the surrender value should decrease. I therefore think it is desirable in all ordinary cases to calculate the surrender value as at the date when the next premium will fall due, and to adhere to this value even if the policy be surrendered before that date. On the other

hand, when a policy has been many years in force, it will be found that ${}_nV_x+\pi_x$ is less than ${}_{n+1}V_x$, so that the value actually increases during the year, the interest on the reserve being greater than the cost of the current mortality risk. In these circumstances it would perhaps be too liberal to allow the surrender value calculated as at the next renewal date, if the policy were surrendered several months prior thereto; and in such cases the Office might allow the value as at the previous renewal date, plus the premium then paid. This would, however, cause a good deal of trouble, and it is a question whether it would be worth while. If in every case the value as at next renewal date were allowed, there would be the advantage of having one rule applicable to all cases; and any loss sustained on the older policies in consequence of this rule would be more than balanced by the gain in the much more numerous cases where the rule operates in favour of the Office. Also, as the proposed surrender values leave a substantial margin for contingencies, the Office can perhaps well afford this small concession.

There are always some policyholders who fancy that premiums paid to an Assurance Office are like deposits in a bank, and that they should be returnable on demand, with the bonuses added and sometimes also with interest. These people will grumble even if their surrender values are calculated on the most liberal scale possible; and those of us whose daily work lies in connection with such matters, know that it is by no means easy to give an explanation which they can understand, and which will satisfy them that the Office is not taking an unfair advantage of them. The appended statement (Appendix III.) was drawn up and printed in leaflet form in order to meet such cases, and it has proved successful in several instances. I think it is desirable that Offices should circulate statements of this kind, giving explanations of the nature of life assurance business, for they cannot fail to have a certain educative effect upon the public, which in its turn ought to prove beneficial to the Offices.

GENERAL REMARKS.

Objection may possibly be taken to some of the methods now proposed, on the ground that in some cases the results are admittedly mere approximations, more or less rough. A little consideration, however, shows that no calculations in connection with life assurance can be strictly accurate, as the data upon which they are based are themselves only approximate. It is impossible

to determine the future rate of interest or expenditure; and even if it be assumed that the past rates of mortality will be unaltered in the future, the fact that our mortality tables require graduation is sufficient to show that they are mere approximations. There is therefore room for considerable difference of opinion even in such a comparatively simple matter as the calculation of premiums, and still more so in the case of surrenders where the question is much more complex. Even the statement that the surrender value should not exceed the reserve, though apparently almost axiomatic, cannot be accepted without qualification. At first sight it appears to be a necessary consequence of the fact that if the total surrender values exceed the total funds, and all the assured were to surrender their policies at once or within, say, a year, the Office would be unable to pay them; but a little consideration will show that this argument is not logically sound. It is undeniably true that if all the policyholders were to die within a year, the Office would be unable to pay the claims if its total funds were less than the face value of the policies; and that this is a risk we all run every day without the least anxiety, because experience teaches us that the chance of that event occurring is so minute as to be incapable of measurement. But experience also teaches us that the chance of all the policies being surrendered within a year, though certainly greater than the chance of all the lives dying in the same time, is also so minute as to be incapable of measurement; so that if we can safely disregard the one risk, it is not necessarily illogical to disregard the other. Again, if it be the case that the lives who withdraw are worse than those of the same age who remain, the true values of the surrendered policies will be greater, and the true values of the remaining policies will be less than the reserves respectively held against them. It might conceivably, on this hypothesis, pay an Office to give surrender values in excess of its reserves, and thus get rid of its undesirable risks; and as a matter of fact Offices do occasionally and quite properly give specially large surrender values in cases where they have reason to believe that the lives assured are in bad health.

It is impossible to frame a mathematically accurate scheme of surrender values based purely on theoretical considerations, and we must therefore be largely guided by considerations of expediency and practice. In fact, it seems to me that though we ought to work out the theory of our business with the utmost accuracy attainable, when we come to apply it in practice it is perhaps of more importance to devise rules which secure simplicity and ease of working, than by the use of elaborate formulas to

strive after a degree of mathematical exactitude which is necessarily unattainable. These rules should, of course, conform as far as practicable with theory, and if they tend to give an error, care should be taken to see that it is on the whole on the right side.

It will probably be generally admitted that (save in exceptional cases where a risk is manifestly undesirable) the surrender value should not exceed the reserve; that in the early years of the policy it should be substantially less—say from one-half to three-quarters thereof; that as the duration increases, a larger proportion may be allowed; and that at very advanced ages it should approximate to the full reserve. The values brought out by the methods I propose, comply on the whole with these conditions; the methods are applicable at all ages from the youngest to the oldest; the paid-up policies and surrender values are consistent with each other; and the necessary calculations are extremely simple, the system probably being less complicated than any other hitherto published in this country, so that it can be applied in practice with a minimum of labour. At the same time it undoubtedly leaves ample scope for improvement, and I hope that the discussion upon it will indicate the directions in which such improvements can best be made.

SUMMARY

Proposed Rules for calculating Paid-up Policies, Surrender Values, Cash Values of Bonus, and Commutations of Premium.

PAID-UP POLICIES.

(1) Construct a table of $1000\left(1-\frac{P_{x+1}}{P_{x+n}}\right)$ and call this Table I. (say).

(2) Construct a table of $\frac{n}{r}$ and call this Table II. (say) (p. 255).

(3) Do not allow any paid-up policy unless two full years' premiums have been paid under the original contract, except in those few cases where payment of the sum necessary to complete two years' premiums, would secure a paid-up policy whose cash value would exceed that sum; and do not allow any, if the amount thereof would be less than £20 (pp. 227, 249).

WHOLE-LIFE ASSURANCES ON SINGLE LIVES.

Find the Office age at entry (x) and the duration of the policy (n), and with these variables enter Table I., multiply the result by the sum assured and divide by 1000. If x be less than 10, enter the table with the variables 10 and n (p. 206).

DISCOUNTED BONUS SYSTEM.

Proceed as above and reduce the result in the ratio

$$\frac{A_{[x+n]}(O^{[M]}\,4\tfrac{1}{2}\%)}{A_{[x+n]}(O^{[M]}\,k\%)}$$

where the rate k is so chosen that $4\frac{1}{2}-k$ is, say, from one-half to one-third of the anticipated annual rate of bonus.

Add to this any existing additions to the sum assured, or the reversionary equivalent of any further reduction of future premiums (p. 226).

WHOLE-LIFE ASSURANCES ON TWO OR MORE (say m) JOINT LIVES.

Find the Office ages at entry x, y, z, . . . (m terms), and from these find (to the nearest integer) an age w such that $c^x+c^y+c^z+\ldots$ (m terms)$=mc^w$. With w as the age at entry and n the actual duration of the policy, enter Table I. Multiply the result by the sum assured and divide by 1000. If there are not more than four lives, w can be got by inspection of the tables of "Uniform Seniority" on pages 244-251 of the "Experience" (O^M and $O^{M(5)}$) Tables (p. 211).

WHOLE-LIFE ASSURANCES ON THE LAST SURVIVOR OF TWO LIVES.

(*a*) *While both the Assured survive.*—Find the Office ages at entry; and with the lesser of these, and the actual duration of the policy, enter Table I. Multiply the result by the sum assured and divide by 1000 (p. 215).

(*b*) *After one of the Assured has died.*—Find the Office age at entry (say y) of the survivor, and the actual duration of the policy (say n). Then find (to the nearest integer) a number r such that $P_{y-r}=P_{x:y}$, and with $y-r$ as the age at entry and $n+r$ as the duration, enter Table I. and proceed as above (p. 215).

WHOLE-LIFE ASSURANCES ON THE LAST SURVIVOR OF THREE LIVES.

(*a*) *While all the Assured survive.*—Find the Office ages at entry, and with the least of these, and the actual duration of the policy, enter Table I. and proceed as above (p. 215).

(*b*) *After one of the Assured has died.*—Find the Office ages at entry of the two survivors (say y and z), and the actual duration (say n). Then find (to the nearest integer) a number s such that $P_{\overline{y-s:z-s}}=P_{\overline{x:y:z}}$, and with the lesser of the two numbers $y-s$ and $z-s$ as the age at entry, and $n+s$ as the duration, enter Table I. and proceed as above (p. 215).

(*c*) *After two of the Assured have died.*—Find the Office age at entry of the survivor (say z) and the actual duration (say n). Then find (to the nearest integer) a number t such that $P_{z-t} = P_{z:y:r}$. With the variables $z-t$, $n+t$ proceed as above (p. 216).

LIMITED-PAYMENT AND ENDOWMENT ASSURANCE POLICIES, DOUBLE ENDOWMENTS, PURE ENDOWMENTS, AND DEFERRED ANNUITIES.

Find the original number of premiums payable (r) and the number actually paid (n), and with these variables enter Table II. Multiply the result by the sum (or sums) assured or endowed, or the annuity, as the case may be (p. 217).

HALF ENDOWMENTS

are to be treated as if they were endowment assurances for the amount payable in the event of survivance. That is to say, no additional paid-up policy is to be granted in respect of the additional temporary assurance.

FOUR PER CENT. DEBENTURE POLICIES.

If it is specially desired that the paid-up policy be on this plan, its amount is to be found by the above rules. In general, however, the paid-up policy should be payable in one sum, and its amount should be 1·15 times the amount given by those rules (p. 219).

FIVE PER CENT. INCOME POLICIES.

If it is specially desired that the paid-up policy be on this plan, its amount is to be found by the above rules. In general, however, the paid-up policy should be payable in one sum, and its amount should be ·7662 of the amount given by those rules (p. 219).

CHILDREN'S DEFERRED ASSURANCES.

(*a*) *At or after the Commencement of the Assurance.*—Divide the premium actually payable, by the Office premium at the attained age for each unit of the benefit as it then exists, and subtract the quotient from the sum assured (p. 220).

(*b*) *Before the Commencement of the Assurance.*—Find the paid-up policy that would be allowed, as above, at its commencement; then multiply this by the number of premiums actually paid, and divide the product by the number of premiums payable under the original contract until the commencement of the assurance (p. 220).

TEMPORARY AND SURVIVORSHIP ASSURANCES SUBJECT TO PERIODICAL PREMIUMS.

Allow no paid-up policy.

BONUSES.

If the original policy be "with profits," add to the amount given by the above rules the amount of the existing bonus additions, including interim bonuses to date (pp. 224, 249).

If the bonus be "compound," increase its amount in the ratio $\frac{A'}{A}$ before adding it to the paid-up policy given by the above rules.

(A′ and A denote respectively the cash values of "compound" and "simple" bonuses; or, what is the same thing, of "participating" and "non-participating" paid-up policies) (p. 256).

FRACTIONS OF A POUND.

In order to avoid as far as possible the insertion of shillings and pence in the lists of sums assured, the paid-up policy is in every case to be calculated to the nearest £ (p. 256).

FUTURE PROFITS.

The paid-up policies found by the above rules are not to share in future profits (p. 223).

If in any case it is specially desired that the paid-up policy should share in future profits, its amount should be got by reducing in the ratio $\frac{A}{A'}$ the amount given by the above rules (p. 256).

SURRENDER VALUES.

WHOLE-LIFE (single, joint, or last-survivor), LIMITED-PAYMENT, ENDOWMENT ASSURANCE, HALF ENDOWMENT, 4% DEBENTURE and 5% INCOME POLICIES.

Find the "non-participating" paid-up policy by the above rules, and multiply it by the net single premium ($O^{[M]}$ $4\frac{1}{2}$% "select" value) for the corresponding benefit. (See Tables in Appendix I.) (p. 228).

(NOTE.—Values of $A_{[x]:[x]}(O^{[M]}\ 4\frac{1}{2}\%)$ will be found in a volume now in course of preparation by Mr. H. H. Austin, F.I.A., and Mr. F. P. Symmons, F.I.A., which will be published in a few months.)

DOUBLE ENDOWMENTS.

When the value given by the above rule exceeds the amount payable under the original contract in the event of death, the latter sum only is to be paid in cash (unless strict evidence of health be furnished). Any excess above this amount is to be converted into a paid-up endowment, payable at the date of maturity of the original policy, and with no return in the event of death before that date.

This conversion is to be made by dividing the excess by $A_{[x]:\overline{n}|}^{1}$ ($O^{[M]}$ $4\frac{1}{2}\%$), where x=age at date of conversion, and $x+n$=age at maturity (p. 240).

PURE ENDOWMENTS, DEFERRED ANNUITIES, AND CHILDREN'S DEFERRED ASSURANCES (before the commencement of the assurance), WITH NON-RETURNABLE PREMIUMS.

No cash surrender value is to be allowed unless strict proof of health be furnished. When this is forthcoming, find the amount of the paid-up benefit by the above rules, and multiply it by the net single premium ($O^{[M]}$ $4\frac{1}{2}\%$ "ultimate" value) for the corresponding benefit (p. 242).

PURE ENDOWMENTS, DEFERRED ANNUITIES, AND CHILDREN'S DEFERRED ASSURANCES (before the commencement of the assurance), WITH RETURNABLE PREMIUMS.

If the duration be less than 11 years, the surrender value is to be equal to the amount of premiums paid after the first year, increased by 10%. If the duration be 11 years or more, the surrender value is to be equal to the whole of the premiums paid (p. 244). Alternatively, the scale on p. 243 might be used.

CHILDREN'S DEFERRED ASSURANCES.

(*a*) At the commencement of the assurance and until the following rule (*b*) would give a larger value, the surrender value is to be 95% of the amount of the pure endowment (with or without return of premiums, as the case may be) that could have been secured by the premiums actually paid; and thereafter

(*b*) Find the paid-up policy by the above rules, and multiply it by the net single premium ($O^{[M]}$ $4\frac{1}{2}\%$ "select" value) for the corresponding benefit (p. 244).

TEMPORARY AND SURVIVORSHIP ASSURANCES.

When the premiums are periodical, allow no surrender value. When the premium is single, allow (1) *for a temporary assurance* the net single premium ($O^{[M]}$ $4\frac{1}{2}\%$ "select" value) for the remainder of the term, omitting therefrom fractions of a year; and (2) *for a survivorship assurance* the net single premium for the benefit ($O^{[M]}$ $4\frac{1}{2}\%$ value, assuming the life assured to be "select" and the counter life to be "ultimate"; and require proof of health of the latter).

"ASCENDING SCALE" POLICIES [*i.e.* policies where the premium for the first few years is just (or little more than) sufficient to cover the risk, expenses, and option to continue the assurance without further proof of health].

Treat these as if they were level premium policies effected at the date the first increased premium fell due.

SINGLE PREMIUM POLICIES.

If "non-participating," allow 85% of the single premium—until such time as the above rules give larger values (p. 245).

If "participating," allow 90% of the single premium—until such time as the net $O^{[M]}$ "select" value at rate j would be larger. [The rate j to be chosen so that $\cdot 045 - j$ is, say, from one-half to two-thirds of the annual rate of bonus (per unit) likely to be maintained in the future] (p. 237).

FRACTIONS OF A POUND.

The calculation of surrender values is not to be carried to more than one place of decimals. That is to say, the value is to be taken to the nearest florin (p 257).

CASH VALUES OF BONUS.

Treat a simple bonus as a "non-participating," and a compound bonus as a "participating" paid-up assurance (pp. 228, 240).

COMMUTATION OF PREMIUMS BY CASH OR SURRENDER OF BONUS.

The amount of cash, or the cash value of the bonus, divided by the net $O^{[M]}$ $3\frac{1}{2}\%$ annuity ("select" value) gives the annual reduction of premium. If the premium be half-yearly or quarterly, divide the annual reduction by 2 or by 4, as the case may be (p. 252).

"RATED-UP" LIVES.

For the purpose of calculating paid-up policies, surrender values, cash values of bonus and commutations of premium, "rated-up" lives are to be treated as if actually of the "rated-up" age; provided they are so treated in the periodical valuations of the Office (pp. 251, 252).

APPENDIX I.

$A_{[x]}$—O[M] $4\frac{1}{2}\%$.

Age.	$A_{[x]}$.	Age.	$A_{[x]}$.	Age.	$A_{[x]}$.	Age.	$A_{[x]}$.
10	·16864	35	·28865	60	·53807	85	·81074
11	·17152	36	·29606	61	·55004	86	·81905
12	·17450	37	·30372	62	·56206	87	·82710
13	·17755	38	·31161	63	·57402	88	·83490
14	·18083	39	·31966	64	·58608	89	·84248
15	·18423	40	·32806	65	·59810	90	·84971
16	·18772	41	·33662	66	·61007	91	·85669
17	·19142	42	·34541	67	·62200	92	·86336
18	·19526	43	·35450	68	·63388	93	·86978
19	·19921	44	·36375	69	·64564	94	·87589
20	·20339	45	·37322	70	·65736	95	·88179
21	·20770	46	·38301	71	·66889	96	·88739
22	·21222	47	·39291	72	·68031	97	·89278
23	·21687	48	·40307	73	·69158	98	·89785
24	·22174	49	·41345	74	·70270	99	·90268
25	·22677	50	·42396	75	·71364	100	·90726
26	·23203	51	·43472	76	·72436		
27	·23750	52	·44566	77	·73491		
28	·24314	53	·45673	78	·74520		
29	·24900	54	·46801	79	·75528		
30	·25502	55	·47938	80	·76518		
31	·26131	56	·49091	81	·77478		
32	·26781	57	·50254	82	·78417		
33	·27453	58	·51430	83	·79330		
34	·28146	59	·52615	84	·80212		

$A_{[x]:\overline{r}|}$.—O[M] $4\frac{1}{2}\%$.

x	$x+r$				x
	30	35	40	45	
20	·65090	·53244	·44122	·37150	20
21	·67854	·55392	·45798	·38460	21
22	·70757	.57648	·47554	·39833	22
23	·73814	·60021	·49398	·41272	23
24	·77026	·62514	·51336	·42787	24
25	·80398	·65137	·53373	·44373	25
26	·83947	·67897	·55513	·46039	26
27	·87671	·70795	·57756	·47787	27
28	·91586	·73839	·60120	·49621	28
29	·95694	·77048	·62605	·51546	29
30	...	·80416	·65215	·53570	30
31	...	·83955	·67962	·55698	31
32	...	·87675	·70847	·57928	32
33	...	·91587	·73887	·60275	33
34	...	·95694	·77078	·62742	34
35	...	...	·80437	·65335	35
36	...	...	·83968	·68065	36
37	...	...	·87684	·70933	37
38	...	...	·91590	·73951	38
39	...	...	·95694	·77130	39
40	...	...	...	·80476	40
41	...	...	...	·83994	41
42	...	...	...	·87697	42
43	..	...	...	·91594	43
44	..	...	..	·95694	44

$A_{[x]:\overline{r}|}$.—O[M] $4\frac{1}{2}$%—*continued.*

x	$x+r$				x
	50	55	60	65	
20	·31867	·27935	·25076	·23074	20
21	·32905	·28762	·25757	·23651	21
22	·33986	·29632	·26463	·24250	22
23	·35122	·30537	·27208	·24878	23
24	·36311	·31488	·27983	·25528	24
25	·37560	·32483	·28792	·26208	25
26	·38869	·33521	·29636	·26923	26
27	·40234	·34610	·30519	·27665	27
28	·41672	·35747	·31441	·28431	28
29	·43179	·36940	·32405	·29236	29
30	·44755	·38184	·33409	·30071	30
31	·46409	·39489	·34463	·30945	31
32	·48144	·40854	·35557	·31850	32
33	·49966	·42284	·36699	·32797	33
34	·51878	·43778	·37895	·33779	34
35	·53880	·45345	·39140	·34804	35
36	·55986	·46986	·40441	·35867	36
37	·58196	·48700	·41801	·36974	37
38	·60516	·50500	·43218	·38129	38
39	·62958	·52382	·44695	·39321	39
40	·65524	·54358	·46241	·40570	40
41	·68224	·56430	·47861	·41862	41
42	·71066	·58604	·49544	·43210	42
43	·74060	·60891	·51310	·44609	43
44	·77211	·63289	·53152	·46069	44
45	·80532	·65813	·55086	·47589	45
46	·84028	·68474	·57110	·49170	46
47	·87714	·71274	·59233	·50814	47
48	·91600	·74223	·61459	·52528	48
49	·95694	·77336	·63802	·54320	49
50	...	·80618	·66260	·56189	50
51	...	·84084	·68853	·58140	51
52	...	·87744	·71592	·60181	52
53	...	·91610	·74477	·62325	53
54	...	·95694	·77526	·64573	54
55	...	...	·80755	·66941	55
56	...	...	·84170	·69439	56
57	...	...	·87787	·72074	57
58	...	...	·91626	·74865	58
59	...	...	·95694	·77823	59
60	...	...	...	·80963	60
61	...	...	...	·84303	61
62	...	...	...	·87861	62
63	...	...	...	·91651	63
64	...	...	...	·95694	64

APPENDIX LI.

SPECIMEN CARD AND CALCULATION.

Policy No. **10,000.**	Life, **George Robinson.**
Rated up, **5** years.	Agency, **H. O.**
Class, **Life.**	Sum assured, **£1000**
Original No. of Premiums, ____	
Born **10 February** 1852.	Vested Bonus, **£330**
Rated up Age at entry (30) **35**	
Entered **1 January** 1882.	Interim do.
Duration, ... **25**	for **3** years, **37·5**
	£367·5
Valued **1 January** 1907.	
Rated up Age at Valuation, **60**	
	Premium (yearly), **£28 : 3 : 4**
Loan, **£ Nil.**	Latest premium paid **1906.**
Reassured, **No.**	Title of **Assured** is clear.

(*If bonus "simple."*)		(*If bonus "compound."*)	
$1000 \times {}_{25}(FP)_{35}$ =	**614**	**Bonus** =	**367·5**
(Simple) Bonus =	**368**	$A'(3\frac{3}{4}\%)$ / $A(4\frac{1}{2}\%)$ =	**890·1** (reversed)
Paid-up policy =	**982**		**368**
$A_{[60]}$ (reversed)	**70835·**		**33**
	4910		**3**
	295		**404**
	79	$1000 \times {}_{25}(FP)_{35}$	**614**
Surrender Value,	**528·4**	**Paid-up policy** =	**1018**
		$A_{[60]}$ (reversed)	**70835·**
			5090
			305
			81
			1
		Surrender Value,	**547·7**

Surrender Value **if bonus "simple"** as at **1 January** 1907, **£528, 8s.**	Total premiums received, **£704.**
Or, if bonus "compound," £547, 14s.	
Quoted to **Assured.**	Date, **31 December 1906.**

APPENDIX III.

Popular Explanation of the Nature of a Surrender Value.

The business of Insurance is based on the principle of averages—a large number of persons contributing sums into a common fund or "pool," which is divided amongst those members who sustain loss by the happening of the event insured against. For example, the owner of a house is liable to the risk of his property being destroyed by fire, and such a loss would be heavy if it fell entirely upon the individual. The risk of fire is small, and by means of Fire Insurance it is spread over so large a number of persons that the burden upon each is trifling. Each holder of a Fire policy pays a small periodical premium to the Office, and the total premiums thus received enable the Office to make good the losses of those policyholders whose property is burnt. That is to say, the losses of the few are met almost entirely out of the premiums paid by the many policyholders whose property is not burnt. By paying the small premium the individual policyholder is to that extent out of pocket, but he receives in exchange the assurance that if a fire does take place the loss will be borne for him. In other words, he incurs the *certainty* of a trifling outlay in order to remove the *possibility* of a serious loss. As the premiums paid to the Office each year are only sufficient to meet the claims arising in that year, and remunerate the Office for its trouble and expense in managing the business, it is clear that no return of premium can be made to those persons who suffer no loss by fire. This is now generally understood by business men who continue to pay fire premiums year after year without getting any tangible return for their money. They recognise the fact that the premiums they have paid have gone to recoup the losses of other policyholders less fortunate than themselves, and to meet the necessary expenses of management.

When Life Assurance, however, is in question, some people still find it difficult to understand why the Office is unable to refund all the premiums paid by any policyholder who wishes to discontinue his policy. They overlook the fundamental principle of the business, which is, as already explained, the payment of claims out of contributions received from other policyholders, in order to spread the individual losses over a large area and thus minimise their effect. It is certain that out of a large number of persons some will die in a year, and, although the *proportion* of deaths can be predicted with comparative accuracy, it is impossible to foresee which *individuals* will die. The Assurance Office collects premiums from a large body of policyholders on the condition that it will pay, as and when they die, certain sums to their representatives. If the probability of death were the same at all ages, the premiums required to enable the Office to pay the claims as they arise would be the same at all ages, and the transaction would be precisely similar to fire insurance—each year's premium meeting each year's claims, expenses, etc., and leaving nothing over which could be returned as a "surrender value." As is well known, however, the probability of death increases with each year that passes, the increase being very marked as old age approaches. The premium paid for life insurance usually remains of the same amount during the whole term of the policy, being "levelled up" in such a way that in the long-run the Office receives enough to pay all the claims. This

"levelling up" has the effect of making the premium greater at the beginning of the assurance but less towards the end of life than is necessary to cover the risk; and therefore the claims of the first few years' assurance are less than the premiums received in that time, so that the Office gradually accumulates a fund or "reserve" as it is called. After the lapse of a number of years, when the lives become old and die more rapidly, the claims will necessarily exceed the premiums, and the difference has therefore to be met out of the fund accumulated in the earlier years. This fund is held by the Office for the benefit of the policyholders as a whole, and if one of them wishes to discontinue his policy it is only equitable that he should be entitled to a share of it. This share is what is known as the "surrender value" of the policy.

Now, as the fund is formed only from the balance of the premiums left over after payment of the claims, expenses, etc., it is clear that the total fund will, as a rule, be much less than the total premiums paid, and during the early years it will be only a very small proportion thereof. It follows that each individual's share of the fund (*i.e.* the surrender value of his policy) must generally be much less than the premiums paid.

Looking at the matter in another way, if a policyholder die within a few years of effecting the policy, his representatives will receive from the Company a sum many times larger than the amount of premiums he has paid. The difference of course is made up out of the premiums of other policyholders, in accordance with the fundamental principle that the claims of those who die early are paid largely by those who live longer. Accordingly if a policyholder does not die but wishes to withdraw from the Company after a few years, he cannot get back all the premiums he has paid, because a part of them has gone in payment of claims under other policies, and part in payment of expenses.

The premiums charged by Assurance Offices leave little or no margin after paying the first year's claims and expenses; and consequently Offices cannot as a rule allow any surrender value during that or even the second year. A rough-and-ready estimate of the surrender value allowed by the best British Offices may be arrived at by taking, say, one-half of the premiums paid after the first year under a whole-of-life policy which carries the right to bonuses, or one-third of the premiums paid after the first year under a policy without bonuses. Many Offices make it a condition that no surrender value shall be allowed unless at least three years' full premiums have been paid.

DISCUSSION.

Mr. CHATHAM.—I regret that I have not been able to devote as much time to the study of Dr. Ernest Sprague's paper as its importance deserves. There are, however, several points I should like to deal with. In the first place, I quite agree with him that none of the methods of calculating surrender values set forth in the papers to which he refers are entirely satisfactory. The method he proposes is simple indeed. He finds the paid-up policy, and values it at 4½ per cent. In the case of an assurance by whole-life premiums his paid-up policy is $1 - \frac{P_{x+1}}{P_{x+n}}$. The substitution of P_{x+1} for P_x, the Office premium for the age at entry, is, as he has stated, due to Dr. T. B. Sprague. I may mention that in practice the latter used the formula $1 - \frac{P_x}{P_{x+n-1}}$, which enabled him to make use of the premium actually payable. On the other hand, Dr. Ernest Sprague's ingenious application of the table of single life paid-up policies to joint and last-survivor assurances is original, and we are indebted to him for such a simple method of dealing with them. In the case of limited-payment and endowment assurance policies, the surrender value is based upon the "proportionate" paid-up policy and I am the more interested in this part of the paper since, as far as I know, I was the first to suggest this method, and apply it in practice. You may remember that in my paper on Endowment Assurances about a year ago I brought it forward and explained its advantages, comparing it with other methods. It is in this matter of comparison that the paper strikes me as being somewhat defective. He does indeed tabulate the net O^M 3 per cent Reserve, but it would have been very desirable if he had given the results of other methods of calculating surrender values and set them alongside his own for comparison. Now I found that in the case of limited-payment and endowment assurances, there was no method which gave such good results as that which I adopted, and naturally I have nothing but praise for it as now recommended in the paper. There is, however, a slight difference between Dr. Ernest Sprague's application of the method and my own. His valuation function is a "select" and mine an "aggregate" single premium. In this connection he refers to Mr. G. F. Hardy's paper (*J. I. A.*, xxiii, 1), in which it was shown that the policyholders of the "British Empire" Office who surrendered their bonuses for cash were better lives than those who did not, and Dr. Sprague puts that forward as a reason why we should use a "select" table in calculating surrender values of policies. Now I cannot see why that should follow. The results of Mr. Hardy's investigation were just what any Actuary would, I think, have expected. I should have been surprised had they been otherwise, because an option is exercised, and wherever that is done, it is against the Office. But in the paper I wrote in 1890 (*J. I. A.*, xxix, 81), and which Dr. Sprague does me the honour of referring to, I showed that withdrawals had a favourable effect on the mortality of assured lives. Since then, moreover, developments have taken place to which no allusion is made in the paper. I refer to the question of "maximum mortality percentages." You will remember the conclusion which Dr. T. B. Sprague originally drew from these, viz. that withdrawing lives were on the whole better than those which remained. Mr. Macaulay, however, in a letter in 1895 (*J. I. A.*, xxxii, 117),

showed that that feature was in all probability due to the table of mortality used in calculating the "expected deaths," and that it was possible to obtain a "maximum" by adding together individual groups in none of which a "maximum" appeared. Dr. T. B. Sprague, in his reply (*J. I. A.*, xxxii, 197), admitted the force of Mr. Macaulay's remarks, and said, "I now entertain considerable doubt on this point," viz. as to whether the figures in question proved that the lives which withdrew were on the average better than those which remained. He then went on to refer to the results of my own investigation already mentioned, in regard to which he said, "Mr. Chatham's conclusion, if correct, is of great importance, but the difference between the rates of mortality in the Scotch and in the total experience is small; and the conclusion seems to me to require confirmation from other sources before it can be accepted without reserve." All this will be found set forth at length in my remarks in the discussion which took place at the Institute on Mr. Fulford's paper in 1900 (*J. I. A.*, xxxv, 236), to which I need only refer. But Mr. Besant, in the same discussion, gave the results of an investigation into the causes of the surrender of policies. He found that, out of 1085 cases, 840, or 77·4 per cent of the whole, were given up entirely from impecuniosity, that is to say, there was no option in the matter; and he thought it might be said that those lives were at least below the average of the general body. The others were probably above the average, and, taking the body as a whole, he thought it was a fair conclusion to come to that their mortality did not differ much from that of the general body of policyholders. The position of matters is therefore this: (1) I found that, in circumstances as nearly similar as possible, the higher rate of withdrawal brought out a lighter rate of mortality; (2) Dr. T. B. Sprague's basis upon which he founded his theory has been shown to be erroneous; (3) Mr. Besant's investigation showed that three-fourths of those withdrawing had no option in the matter, and were likely to be worse lives than those that remained. So far as I know, nothing has been done since then to disprove my conclusion that those who surrender their policies are certainly not better lives than those who remain. Therefore I do not agree with Dr. Ernest Sprague in his use of a "select" table, and I think an "aggregate" table such as the H^M or O^M is more correct. I see he recommends the adoption of loose cards or sheets for the calculation of surrender values. This plan has been in operation in a number of Offices for many years past. In some Offices the calculations are put up with the proposal papers, and are thus easy of access. With regard to the desirability of educating the public in Life Assurance matters I quite agree with the author, and I welcome the model "explanation of a surrender value" which he offers with that object in view. I prefer, however, and have been in the habit of using, a much simpler form; still his suggestion is a step in the right direction. Taking the paper as a whole, I think one cannot help being struck with its thoroughness; and the clear and lucid manner in which the details are worked out reminds me of his father's contributions. The paper is a very valuable one indeed, and we are all greatly indebted to Dr. Ernest Sprague for it.

Mr. Gunn.—Dr. Sprague's paper is an eminently suggestive one, and is well worthy of careful study. I agree with him when he says that surrender values should be calculated on the footing that the policyholder is entitled to his equitable share of the funds of the Assurance Office. In former days the view was fairly prevalent that the Office was entitled to make a handsome profit out of surrenders, but I am glad to observe that we are approximating to the opinion held in America that the reserve value should be paid as surrender value. This, of course, is going too far, and the question is, what constitutes the policyholder's share of the funds in the event of surrender? Dr. Ernest Sprague says we have to take

into account the cost which has been incurred by the Office, and I think, notwithstanding the remarks of Mr. Chatham, that we must also assume that a certain amount of selection against the Office exists. Subject to these qualifications, therefore, the surrender value should have some relation to the table of mortality and rate of interest on which the Office values its liabilities. Dr. Ernest Sprague's explanation of his system is admirable, but there is one point on which I should have been glad of some explanation: both the paid-up policy and its value are calculated at an arbitrary rate of interest—*i.e.* a rate not actually earned by the Office. Is not the policyholder unduly taxed by the double calculation? If the surrender value were based directly on an actual surrender-value formula which took into account the various elements, would not the resulting surrender value be larger? In conclusion, I wish to re-echo what Mr. Chatham has said with reference to the value of the paper, and to the great care and thoroughness with which it has been prepared.

Mr. Low.—I was a little afraid when Mr. Gunn was speaking that he was going on to say he approved of allowing the full reserve as surrender value, but I was relieved to find he did not go so far. Something of the kind was done, I think, by State legislation in America. It is, of course, quite out of the question. I think that the surrender value should bear some reasonable relationship to the reserve value. And I agree with Dr. Sprague that it ought also to bear some relation to the value of the paid-up policy to which the assured may be entitled. In regard to the question of "breach of contract," I cannot help thinking that Dr. Sprague rather too hastily rejects the idea that the Company is entitled to some compensation on this account. I admit that the expression "breach of contract" is not quite appropriate. But barring the defect of that particular expression, there is an idea underlying it which I think an Office is quite entitled to take into account. The Society is formed for the particular object of granting Assurances, and if a member, for his own advantage or from his own necessity, wishes to resile from the contract the Society has made with him, I do think this question of "compensation" is one which they may legitimately take into consideration in fixing the surrender value. I was rather puzzled with the figures given in the paper with reference to "select" and "mixed" lives. Dr. Sprague seems to assume that the possession of a Select Life Table enables him to say what proportion of "select" lives will be in existence out of a given number of "mixed" lives. I am unable to follow that. I do not think, for example, it is *a priori* reasonable to assume that out of 94974 lives who assured at 10 and are alive at 20 there are 93696 "select" and 1278 "damaged." I doubt whether a Select Table warrants such an assumption, and I should like to hear something on that point, because it seems to me impossible to base any argument on such figures. I think it is putting an unwarranted interpretation on the figures a Select Table contains.* As to the general object of the paper—if one can find on further investigation that it solves the question of the proper relation between the paid-up policy and the surrender value, and at the same time gives suitable values for practical purposes, it will prove a very valuable contribution to the *Transactions*, the more so as Dr. Sprague goes so fully into detail and works out his methods so thoroughly.

Mr. M'Lauchlan.—This will, I am sure, be a most useful paper. One cannot help feeling that the question of surrender values is much more important than it used to be. Many intending assurers are careful to inquire before taking out a policy what their position will be in the event of their desiring to discontinue their payments, and the information supplied by the Office would no doubt be regarded as part of the contract. This being so,

* We understand that the author of the paper is giving consideration to this point.—Ed.

it is very desirable that the system of surrender values and paid-up policies should be a consistent one. I agree with Mr. Chatham as to the importance, in the case of surrender values or paid-up policies, of having the results of various methods of calculation set down and compared. In coming to a decision on questions such as those dealt with in this paper, it is important to look at them from different points of view, including, possibly, that of the man in the street. But the fact that Dr. Sprague's paper is a long and elaborate one may be sufficient reason for his omitting any comparison of his results with those obtained by other methods.

Mr. J. R. Hart, F.I.A.—Dr. Sprague has presented what appears to be a very complete and satisfactory system of dealing with surrenders and paid-up policies, based, as I understand it, upon the consideration that the formulæ should allow for adverse selection and expenses. The method is one that can be applied with great facility. I notice, however, that at the end of the paper the author states, "It is impossible to frame a mathematically accurate scheme . . . and we must therefore be largely guided by considerations of expediency and practice." Now it has always appeared to me that one of the most striking features of our business is the huge expenditure of energy and time to secure a proposal compared with the ease with which a policy is allowed to slip off the books. Would it not be possible to deflect some of the force employed in the shape of our agency organisations towards maintaining a policy instead of allowing the question of surrender to be dealt with almost entirely by the Actuary? There is indeed the renewal commission, which acts as an incentive to the agent to work for the up-keeping of the policy, but for the main part of our agency machinery new business is the all-absorbing question. Interest in a proposer is apt to cease as soon as his policy has been placed on the books. In my opinion it should be our endeavour to get into closer touch with policyholders contemplating surrender, with the view of ascertaining more accurately the circumstances under which the policy is being dropped. These circumstances are probably different in earlier and in later years. As regards "selection," I should say that in cases where paid-up policies are taken, the assured, as a whole, are probably in worse health than those who actually surrender, and a similar conclusion might reasonably be drawn in the case of policyholders taking loans. I may mention one small point in connection with policy loans. It may happen that the assured has borrowed on two or more policies and wishes to repay so much of the loan as will release one of them. In such a case it is advisable to get payment at the same time of the accrued interest on the principal repaid, or if not, to make sure that there is sufficient cover for it in event of the other policy lapsing. As regards guaranteed surrender values, I agree with Dr. Ernest Sprague that one-third or 40 per cent of the premiums is hardly likely to appeal to the public as an attractive feature of life assurance. From my experience of outside work, it is better to keep such a feature in the background, and when an intending assurer raises the question, to explain that after a policy has been a number of years in force, the surrender value will be a good deal more. A statement like that in Appendix III. might very usefully be put into our prospectuses. In conclusion, I may mention the relationship, if any, between the market value and the surrender value of a policy as a point on which it would have been interesting to have had Dr. Ernest Sprague's views.

Mr. J. A. Rankin.—Dr. Ernest Sprague's plan of finding the surrender value from a tabulated paid-up policy has the advantage of shortening work, and, as he shows, the rule can be applied with good results in all ordinary, as well as in many special cases. The effect of substituting P_+ for P_x in the formula to allow for initial expenditure is, as he points out, to allow no paid-up policy or surrender value for assurances of only one year's

duration. That it also has the advantage of making the charge decrease with the duration, can be shown as follows: Using net functions throughout, after n years the value of the deduction is of course $(\pi_{x+1} - \pi_x)a_{x+n}$ or (multiplying and dividing by a_{x+1}), ${}_1V_x\,(1 - {}_{n-1}V_{x+1})$, which is obviously a decreasing quantity. For a paid-up policy in the case of a whole-life assurance Dr. T. B. Sprague suggested the alternative formula $1 - \frac{P_x}{P_{x+n-1}}$ less 10 per cent. I calculated a few examples by this formula and found that, apart from the percentage deduction, the results only differed from about 4 to 1 per cent when compared with those given in the paper.

It is shown that the "without profit" premiums in Table A obtained from the O[NM] Table, with a loading of 10 per cent and 2s. per £100 of sum assured, represent fairly well the average "without profit" rates actually charged by Offices. I may point out as an interesting coincidence that if the "without profit" premiums so obtained be similarly loaded, the results, up to at least age 65, are remarkably near the average Office premiums "with profits." For example, column (1) below contains the values of $100(1{\cdot}1P_x + {\cdot}001)$, P_x being Dr. Ernest Sprague's "without profit" premium, and column (2) gives the average published Office rates with profits:

Age.	(1)	(2)	(1)−(2)
20	1·920	1·967	−·047
30	2·374	2·467	−·093
40	3·219	3·258	−·039
50	4·650	4·567	+·083
60	7·159	7·042	+·117
65	9·090	8·996	+·094

Now it is shown in the paper (p. 206) that the paid-up policies deduced from the "without profit" Office premiums (as loaded with a uniform percentage and constant) are less than those deduced from the corresponding net premiums. If, then, the "with profit" premiums could be obtained from the "without profit" ones similarly loaded, the same proof (apart from the ratio brought out in Tables F, G, and H) would hold for the relationship between paid-up policies based upon "with" and "without" profit premiums respectively, viz. that excluding existing bonuses the former are less than the latter.

In regard to double endowments, Dr. Ernest Sprague sees no objection to paying a surrender value in excess of the sum payable at death provided the assured be in good health. As the endowment portion is payable only on survivance, it would appear at first sight that the Office must lose in those cases where the assured dies before the date of maturity. The proviso as to medical examination, while lessening the loss, cannot entirely obviate it, as some of the lives, even though select at the time of surrender, will fail in the interval before maturity. This may be counterbalanced by the profit in respect of those who surrender and survive, but it is doubtful if the Office will gain on the whole by paying the excess surrender value in such cases, for it must be borne in mind that the self-selection by the assured is possibly more effectual than the selection exercised by the Office. I notice that for an ordinary pure endowment without return the O[M] 4½ per cent "ultimate" value is recommended, subject to proof of good health, while in the case of a double endowment the value of the excess portion R is obtained by using an O[M] "select" basis. I take it, however, that the "select" value is conditional on the whole double endowment being surrendered at once, and that if the pure endowment portion were not surrendered till afterwards, the "ultimate" value would apply.

Dr. Sprague would be inclined to allow interim bonus in cases of surrender.

If, however, a fair surrender value for the policy and existing bonuses is given, it is quite conceivable that it might be better for the Office not to include interim additions. As a consequence of such allowance the Office's quinquennial rate of bonus would be slightly reduced. This might cause a greater number of policies to be surrendered, and it is possible that the offer of larger surrender values would then act in the same direction.

Mr. MAXWELL.—Dr. Sprague draws our attention (p. 237) to the facts that, in certain cases, his surrender values for limited-payment and endowment assurance policies exceed the 3 per cent reserve values, and that the surrender values of some long term endowment assurances are less than those proposed for whole-life policies effected at the same age. He characterises these as undoubted defects in his methods, but he does not propose to get rid of them. My view is that such defects as these are sufficient in themselves to condemn the structure on which they grow. It is obvious that either the surrender values of the whole-life policies are too large, or those of the endowment assurances are too small. Taking for example a whole-life policy effected at age 30, the paid-up policy which Dr. Spague would allow on attainment of age 70 is no less than 97½ per cent of the same function calculated on the O^M 3 per cent basis. If we look on the difference as profit, the Office may thus be said to benefit by only 2½ per cent on the cancellation of all future premiums, and if it is a proprietary Company the shareholders would derive a mere tithe of this. Such treatment (of which this is by no means an isolated instance) appears to me to be too generous, and a much larger deduction should, in my opinion, be made from the reserve held by the Office. If such deduction took in practice the form of a percentage of the reserve, it is not necessary that it should be a fixed percentage, as mentioned by the author; it might be a varying proportion according (say) to the age attained.

A more detailed comparison of the values in the various tables shows the inconsistency between the treatment of the whole-life and the endowment assurances to be spread over a much wider area, as will be seen from the following specimen percentage ratios of the values proposed to those calculated at O^M 3 per cent and quoted in the paper—evidently for comparison :—

x	n	Paid-up Policy, per cent of O^M 3%.		Difference in favour of Whole-Life.	Surrender Value, per cent of O^M 3%.		Difference in favour of Whole-Life.
		Life	E. A. 40 yrs.				
20	20	88	84	4	62	65	−3
		Life	E. A. 50 yrs.				
20	10	77	68	9	51	47	4
,,	20	88	77	11	62	55	7
,,	30	93	85	8	70	67	3
,,	40	96	93	3	77	81	−4
		Life	E. A. 30 yrs.				
40	15	94	90	4	73	75	−2
		Life	E. A. 40 yrs.				
40	20	95	84	11	77	68	9

The relative injustice done to the general body of endowment assurance policyholders is clearly indicated in the above examples, which do not seem to call for further comment.

Mr. M'Intosh.—In considering the suggestion that surrender values ought to be based on reserve values, the reasons for adopting any particular basis in estimating the Office liability should be borne in mind. An Office might, for instance, use assurance values and net premiums both loaded for bonus, in conjunction with a rate of interest near what is actually earned. But various writers have shown that much the same result can be obtained by valuing at a sufficiently low rate of interest, and using the ordinary assurance values and net premiums; and practically all offices adopt the latter plan. The Office, for the sake of uniformity, also employs the lower rate of interest in valuing "without profit" policies, and thereby makes a specially strong reserve for them. If, therefore, it be thought desirable to calculate surrender values by means of tables of reserve values, it would be quite proper in the case of "without profit" policies to use a higher rate of interest than the valuation rate—in fact, 4 or 4½ per cent, as indicated in the paper. This is apart from any deductions which may be made for other circumstances. The cost of putting a policy on the books and covering the first year's risk leaves little over to provide any paid-up policy in respect of the first year's premium. In view of this, it should be noted that, by taking a proportionate paid-up policy and assuring afresh for the balance, a policyholder may often be able to obtain assurance for the full original amount at a lower premium than he has hitherto been paying. The possibility of this could be shown from the published tables of various Offices. The proportionate paid-up policy for the early years may be looked upon as a popular concession, but when the surrender value is required, it would appear to be unnecessary to give the same advantage. An Office might easily fix its scale of surrender values for the early years so as to allow of its being recouped for the heavy initial expense, without much risk of being considered inconsistent. Again, in the case of an endowment assurance nearing maturity where the surrender value increases each year by more than the premium, it may possibly be found that under the proposed scale of surrender values it would be more advantageous for a person intending to surrender, to borrow the full loan value of his policy. He will thus get, say, 95 per cent of the surrender value at once, be able to borrow future premiums and interest as they fall due, and, in addition to having the advantage of assurance for the remainder of the term, ultimately have a small cash surplus in his favour. Thus, on the assumption that the option would be exercised against the Office, there would be little profit from surrenders of old policies to compensate the Office for any liberality in the early years.

Mr. Evans.—As one not now officially connected with life assurance I should like to say that I am glad to see there is a growing tendency in favour of allowing a surrender value as nearly as possible equal to the reserve value. Having talked a good deal with policyholders on the subject, I cannot help thinking that liberality in the matter of surrender values would materially increase the popularity of life assurance. Twenty years ago I favoured the allowance, in surrenders, of the values brought out by a "select" mortality table at a higher rate of interest than that used in calculating the reserves. This assumed, of course, that there were good grounds for the view that withdrawing policyholders were, on the whole, better lives than those who remained on the books. But even at that time I had doubts as to the soundness of that view, and I am interested to hear from Mr. Chatham that there is some evidence against it.

Mr. Finlay J. Cameron wrote—The paper is a brilliant example of

that combination of theoretical knowledge with practical application which is so necessary in actuarial work. I feel sure that Dr. Ernest Sprague's descriptions of his suggested methods of calculating paid-up policies and surrender values will be of great service, not only to students, but to all engaged in the duties of an actuarial department. With regard, however, to his general scheme of calculating the paid-up policy in the first instance, and then deriving the surrender value from it, there is room for difference of opinion. Such a plan is, I think, applicable rather to the case of policies with premiums limited to a term of years than to the case of policies with premiums payable during life. In the former the paid-up policy is usually "proportionate," and it is therefore desirable as well as convenient to obtain the surrender value from the paid-up policy. There are, however, comparatively few British Offices which guarantee fixed paid-up policies in the case of whole-life assurances with premiums payable during life, and I think that the better plan in this class is to derive the paid-up policy and the surrender value independently, taking care that the two bases of computation are as consistent as possible. For the surrender value of an ordinary whole-life policy a convenient and equitable plan is to use the formula ${}_{n-1}V_{x+1}$ (where x=age at entry, and n=duration). In this way allowance is made for the heavy initial expenditure. A "select" table such as the $O^{[M]}$ should, I think, be used when practicable. If an "aggregate" table such as the O^{M} be used, a small deduction might be made from the value on account of selection. With regard to the rate of interest, it is, I think, equitable to value at 4½ per cent (a good investment rate) in the case of non-participating policies, and in the case of participating policies to reduce the rate according to the bonus expected. I should like to mention that I have found the system advocated by Dr. Ernest Sprague, of computing surrender values on cards, both convenient and time-saving.

The PRESIDENT, in closing the discussion, said—I am glad Dr. Ernest Sprague has stated that the practice of Offices, based upon sound actuarial principles, is such that it commends itself to the "common-sense" of the assuring public. With reference to the brief explanatory leaflet about surrender values, I happen to know that where it is used it is having the desired effect—in "putting on the closure" to much unnecessary correspondence; and, as it is not "copyright," all Offices are free to make use of it. I am one of those who believe that the better informed the public are in all matters relating to Life Assurance the better it will be for the public—and for the Life Offices. Dr. Ernest Sprague's methods, while theoretically sound, are eminently practical and easily worked out in the actuarial department, not only in respect to time and labour, but, more important still, in respect to accuracy—the risk of error being greatly reduced. With regard to the relation which should subsist between the Office surrender value and the actuarial reserve for an individual policy we have, I think, an analogy in the redemption of a rent-charge, *i.e.* the buying back of a self-redeeming annuity granted in repayment—principal and interest—of a loan. When the borrower desires to bring the transaction to a close before its natural termination, he is not at liberty to do so by merely sending a cheque for the outstanding balance as per table. The Office in granting the loan practically *bought* the annuity on, say, a 4 per cent basis—now it is asked to *sell* it or what remains of it; and this it might possibly do on, say, a 3 or 3½ per cent basis, the difference being compensation for giving up, at the call of the borrower, a good 4 per cent investment. I agree with Dr. Sprague that, in these days, it is useless to talk of the Office having a contract with the policyholder to continue the payment of premiums, and that on surrender he seeks to depart from his part of the

contract. At the same time, as it is he who seeks to disturb matters, whether it suits the Office or not, it is only fair that the Office should consider the terms on which he may be permitted to withdraw, and, in doing so, allow for the labour and cost of placing his business on the books and of finding a new "member" to take his place.

With these considerations it is obvious that the full actuarial reserve ought not to be allowed on surrender. While I am sure Dr. Ernest Sprague would be the last to claim to have solved all the difficulties involved in this interesting and thoroughly practical question, I think we shall all admit that he has made a valuable contribution towards their solution. The time is undoubtedly come when in this world-wide business of Life Assurance we actuaries, educating the public, should bring our knowledge and practice into line with popular requirements—without, of course, departing from sound principles. I have very great pleasure in proposing that we give Dr. Ernest Sprague a most hearty vote of thanks for his extremely interesting and useful contribution to our *Transactions*: it is a paper of permanent value which we, and many others, will do well to refer to and carefully study at our leisure.

Dr. Sprague's Reply.

The principles advocated in the paper are for the most part neither new nor original; and the paper is therefore to be regarded not as an original investigation, but rather as an attempt to evolve a simple and consistent practical scheme out of a somewhat chaotic mass of different formulas invented by different people at different times. The formula for a paid-up policy, $1-\frac{P_x}{P_{x+n}}$, was invented by Dr. T. B. Sprague (*J. I. A.*, vii, 59), and first applied by him, with modifications, to the calculation of surrender values (*J. I. A.*, xxiv, 365); and he was the first to advocate the use of the "select" policy value ${}_n(hV)_x$ in their calculation (*J. I. A.*, xxii, 417 *et seq.*). The principle of getting surrender values direct from the "proportionate" paid-up policies in limited-payment and endowment assurance cases, was adopted by me early in 1901, and for a long time I thought it was original; but I have heard recently that Mr. Chatham had adopted it some years previously, though it was not published till his paper of last session. To him, therefore, is due the credit of being the first to devise the method and to put it into practical operation. The fact that the value of a benefit carrying the right to bonus at a fixed rate is equal to the value of a uniform benefit at a certain lower rate of interest, must have been known to many actuaries, and the point has been touched upon by Dr. T. B. Sprague and Mr. G. F. Hardy; but I do not know that the demonstration has been published hitherto.*

In reply to those gentlemen who wish to compare the values now proposed, with those given by other methods, I have to point out that elaborate tables of values by different methods are given in the papers to which I have referred by Mr. Crisford, Mr. MacFadyen, Mr. Fulford, and Mr. Chatham; and I do not think that much would have been gained by repeating the tables there given, for any one who wishes to study the subject will, naturally, not be content to accept the views of one writer without ascertaining those of the others who have worked at the subject, and comparing the different conclusions. The references to former papers are in-

* A demonstration similar to that on p. 238 was given by Mr. H. W. Brown in his paper (1899) on the Valuation of Special Class Policies (*Transactions of the Actuarial Society of Edinburgh*, Vol. iv. No. 12). A similar demonstration will also be found in an unsigned review (1904), *J. I. A.*, xxxviii. 375.—Ed.

serted with the object of facilitating such reference and comparison, and thus avoiding a great deal of unnecessary repetition.

With regard to the rate of mortality among lives who surrender their policies, I find myself unable to agree with some of Mr. Chatham's conclusions. He holds (I understand) that because 840 withdrawals out of 1085 were due to impecuniosity, therefore no option is exercised in the matter of withdrawal. This conclusion seems to me to be logically unsound, firstly because the 245 surrenders due to other causes might be sufficient to cause an adverse selection, and secondly for the following reason: Out of a number of impecunious persons, say 100, it is possible that 99, say, might be select lives and surrender their policies, and that the remaining person might be in very bad health and either keep his policy in force by means of a loan, or sell it to a speculative purchaser for more than the surrender value. In these circumstances every surrender would be due to impecuniosity, and yet the option would be exercised to the maximum extent possible against the Office. Therefore, even if it were proved that every surrender was due to impecuniosity, it would not necessarily follow that no option is exercised, because the damaged lives might have exercised the option by continuing their policies or selling them to a third person who would do so. Mr. Chatham states that Mr. Besant's investigation showed that the abovementioned 840 withdrawals were likely to be worse lives than those who remained, but from the report of Mr. Besant's remarks it would appear that this was a mere opinion, and no facts are cited in support of it. It seems to me that if a man in bad health is reluctant to surrender his bonus, he will be still more reluctant to surrender his whole policy, and that for this reason Mr. G. F. Hardy's figures are evidence (though not conclusive proof) of a selection adverse to the Office. On the other hand, Mr. Chatham's figures, taken from the Scottish and British Experiences respectively, seem to me to be inconclusive, because it is possible that the rates of mortality in the two sections (apart from withdrawals) might be different. The fact that the rates of withdrawal are different indicates some difference in the circumstances of the lives in the two sections, and in view of this I do not see what right we have to assume that the rates of mortality would be the same were it not for the influence of the withdrawals. The evidence, therefore, seems to me conflicting, and until conclusive evidence is produced, I think it is only prudent to give the Office the benefit of the doubt; but this is one of the controversial points on which we cannot expect at this stage to secure agreement. The rate of mortality among withdrawing lives is, however, a matter of fact, not of opinion, and it should therefore be capable of proof, one way or the other. In the transactions of the recent Actuarial Congress there is a reference to an investigation by Dr. Fredholm into the mortality among the withdrawals from the "Skandia," which is stated to have shown that the existence of an adverse selection is improbable; but I have not yet seen Dr. Fredholm's report. A larger investigation is being made by a number of Scandinavian Companies into the same question, but I do not think that this has been completed yet, and it seems to me that the only way of settling the question would be to make a similar investigation among British Companies—provided it were found possible to trace the withdrawals with sufficient accuracy.

Assuming the existence of an adverse selection when bonuses are surrendered for cash, it follows that the use of an aggregate table for finding the cash values is wrong in principle, and would tend to give too small values at young ages and too large values later in life. Therefore it seems to me that the use of the O^M table as advocated by Mr. Chatham leads to this difficulty, namely, that the cash values of bonus would tend in most cases to be too large; or if this tendency be corrected by using, say, select values

for bonuses, and aggregate values for paid-up policies, then in most cases a bonus would have a smaller value than a paid-up policy of the same amount, which would almost certainly cause complaints by policyholders, and be very difficult of explanation.

I quite agree with Mr. Chatham that the explanatory leaflet appended to the paper could be simplified, and my object in giving it was not that it might serve as a model, but that I hoped to receive suggestions for its improvement; and if any of you have similar leaflets I shall be grateful if you will favour me with copies. With regard to the expression "breach of contract," I may say that the prospectuses of all the 59 Companies mentioned in the paper, without exception, promise "liberal surrender values," or words to that effect, and in these circumstances I think it is clear that the right to surrender is part of the contract, and therefore that no deduction should be made on that ground. Whether a deduction should be made on account of disturbance of the Office's arrangements is quite a different question, and "breach of contract" is not the correct expression to use in this connection.

I do not follow Mr. Gunn's remarks about the rate of interest used in the calculations. In whole-life assurance the paid-up policy is derived from the Office premiums, and the rate of interest involved is, I suppose, usually about 3½ per cent, which is not widely different from the rate earned; and in limited-payment and endowment assurances the paid-up policy is arbitrary, and entirely independent of the rate of interest.

When the surrender value of a double endowment exceeds the sum payable at death, there is, as Mr. Rankin points out, a slight inconsistency between it and the surrender value proposed for a pure endowment, in that "select" functions are used throughout for the former, but an "ultimate" function for the latter. This had not escaped my notice; but I consider the proposed methods sufficiently good for practical purposes. The inconsistency could be removed by using a combination of "select" and "ultimate" functions in the former case, but I think that the disadvantage of this complication would outweigh any advantage gained thereby.

I would remind Mr. Maxwell that my reason for recommending a "proportionate" paid-up policy is the simplicity and popularity of the system, combined with the fact that, as a rule, it gives results which are sufficiently accurate for practical purposes. I do not claim that it is theoretically correct. Unless he is prepared to discard the system altogether and thus run counter to the general practice of most Offices, I think he will find that it is impossible to avoid some anomalies, and that it is difficult to confine them within smaller limits than I have done. It is, of course, a matter of opinion whether the advantages of the system outweigh its disadvantages, and therefore we cannot expect universal agreement regarding it; but I think there is no doubt that the discrepancies to which Mr. Maxwell draws attention are of very rare occurrence, and that in most cases which occur in practice the system works fairly well. In comparing the proposed values with the O^M 3 per cent reserves, it must be borne in mind that those reserves are based on three assumptions, namely: (1) that the rate of interest is 3 per cent; (2) that the future rates of mortality will be those shown by the aggregate O^M table; and (3) that the expenses are uniform throughout the duration of the policy and equal in amount to the difference between the Office and the O^M 3 per cent net premium. Now the rate of interest earned is nearer 4 per cent than 3 per cent; the rates of mortality are better represented by the $O^{(M)}$ and $O^{(MM)}$ (select) tables than by the O^M; and the expenses are certainly not uniform, but are very much heavier in the first than in subsequent years. That is to say, every assumption on which the O^M reserves are based is incorrect, and as the three

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Abstract of Discussion thereon

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Notes on the British Offices Life Annuity Tables (1893); *with special reference* (*a*) *to the persistence of Selection, in relation to the Graduated Tables;* (*b*) *to the computation of Joint Annuities, by special applications of the Law of Uniform Seniority.* *By* THOMAS G. ACKLAND, *Fellow, and one of the Vice-Presidents, of the Institute of Actuaries; Honorary Fellow of the Faculty of Actuaries.*

[Read before the Faculty 11 March 1907.]

IN response to the invitation of your President to prepare and submit a Paper to the Faculty during the present session, I thought that it would be appropriate, and I hope also of some interest to the Members, if I endeavoured to deal with a section of that important Investigation, recently completed, into the mortality of Assured Lives and Annuitants, in which the Faculty and the Institute so happily collaborated, to the great advantage of the actuarial profession.

I have selected, for the particular subject of these Notes, the British Offices Life Annuity Tables. I propose to advert, in the first place, to the indications and persistence of selection in the unadjusted data of that experience, and the consequent difficulties found in forming graduated Select Tables which should pass, after a convenient number of years, into an Ultimate Table; and to consider some of the causes of these difficulties. I then propose briefly to review the processes of graduation followed by Mr. G. F. Hardy, and in particular the endeavours made, under unfavourable conditions, to retain, to the fullest possible extent,

the advantages resulting from a graduation by Makeham's law. This will lead up to my main practical object in these Notes, namely, to consider the suggestions made by Mr. Hardy for the calculation of joint-annuity values by the employment of the law of uniform seniority, under the particular conditions and disabilities arising in the graduated Female Annuitant Table; and to compute and submit, for the consideration and use of the Profession, a specimen set of the Tables required, in order practically to illustrate and apply Mr. Hardy's suggestions, in deducing annuity values on two joint lives.

UNADJUSTED DATA: THE PERSISTENCE OF SELECTION, IN RELATION TO THE GRADUATED TABLES.

It will be remembered that Mr. Hardy found considerable difficulty in effecting a good and satisfactory junction between the Select mortality of the first five, or ten years, and the mortality of the Aggregate Table, after exclusion of the experience of those early years. The object of such a junction was, of course, to set out the graduated results in a form suited for practical use, by the employment of Select Tables, preferably for a term of five, and in any case for a term not exceeding ten, years, with an ultimate Table, representing at each age the combined experience of later durations.

In Tables I to IV of the Appendix to this Paper, I show the nature, and to some extent the causes, of this difficulty. Columns (1), (2), (4), (5), and (6), of these Tables are extracted from the two tables given by Mr. Hardy, in the brief statement of his method of graduation, on pp. 124 and 125 of the *Account of the Principles and Methods adopted in the Graduation of the Experience, etc.*, and show the difficulty in forming a junction between the Select Tables and the ultimate Aggregate Tables after five, or even ten, years. Columns (3), (7), and (8), of Tables I to IV of the Appendix are here included for the first time, and are intended to show how far a good junction could be made, if the ultimate Table were based on *Select*, instead of on *Aggregate* data.

Before considering as to the indications given by these Tables, it is necessary clearly to discriminate between the different forms of expectations of life set out in Tables I to IV, and also between the Select and Aggregate data upon which they are based. I have followed Mr. Hardy in considering comparisons of the expectations of life as far better and more conclusive than comparisons of the rates of mortality, which, on account of the paucity of the data when differentiated for particular entry ages and

durations, exhibit perplexing, and even violent, fluctuations. In column (4) of Tables I to IV are given, for the groups of ages indicated, the average values of $e_{[x-5]+5}$ and $e_{[x-10]+10}$, that is to say, the curtate expectation of life at age x, in respect of select data entering at age $[x-5]$, or $[x-10]$, observed after precisely five or ten years' duration. In column (2) are given the average expectations of life, deduced from the Aggregate Tables, after excluding the experience of the first five, or ten, years. These are the average values of the expectations, over the groups of ages indicated, given in the volume of *Unadjusted Data*, on pp. 188-9 and 190-1 for Male Lives, and on pp. 198-9 and 200-1 for Female Lives, after five years and ten years respectively, and may conveniently be represented by the symbols $e_x^{(5)}$ and $e_x^{(10)}$. (The expectations actually tabulated in the volume of *Unadjusted Data* being "complete," have been reduced by ·5 to make them "curtate" for purposes of comparison.) The values in column (2) of Tables I to IV thus represent, at age x, the average expectations, computed from mixed aggregate data of upwards of five, or ten, years' duration.

In column (3) I have inserted average expectations of life deduced from *Select* data, after eliminating the first five, or ten, years' experience. These expectations, which may be designated $e_x^{[5]}$ or $e_x^{[10]}$, are deduced from the experience set out for Male Lives on pp. 1 to 74, and for Female Lives on pp. 76 to 166, of the volume of *Unadjusted Data*. The data, there set out in respect of particular entry ages and durations, have here been brought together at ages attained, but with exclusion of the experience of the first five (or ten) years from entry, so that

$$E_x^{[5]} = E_{[x-5]+5} + E_{[x-5]+6} + \ldots + E_{[1]+x-1} + E_{[0]+x}.$$

$$\theta_x^{[5]} = \theta_{[x-5]+5} + \theta_{[x-5]+6} + \ldots + \theta_{[1]+x-1} + \theta_{[0]+x}.$$

$$q_x^{[5]} = \frac{\theta_x^{[5]}}{E_x^{[5]}},$$

$$\text{and } e_x^{[5]} = p_x^{[5]} + {}_2p_x^{[5]} + {}_3p_x^{[5]} + \ldots + {}_{\omega-x-1}p_x^{[5]}.$$

x having, throughout, values of 5 and upwards. The values of $e_x^{[5]}$ are thus derived from mixed Select data of upwards of five years' duration. The resulting numbers exposed to risk, deaths, rates of mortality, and curtate expectations of life, are set out at ages attained in columns (6) to (9) of Tables V to VIII for Male and Female Annuitants, excluding the first five, and also the first ten, years' experience. In columns (2) to (5) of these Tables are set out the numbers exposed to risk, deaths, rates of mortality, and curtate expectations of life at ages attained, as deduced from the Aggregate

Data, excluding the experience of the first five, and of the first ten, years; these latter figures being extracted from the Tables given for Male Lives on pp. 188-91, and for Female Lives on pp. 198-201, of the volume of *Unadjusted Data.*

The distinction between the Aggregate and Select data is, as will be remembered, in respect of the different methods of eliminating duplicates adopted in the two sets of data. In the Select data, duplicates were only eliminated, when arising at the same entry age; so that, for example, an annuitant effecting two contracts at age 20, a third at age 40, and a fourth at age 60 at entry, would be included in the Select data once at each of the ages at entry, 20, 40, and 60. In the Aggregate data, however, duplicates were eliminated at all ages attained or passed through; so that, in the above example, only the first contract, effected at age 20, and continuing until death, or until the termination of the period of observation, would be included in the Aggregate data. The effect is that the cases of *re-selection* were included in the Select data, but not in the Aggregate data; and it will be seen, on comparing the rates of mortality in columns (4) and (8), and more conclusively on comparing the expectations of life in columns (5) and (9), of Tables V to VIII, that the mortality is materially lighter, at almost all ages, by the Select data, than by the Aggregate data. The Select expectations are, in fact, greater than the Aggregate at all ages, excepting only ages 80-85 for Male Lives, and ages 93-100 for Female Lives. It follows that the inclusion in the Select data of the cases of re-selection has the effect of somewhat materially increasing the vitality of the lives observed.

Reverting now to Tables I to IV of the Appendix, it will be seen that the average values[1] of the select expectations, computed after precisely five (or ten) years from entry, $e_{[x-5]+5}$ (or $e_{[x-10]+10}$) are, at all groups of ages between 55 and 89 (excepting only at ages 55-64 in the ultimate Female Table excluding ten years), persistently greater than the average expectations deduced from Aggregate data at the same groups of ages, in respect of

[1] It should perhaps be noted that, in following the plan adopted by Mr. Hardy of combining the values of $e_{[x-5]+5}$ and $e_{[x-10]+10}$ for ten consecutive values of x, and taking the average of such ten values, the vitality of the lives is probably somewhat overstated (as compared with expectations taken at an individual age) by the inclusion of such cases of re-selection as fell within the period of ten years. Thus the successive values of $e_{[x-5]+5}$, in the combined group $x=55$ to 64, include all entry ages 50 to 59 inclusive, and if annuity contracts were effected on the same life (for instance) at entry ages 50, 53, and 56, the value of the average expectation deduced from the group would be enhanced, by any superior vitality included in respect of such cases of re-selection.

durations of upwards of five years, $e_x^{(5)}$ (or $e_x^{(10)}$). The effect is, as pointed out by Mr. Hardy, that if a junction were made with an ultimate aggregate Table after five, or even after ten, years, the graduated expectations at those points, and also those deduced therefrom as at entry, would understate the true expectations, with a corresponding understatement of the annuity values, as at five, or ten, years after entry, and as at entry. This would be a serious defect in a Table intended mainly to be employed in dealings with and valuations of annuity business; and Mr. Hardy, in view of this practical difficulty, resorted to the ingenious device of a junction with a *hypothetical* mortality Table, so constructed as closely to reproduce the expectations five years after entry, $e_{[x-5]+5}$, and which, when combined with the graduated select mortality table for the first five years, should also closely reproduce the expectations of life as at entry. That he attained these objects with conspicuous success is sufficiently shown by the Tables on pp. 144-5 of the *Account of Principles and Methods, etc.*

It is probable that an alternative course might have been followed, and that if the select data given in columns (6) to (9) of Tables V to VIII had been available when the graduation was under consideration, an ultimate Table, deduced from such data, with exclusion of five, or ten, years, would have been found to give a satisfactory junction with the select mortality rates of the earlier years. This will, I think, be demonstrated on reference to columns (3), (7), and (8) of Tables I to IV, in which the average values of the select expectations after precisely five (or ten) years, $e_{[x-5]+5}$ or $e_{[x-10]+10}$, are compared with the average expectations deduced from mixed Select data, excluding the first five years' experience, $e_x^{[5]}$, or $e_x^{[10]}$. It will be seen that the differences are much reduced in magnitude, as compared with those given in columns (5) and (6); and that, instead of being persistently in excess, they now exhibit several fluctuations as to sign; and it is on the whole evident that a good junction could be made with the select ultimate Table after five years, and a practically perfect one after ten years, from entry. In fact, the substitution of the select ultimate Table for the aggregate ultimate Table, appears precisely to meet the practical necessities of the graduation. The persistence of selection after five years introduces an awkward breach of continuity between the experience at that point of duration, and the experience shown in the aggregate ultimate Table, with its lower average vitality, caused by the exclusion from that Table of all cases of re-selection; whilst, on the other hand, the continuity is practically

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or fractional; the logarithm of the combined probability is then equal to

$$-r(a^{(a)}+a^{(b)})-c^{x+t}\frac{c^r-1}{c-1}\beta^{(a)}(1+c^{h+n})-(f^{(a)}_{t+r}-f^{(a)}_t)-(f^{(b)}_{t+r}-f^{(b)}_t)$$
$$-c^x(\psi_{t+r}-\psi_t)\beta^{(a)}(1+c^{h+n}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

There are now two different methods by which probabilities and annuities on two joint lives of substituted ages can be employed: (i) by the substitution of two lives of equal age; (ii) by the substitution of two lives with a constant difference of n years between the ages (the value of n being determined by the relation of $\beta^{(a)}$ to $\beta^{(b)}$ as above). It will be desirable to demonstrate separately each of these two methods.

(i) The logarithm of the joint probability, in respect of two lives of equal age $[x+s]+t$, is equal to

$$\log {}_rp^{(a)}_{[x+s]+t}{}^{(b)}_{[x+s]+t}=-r(a^{(a)}+a^{(b)})-c^{x+t}\frac{c^r-1}{c-1}\beta^{(a)}(c^s+c^{s+n})-(f^{(a)}_{t+r}-f^{(a)}_t)$$
$$-(f^{(b)}_{t+r}-f^{(b)}_t)-c^x(\psi_{t+r}-\psi_t)\beta^{(a)}(c^s+c^{s+n}) \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and this is identical with formula (3) above, provided that

$$c^s+c^{s+n}=1+c^{h+n}$$

that is, if
$$c^s=\frac{1+c^{h+n}}{1+c^n}$$

whence
$$s=\frac{\log\frac{1+c^{h+n}}{1+c^n}}{\log c} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The above joint probabilities being equal for all values of r, it follows that the joint annuities are also equal at all rates of interest, so that

$$a^{(a)}_{[x]+t}{}^{(b)}_{[x+h]+t}=a^{(a)}_{[x+s]+t}{}^{(b)}_{[x+s]+t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where
$$s=\frac{\log\frac{1+c^{h+n}}{1+c^n}}{\log c}.$$

(ii) The logarithm of the joint probability in respect of two lives of substituted ages $[x+s]+t$, and $[x+s-n]+t$ (where the difference between the ages is n years) is equal to

$$\log {}_rp^{(a)}_{[x+s]+t}{}^{(b)}_{[x+s-n]+t}=-r(a^{(a)}+a^{(b)})-c^{x+t}\frac{c^r-1}{c-1}\beta^{(a)}(c^s+c^s)-(f^{(a)}_{t+r}-f^{(a)}_t)$$
$$-(f^{(b)}_{t+r}-f^{(b)}_t)-c^x(\psi_{t+r}-\psi_t)\beta^{(a)}(c^s+c^s) \quad . \quad . \quad . \quad . \quad . \quad (4a)$$

and this expression is identical with formula (3) above, provided that

$$2c^s=1+c^{h+n}$$

that is, if
$$c^s=\frac{1+c^{h+n}}{2}$$

whence
$$s = \frac{\log \frac{1+c^{h+n}}{2}}{\log c} \quad . \quad . \quad . \quad . \quad . \quad (5a)$$

Thus, by this formula,
$$a^{(a)}_{[x]+t:[x+h]+t} = a^{(a)}_{[x+s]+t:[x+s-n]+t} \quad . \quad . \quad . \quad . \quad . \quad (6a)$$
at all rates of interest, the value of s being deduced by formula (5a).

The above formulas are throughout true for all values of t, and are therefore applicable to select, or ultimate, probabilities or annuities. For ultimate annuities, writing (x) for $[x]+t$ and $(x+h)$ for $[x+h]+t$ in formulas (6) and (6a) above, we have
$$a^{(a)\ (b)}_{x\,.\,x+h} = a^{(a)\ (b)}_{x+s.x+s}$$
and $$a^{(a)\ (b)}_{x\,.\,x+h} = a^{(a)\ (b)}_{x+s.x+s-n}$$
respectively.

We have thus the choice between two methods, one of which involves the quantity n (arising from the relation of $\beta^{(a)}$ to $\beta^{(b)}$ in the two mortality Tables) in the construction of the uniform seniority Table, but deduces joint annuity values on lives of equal age; whilst the other method proceeds on the basis of equal substituted ages, in the construction of the uniform seniority Table, but with a difference of n years between the substituted ages upon which the joint annuity values are calculated.

The method (i) requires the calculation of a distinct Table of uniform seniority by formula (5) for each value of n involved, that is, for each combination of mortality Tables; but proceeds by the usual and convenient course of computing joint annuities on substituted lives of equal age. I have adopted this method in preference to the other, partly because it is clearly that suggested by Mr. Hardy (see p. 137, *Account of Principles and Methods, etc.*), and partly because the laborious calculations involved in the computation of select joint annuities on two lives of equal age were already well in hand when these alternative methods came up for consideration.

The method (ii) has the advantage that a single Table of uniform seniority, computed by the ordinary formula
$$s = \frac{\log \frac{1+c^{h'}}{2}}{\log c}$$
(where the ordinary limits of the values of $h' = (h+n)$ are extended, according to the value of n in the two particular mortality Tables taken in combination); but it has the slight inconvenience of involving the calculation of joint annuities on two substituted lives, with a constant difference of n years between the ages.

It should be added that, in lieu of constructing a special Table,

or tables, of uniform seniority, the alternative course may be adopted of deducing the ages of the substituted lives from a Table of the force of mortality; and it will now be shown that the value of μ may be taken from *either* of the mortality Tables which are to be combined for joint annuities, or indeed from any single Table whatever, graduated by Makeham's first modification of Gompertz's law, with a value of c identical with that adopted in the graduation of the Tables which are to be combined. Thus, where it is desired to compute the value of a joint annuity on two lives $[x]+t$ and $[x+h]+t$ by Tables (a) and (b) respectively (in which $\beta^{(b)}=\beta^{(a)}c^n$) the values of $\mu_{[x]+t}$ and of $\mu_{[x+h+n]+t}$ can be taken from either of the Tables (a) or (b), or from any other Table conditioned as above, and their sum equated either (i) to $\mu_{[x+s]+t}+\mu_{[x+s+n]+t}$ or (ii) to $2\mu_{[x+s]+t}$.

We have

$$\mu_{[x]+t}=A_t+B_tc^{x+t}$$

and

$$\mu_{[x+h+n]+t}=A_t+B_tc^{x+h+n+t}$$

where A_t and B_t are functions of t only. Summing these expressions, we have

$$\mu_{[x]+t\cdot[x+h+n]+t}=2A_t+B_tc^{x+t}(1+c^{h+n}) \quad . \quad . \quad . \quad . \quad . \quad (7)$$

But (i), substituting ages $[x+s]+t$ and $[x+s+n]+t$,

$$\mu_{[x+s]+t\cdot[x+s+n]+t}=2A_t+B_tc^{x+t}(c^s+c^{s+n}) \quad . \quad . \quad . \quad . \quad (8)$$

which is identical with (7) if

$$c^s=\frac{1+c^{h+n}}{1+c^n}$$

whatever values are assigned to A_t and B_t.

Also (ii), substituting two lives of equal age $[x+s]+t$,

$$\mu_{[x+s]+t\cdot[x+s]+t}=2A_t+B_tc^{x+t}(c^s+c^s) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

which is identical with (7) if

$$c^s=\frac{1+c^{h+n}}{2}$$

Method (ii) lends itself readily to the deduction of substituted equal ages by the use of a Table of $\mu_{[x]+t}$; but by method (i), it would be somewhat troublesome in practice to find the two values of μ (with ages differing by n years) the sum of which was identical with the sum of the two values of μ taken out in respect of the true ages of the lives; and in this case a Table of uniform seniority would be more useful.

APPLICATION OF ABOVE PRINCIPLES AND FORMULAS TO BRITISH OFFICES ANNUITY EXPERIENCE.

Mr. Hardy's method involves the calculation of joint annuities for two lives upon all combinations (in pairs) of the male annuity

Table, and the first, and second, series of the female annuity Table. In the construction of these three Tables, the following relations of the constant β hold good, for all values of t:—

$$\beta_t^{(m)} = \beta_t^{(f_1)} c = \beta_t^{(f_2)} c^{-24} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$\beta_t^{(f_1)} = \beta_t^{(m)} c^{-1} = \beta_t^{(f_2)} c^{-25} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$\beta_t^{(f_2)} = \beta_t^{(m)} c^{24} = \beta_t^{(f_1)} c^{25} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

identical values of the constant c and of the function ψ_t having been adopted in the graduation of the three Tables. If it be then desired to substitute, for an annuity on two lives $[x]+t$ and $[x+h]+t$, having mortality respectively according to the Tables entering into the several combinations specified in the scheme set out below, an annuity on two lives of equal age $[x+s]+t$, according to the same combination of mortality, the equivalent equal ages will be arrived at by the employment of formula (5), with values of n, as shown in the formulas of relation (10) to (12) above:—

Case No.	Mortality of Younger Life according to	Mortality of Older Life according to	Value of n	Value of s	Formula No.
(1)	Male Table	Female (1st Series)	-1	$\dfrac{\log \dfrac{1+c^{h-1}}{1+c^{-1}}}{\log c}$	(13)
(2)	Female (1st Series)	Male Table	1	$\dfrac{\log \dfrac{1+c^{h+1}}{1+c}}{\log c}$	(14)
(3)	Male Table	Female (2nd Series)	24	$\dfrac{\log \dfrac{1+c^{h+24}}{1+c^{24}}}{\log c}$	(15)
(4)	Female (2nd Series)	Male Table	-24	$\dfrac{\log \dfrac{1+c^{h-24}}{1+c^{-24}}}{\log c}$	(16)
(5)	Female (1st Series)	Female (2nd Series)	25	$\dfrac{\log \dfrac{1+c^{h+25}}{1+c^{25}}}{\log c}$	(17)
(6)	Female (2nd Series)	Female (1st Series)	-25	$\dfrac{\log \dfrac{1+c^{h-25}}{1+c^{-25}}}{\log c}$	(18)
(7)	(a) Male Table (b) Female (1st Series) (c) Female (2nd Series)	Male Table Female (1st Series) Female (2nd Series)	0	$\dfrac{\log \dfrac{1+c^{h}}{2}}{\log c}$	(19)

I have computed, for all required values of h, the seven distinct

Tables of Uniform Seniority given by formulas (13) to (19) above. These are given in Table XIV of the Appendix, and are applicable at all rates of interest, and in the calculation of joint annuities on two lives, at all identical durations from entry.

Tables of select and ultimate annuity values on two lives of equal age have also been computed, at 3 and 4 per cent., according to each of the above combinations of mortality, involving one or more female lives. The combinations (1), (3), (5) being complementary to the combinations (2), (4), (6), are, of course, respectively identical, where annuity values on equal ages are tabulated; a single Table of annuity values therefore serves for each of these two combinations. The whole of the annuity values required for the different combinations including one or two female lives, are given in Tables XV to XVIII of the Appendix.

For the purpose of computing these Tables, it was necessary to calculate the fundamental Tables of the First and Second Series, which constitute, when combined, the graduated Female Annuity Table. The logarithms of the numbers living were worked out for each series by the formula

$$\log l_{[x]+t} = \log k_t + (x+t)\log s + c^{x+t}\log g_t$$

for all values of $[x]$, and all values of t from 0 to 5 inclusive; the appropriate values of the constants for the First Series, and the Second Series, for the above values of t, being taken from p. 43 of the volume of *Tables deduced from the Graduated Experience of Life Annuitants.* The resulting values of $\log l_{[x]+t}$ are given to six places for the First Series in Table XII, and for the Second Series in Table XIII of the Appendix. The natural numbers of these logarithms, when summed in respect of particular values of $[x]$ and t, give the numbers living according to the graduated $O^{[a/]}$ Table. In a few cases, a difference of ± 1 was found in the resulting numbers living, as compared with the graduated Table; and at entry ages 20 to 24 there were somewhat greater differences (never, however, exceeding 15 in a radix of 100,000), arising from arbitrary alterations made by Mr. Hardy, in order to preserve the progression of the values of $p_{[x]+t}$ at these early entry ages.

The logarithms of the numbers living given in Tables XII and XIII, in respect of the First, and of the Second, Series of the Female Table, were then combined with one another, and with the logarithms of the numbers living in the Male Table (given on pp. 4, 5 of the volume of *Tables deduced, etc.*), in order to form the required joint mortality tables, from which the joint annuities for the different combinations were deduced. The

logarithms of the numbers living for the Male Table being given to five figures only in the published volume, the combinations into which the Male Table entered were necessarily cut down to five-figure logarithms. The ultimate annuity values for each combination were first computed, by the usual method of D and N columns, and the ultimate annuity value a_{x+5} at each age was then extended back through the successive select values $a_{[x]+4}$, $a_{[x]+3}$, $a_{[x]+2}$, $a_{[x]+1}$, and $a_{[x]}$; a somewhat long and troublesome operation, occupying at least sixteen hours for each combined mortality table, at a single rate of interest. In order to economise space, the select values, $a_{[x]}$ and the ultimate values, a_{x+5}, for each combination, are alone tabulated for all values of x, at 3 and 4 per cent. interest. I had proposed to include also select and ultimate annuity values at 5 per cent., but time did not permit of their computation; should there be any desire for the inclusion of annuity values at other rates of interest they can be computed and published as an Appendix to the present Paper.[1]

FORMULAS FOR DETERMINING JOINT ANNUITY VALUES ACCORDING TO THE BRITISH OFFICES ANNUITY TABLES, ON TWO FEMALE LIVES, OR A MALE AND FEMALE LIFE, IN TERMS OF ANNUITY VALUES BASED ON THE MALE TABLE AND THE FIRST, OR SECOND, SERIES OF THE FEMALE TABLE.

I now proceed to set out the very simple demonstration required to prove the accuracy of the relation shown by Mr. Hardy in formulas (14) and (15) on p. 137 of the *Account of Principles and Methods, etc.*, as subsisting between the desired joint annuity according to the British Offices Annuity Tables, on a male and a female life, or on two female lives, and the annuity values deduced from combinations between the Male Table and the First and Second Series of the Female Table.

Taking first the case of a joint annuity on a male life (x) and a female life (y) we have

$$a_{x\,y}^{(m)(f)} = \frac{1}{l_x^{(m)} l_y^{(f)}} \left[v l_{x+1}^{(m)} (l_{y+1}^{(1)} + l_{y+1}^{(2)}) + v^2 l_{x+2}^{(m)} (l_{y+2}^{(1)} + l_{y+2}^{(2)}) + \quad . \quad . \quad . \quad \right]$$

$$= \frac{1}{l_x^{(m)} l_y^{(f)}} \left(l_x^{(m)} l_y^{(1)} a_{x\,.\,y}^{(m)\,(1)} + l_x^{(m)} l_y^{(2)} a_{x\,y}^{(m)(2)} \right)$$

where the bracketed symbols (m), (f), (1), and (2) distinguish the

[1] The Select and Ultimate annuity values on two joint lives of equal age have since been computed at 5 per cent. interest, and are given in Tables XIX and XX of the Appendix.

Male and Female Tables, and the First and Second Series of the Female Table. The above expression reduces to

$$\frac{l_y^{(1)}}{l_y^{(f)}}a_{x\ y}^{(m)(1)}+\frac{l_y^{(2)}}{l_y^{(f)}}a_{x\ y}^{(m)(2)}.$$

If now $\quad r_y=\frac{l_y^{(2)}}{l_y^{(1)}}, \quad \frac{1}{1+r_y}=\frac{l_y^{(1)}}{l_y^{(f)}}, \quad$ and $\frac{r_y}{1+r_y}=\frac{l_y^{(2)}}{l_y^{(f)}}$

we have finally $\quad a_{x\ y}^{(m)(f)}=\frac{a_{x\ y}^{(m)(1)}+r_y a_{x\ y}^{(m)(2)}}{1+r_y} \quad . \quad . \quad . \quad . \quad . \quad . \quad (20).$

which is Mr. Hardy's formula (14), referred to above.

If it is desired to avoid the double processes of multiplication and division, we may alternatively put

$$\rho_y=\frac{l_y^{(1)}}{l_y^{(f)}}, \quad (1-\rho_y)=\frac{l_y^{(2)}}{l_y^{(f)}}$$

and we then have

$$a_{x\ y}^{(m)(f)}=\rho_y a_{x\ y}^{(m)(1)}+(1-\rho_y)a_{x\ y}^{(m)(2)} \quad . \quad . \quad . \quad . \quad (21).$$

Taking now the case of a joint annuity on two female lives (x) and (y), we have

$$a_{x\ y}^{(f)(f)}=\frac{1}{l_x^{(f)}l_y^{(f)}}\left[v(l_{x+1}^{(1)}+l_{x+1}^{(2)})(l_{y+1}^{(1)}+l_{y+1}^{(2)})+v^2(l_{x+2}^{(1)}+l_{x+2}^{(2)})(l_{y+2}^{(1)}+l_{y+2}^{(2)})+\ldots\right]$$

$$\begin{aligned}
&=\frac{1}{l_x^{(f)}l_y^{(f)}}\Big[vl_{x+1}^{(1)}l_{y+1}^{(1)}+v^2l_{x+2}^{(1)}l_{y+2}^{(1)}+ \quad . \quad . \quad . \quad . \quad . \quad . \\
&\qquad +vl_{x+1}^{(1)}l_{y+1}^{(2)}+v^2l_{x+2}^{(1)}l_{y+2}^{(2)}+ \quad . \quad . \quad . \quad . \quad . \quad . \\
&\qquad +vl_{x+1}^{(2)}l_{y+1}^{(1)}+v^2l_{x+2}^{(2)}l_{y+2}^{(1)}+ \quad . \quad . \quad . \quad . \quad . \quad . \\
&\qquad +vl_{x+1}^{(2)}l_{y+1}^{(2)}+v^2l_{x+2}^{(2)}l_{y+2}^{(2)}+ \quad . \quad . \quad . \quad . \quad . \quad . \quad \Big] \\
&=\frac{1}{l_x^{(f)}l_y^{(f)}}\left(l_x^{(1)}l_y^{(1)}a_{x\ y}^{(1)(1)}+l_x^{(1)}l_y^{(2)}a_{x\ y}^{(1)(2)}+l_x^{(2)}l_y^{(1)}a_{x\ y}^{(2)(1)}+l_x^{(2)}l_y^{(2)}a_{x\ y}^{(2)(2)}\right) \\
&=\frac{a_{x\ y}^{(1)(1)}+r_y a_{x\ y}^{(1)(2)}+r_x a_{x\ y}^{(2)(1)}+r_x r_y a_{x\ y}^{(2)(2)}}{(1+r_x)(1+r_y)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22).
\end{aligned}$$

which is identical with Mr. Hardy's formula (15).

Expressed in terms of ρ and $(1-\rho)$, this becomes

$$\begin{aligned}
a_{x\ y}^{(f)(f)}=&\rho_x\rho_y a_{x\ y}^{(1)(1)}+\rho_x(1-\rho_y)a_{x\ y}^{(1)(2)}+\rho_y(1-\rho_x)a_{x\ y}^{(2)(1)} \\
&+(1-\rho_x)(1-\rho_y)a_{x\ y}^{(2)(2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23).
\end{aligned}$$

It should be added that formulas (20) to (23) apply for deducing the value of $a_{[x]+t[y]+t}$, whatever value be assigned to t; the sectional joint annuity values being, of course, computed and employed for the assigned value of t. In Table XXI of the Appendix are given select and ultimate values of $\rho_x=\frac{l_x^{(1)}}{l_x^{(f)}}$, and $(1-\rho_x)=\frac{l_x^{(2)}}{l_x^{(f)}}$ (these

values being, of course, complementary to one another). As the values of $l_{ix}^{(2)}$ become practically negligible after age 79, it is not necessary to extend the Table beyond that age.

It may be convenient to add the formulas for joint annuities on three lives, although Tables of uniform seniority, and subsidiary Tables of joint annuities on three lives of equal age, are not included in this Paper. In the case of a joint annuity on a male life, (x), and two female lives, (y) and (z), we have—

$$a_{x\,y\,z}^{(m)(f)(f)}=\frac{a_{x\,y\,z}^{(m)(1)(1)}+r_y a_{x\,y\,z}^{(m)(2)(1)}+r_z a_{x\,y\,z}^{(m)(1)(2)}+r_y r_z a_{x\,y\,z}^{(m)(2)(2)}}{(1+r_y)(1+r_z)} \quad . \quad . \quad . \quad (24).$$

This is Mr. Hardy's formula (16), and may be otherwise expressed as—

$$=\rho_y\rho_z a_{x\,y\,z}^{(m)(1)(1)}+\rho_z(1-\rho_y)a_{x\,y\,z}^{(m)(2)(1)}+\rho_y(1-\rho_z)a_{x\,y\,z}^{(m)(1)(2)}$$
$$+(1-\rho_y)(1-\rho_z)a_{x\,y\,z}^{(m)(2)(2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25).$$

In the case of an annuity on three female lives, (x) (y) (z), we have—

$$a_{x\,y\,z}^{(f)(f)(f)}=\left[a_{x\,y\,z}^{(1)(1)(1)}+r_x a_{x\,y\,z}^{(2)(1)(1)}+r_y a_{x\,y\,z}^{(1)(2)(1)}+r_z a_{x\,y\,z}^{(1)(1)(2)}+r_x r_y a_{x\,y\,z}^{(2)(2)(1)}+r_x r_z a_{x\,y\,z}^{(2)(1)(2)}\right.$$
$$\left.+r_y r_z a_{x\,y\,z}^{(1)(2)(2)}+r_x r_y r_z a_{x\,y\,z}^{(2)(2)(2)}\right]\div(1+r_x)\,(1+r_y)\,(1+r_z)$$
$$=\rho_x\rho_y\rho_z a_{x\,y\,z}^{(1)(1)(1)}+(1-\rho_x)\rho_y\rho_z a_{x\,y\,z}^{(2)(1)(1)}+\rho_x(1-\rho_y)\rho_z a_{x\,y\,z}^{(1)(2)(1)}$$
$$+\rho_x\rho_y(1-\rho_z)a_{x\,y\,z}^{(1)(1)(2)}+(1-\rho_x)(1-\rho_y)\rho_z a_{x\,y\,z}^{(2)(2)(1)}+(1-\rho_x)\rho_y(1-\rho_z)a_{x\,y\,z}^{(2)(1)(2)}$$
$$+\rho_x(1-\rho_y)(1-\rho_z)a_{x\,y\,z}^{(1)(2)(2)}+(1-\rho_x)(1-\rho_y)(1-\rho_z)a_{x\,y\,z}^{(2)(2)(2)}.$$

This formula is not deduced by Mr. Hardy.

APPLICATION AND USE OF THE TABLES GIVEN IN THE APPENDIX, IN THE CALCULATION OF JOINT ANNUITY VALUES ON TWO LIVES.

It only remains practically to illustrate the use of the above methods, and of the Tables given in the Appendix, in computing the values of joint-annuities on two lives. This can best be done by a few examples, showing precisely the method proposed to be followed, and the amount of labour involved; but, before setting these out, it will be useful to refer to the two-life Tables already published, on the basis of the British Offices Life Annuity Experience.

The monetary *Tables deduced from the graduated experience of Life Annuitants* (*Male and Female*) include joint-annuity values (select and ultimate) on two lives at $2\frac{1}{2}$, 3, and $3\frac{1}{2}$ per cent. interest, separately tabulated for two male lives; a male and a female life; and two female lives. These values are tabulated only for equal ages, and for differences of 5, 10, 15 . . . up to 75

years, between the ages of the lives. For intermediate differences of age, the values must be approximately deduced by interpolation.

At 4, and also at 5, per cent. interest, select and ultimate annuity values, discriminated as above as to sex, are tabulated in respect of *two equal ages only*. These annuity values are computed for each tenth of a year of age; and a Table of uniform seniority is added (identical with column (6) of Table XIV in the Appendix to the present paper), which is applicable only (so far as the volume of monetary Tables is concerned) to the deduction of joint-annuities on two male lives. The values of joint-annuities on two male lives of equal age, interpolated for tenths of a year of age, and intended for use with the uniform seniority Table, are also included at 2½, 3, and 3½ per cent. interest.

As regards two male lives of any age, therefore, select and ultimate joint-annuity values can be computed at 2½, 3, and 3½ per cent., either (i) by the use of the tabular values on pp. 86-109 of the volume of *Tables deduced*, interpolating where the difference between the ages is other than a multiple of 5; or (ii) by the employment of the uniform seniority Table on page 190, and the Tables of joint-annuity values on two male lives of equal age on pp. 192-203 of the same volume. At 4 and 5 per cent., select and ultimate annuity values on two male lives can be computed by the use of the uniform seniority Table on p. 190, and the Tables of annuity values on two male lives of equal age given on pp. 204-211 of the volume of *Tables deduced*.

In the case of a male and a female life, or of two female lives, the Tables of select and ultimate annuity values at 2½, 3, and 3½ per cent., given on pp. 112-135 for two females, on pp. 138-161 for a male and female life (female the elder), and on pp. 164-187 for a male and female life (male the elder), can be employed, interpolating where the difference between the ages is other than a multiple of 5.

The select and ultimate annuity values given at 4 and 5 per cent. interest on pp. 212-19 in respect of two females, and on pp. 220-7 in respect of a male and female life, respectively of equal age, cannot, however, be used with the Table of uniform seniority on p. 190, because the law of uniform seniority does not hold (in its normal form) where a female life is involved. The ingenious method suggested by Mr. Lidstone, and explained and illustrated on pp. 230-3 of the volume of Tables, can, however, be employed in these cases. The method involves (i) the ascertainment (by interpolation where necessary) of the joint annuity values at the desired ages at 3, and at 3½ per cent., from the values given

on pp. 112-87 of the volume; (ii) the calculation, from the Tables given on pp. 112-3, 138-9, and 164-5, of the *equal ages* giving identical annuity values at 3 and at $3\frac{1}{2}$ per cent.: (iii) the calculation, by comparison of the equal ages deduced as above at 3 and $3\frac{1}{2}$ per cent., of the modified equal ages, approximately applicable at 4 (or 5) per cent.; (iv) the computation of the annuity value at the modified equal ages thus deduced, from the 4 (or 5) per cent. Tables given on pp. 212-27 of the volume of Tables.

The Tables appended to this Paper are intended for the practical application of the alternative method demonstrated earlier in the Paper, based upon the law of uniform seniority, in its application to the first and the second series of the Female Annuitants Table. I now proceed to illustrate the application of the method, and of the Tables, in a few practical examples.

EXAMPLE I.—Required the value of $a^{(m)(f)}_{[28][40]}$ (Select) and of $a^{(m)(f)}_{28:40}$ (Ultimate) at 3, and at 4, per cent. interest.

(*a*) *Select* annuity value at 3 per cent.

By formula (21) this is equal to

$$\rho_{[40]}a^{(m)(1)}_{[28][40]}+(1-\rho_{[40]})a^{(m)(2)}_{[28][40]}$$

From Table XIV column (2) (male and female life—male the younger) we find that

$$a^{(m)(1)}_{[28][40]}=a^{(m)\ (1)}_{[35\cdot26][35\cdot26]}$$

and from column (3) of the same Table that

$$a^{(m)(2)}_{[28][40]}=a^{(m)\ (2)}_{[39\cdot16][39\cdot16]}$$

Referring now to Table XV (Select annuity values at 3 per cent.) we have

$$a^{(m)(1)}_{[35][35]}=18\cdot601 \qquad a^{(m)(2)}_{[39][39]}=8\cdot269$$

$$a^{(m)(1)}_{[36][36]}=\underline{18\cdot224} \qquad a^{(m)(2)}_{[40][40]}=\underline{7\cdot995}$$

whence $a^{(m)\ (1)}_{[35\cdot26][35\cdot26]}=18\cdot503$ and $a^{(m)\ (2)}_{[39\cdot16][39\cdot16]}=8\cdot225$

Finally, taking out the values of $\rho_{[40]}$ and $(1-\rho_{[40]})$ from the Select columns in Table XXI,

$$\rho_{[40]}=\cdot74553 \text{ and } (1-\rho_{[40]})=\cdot25447$$

and the desired annuity value

$$a^{(m)(f)}_{[28][40]}=(\cdot74553\times18\cdot503)+(\cdot25447\times8\cdot225)$$
$$=\quad 13\cdot795 \quad +2\cdot093=15\cdot888$$

(*b*) *Select* annuity value at 4 per cent.

Here, the equivalent ages, deduced above, will apply, also the values of $\rho_{[40]}$ and $(1-\rho_{[40]})$; and we have from Table XVII (Select annuity values at 4 per cent.),

$$a^{(m)\,(1)}_{[35][35]} = 16{\cdot}345 \qquad a^{(m)\,(2)}_{[39][39]} = 7{\cdot}734$$
$$a^{(m)\,(1)}_{[36][36]} = 16{\cdot}053 \qquad a^{(m)\,(2)}_{[40][40]} = 7{\cdot}492$$
$$a^{(m)\,(1)}_{[35{\cdot}26][35{\cdot}26]} = 16{\cdot}270 \quad a^{(m)\,(2)}_{[39{\cdot}16][39{\cdot}16]} = 7{\cdot}695$$
$$\text{and } a^{(m)(f)}_{[23][40]} = ({\cdot}74553 \times 16{\cdot}270) + ({\cdot}25447 \times 7{\cdot}695)$$
$$= \quad 12{\cdot}130 \quad + 1{\cdot}958 = 14{\cdot}088.$$

These examples are worked out by Mr. Lidstone, on p. 231 of the volume of *Tables deduced.* The value at 3 per cent. is deduced by him by interpolation as 15·885 (comparing with 15·888 above), whilst at 4 per cent. the value by his approximate method would be 14·083 (comparing with 14·088 above).

(*c*) *Ultimate* annuity value at 3 per cent.

In this case, the equivalent ages deduced under (*a*) still apply; and we have from Table XVI

$$a^{(m)\,(1)}_{35:35} = 18{\cdot}268 \qquad a^{(m)\,(2)}_{39:39} = 7{\cdot}913$$
$$a^{(m)\,(1)}_{36:36} = 17{\cdot}893 \qquad a^{(m)\,(2)}_{40:40} = 7{\cdot}634$$
$$a^{(m)\,(1)}_{35{\cdot}26:35{\cdot}26} = 18{\cdot}170 \quad a^{(m)\,(2)}_{39{\cdot}16:39{\cdot}16} = 7{\cdot}868$$

The values of ρ_{40}, and of $(1-\rho_{40})$ must now be taken from the ultimate columns in Table XXI; and we have

$$a^{(m)\,(f)}_{23:40} = ({\cdot}73963 \times 18{\cdot}170) + ({\cdot}26037 \times 7{\cdot}868)$$
$$= \quad 13{\cdot}439 \quad + 2{\cdot}049 = 15{\cdot}488$$

as compared with 15·489, deduced by interpolation from the published 3 per cent. joint annuity values.

(*d*) *Ultimate* annuity value at 4 per cent.

Employing still the equivalent ages deduced under (*a*), we have from Table XVIII

$$a^{(m)\,(1)}_{35:35} = 16{\cdot}054 \qquad a^{(m)\,(2)}_{39:39} = 7{\cdot}403$$
$$a^{(m)\,(1)}_{36:36} = 15{\cdot}764 \qquad a^{(m)\,(2)}_{40:40} = 7{\cdot}156$$
$$a^{(m)\,(1)}_{35{\cdot}26:35{\cdot}26} = 15{\cdot}979 \quad a^{(m)\,(2)}_{39{\cdot}16:39{\cdot}16} = 7{\cdot}363$$

Applying now the ultimate values of ρ_{40} and $(1-\rho_{40})$ from Table XXI, we have

$$a^{(m)\,(f)}_{23:40} = ({\cdot}73963 \times 15{\cdot}979) + ({\cdot}26037 \times 7{\cdot}363)$$
$$= \quad 11{\cdot}819 \quad + 1{\cdot}917 = 13{\cdot}736$$

The value deduced by Mr. Lidstone's method being 13·735.

EXAMPLE II.—Required the value of $a^{(f)\,(f)}_{[23][50]}$ at 3, and 4, per cent. interest.

(*e*) *Select* annuity value at 3 per cent.

By formula (22) the desired annuity value is equal to

$$\rho_{[23]}\rho_{[50]}a^{(1)\,(1)}_{[23][50]} + \rho_{[23]}(1-\rho_{[50]})a^{(1)\,(2)}_{[23][50]} + \rho_{[50]}(1-\rho_{[23]})a^{(2)\,(1)}_{[23][50]}$$
$$+ (1-\rho_{[23]})(1-\rho_{[50]})a^{(2)\,(2)}_{[23][50]}$$

Entering successively columns (6) (7) (8) and (6) of the Table XIV of uniform seniority, we have

$$a^{(1)\,(1)}_{[23][50]} = a^{(1)\,(1)}_{[43\cdot11][43\cdot11]}\,;\quad a^{(1)\,(2)}_{[23][50]} = a^{(1)\,(2)}_{[48\cdot90][48\cdot90]}$$

$$a^{(2)\,(1)}_{[23][50]} = a^{(2)\,(1)}_{[30\cdot75][30\cdot75]}\,;\quad a^{(2)\,(2)}_{[23][50]} = a^{(2)\,(2)}_{[43\cdot11][43\cdot11]}$$

and taking out the several annuities at integral ages from Table XV (Select annuities, 3 per cent.), we have

$$\begin{array}{rll}
a^{(1)\,(1)}_{[43][43]} & =18\cdot271 & a^{(1)\,(2)}_{[48][48]} \quad =6\cdot272. \\
a^{(1)\,(1)}_{[44][44]} & =17\cdot734 & a^{(1)\,(2)}_{[49][49]} \quad =5\cdot984. \\
\text{whence } a^{(1)\,(1)}_{[43\cdot11][43\cdot11]} & =18\cdot212 & a^{(1)\,(2)}_{[48\cdot90][48\cdot90]} =6\cdot022. \\
\text{also } a^{(2)\,(1)}_{[30][30]} & =12\cdot010 & a^{(2)\,(2)}_{[43][43]} \quad =5\cdot135. \\
a^{(2)\,(1)}_{[31][31]} & =11\cdot686 & a^{(2)\,(2)}_{[44][44]} \quad =4\cdot912. \\
\text{whence } a^{(2)\,(1)}_{[30\cdot75][30\cdot75]} & =11\cdot767 & a^{(2)\,(2)}_{[43\cdot11][43\cdot11]} =5\cdot110.
\end{array}$$

Taking then the select values of $\rho_{[23]}$, $(1-\rho_{[23]})$, $\rho_{[50]}$, and $(1-\rho_{[50]})$ from Table XXI, we have, for the desired value

$$\begin{array}{l}
(\cdot59505\times\cdot85854\times18\cdot212)+\ (\cdot59505\times\cdot14146\times\ 6\cdot022) \\
+(\cdot85854\times\cdot40495\times11\cdot767)+\ (\cdot40495\times\cdot14146\times\ 5\cdot110) \\
=\qquad 9\cdot304\ +\ \cdot507\ +\ 4\cdot091\ +\cdot292\ =14\cdot194.
\end{array}$$

(*f*) *Select* annuity value at 4 per cent.

Using the same equivalent ages as before, we have, from Table XVII,

$$\begin{array}{rll}
a^{(1)\,(1)}_{[43][43]} & =16\cdot218 & a^{(1)\,(2)}_{[48][48]} \quad =5\cdot954. \\
a^{(1)\,(1)}_{[44][44]} & =15\cdot789 & a^{(1)\,(2)}_{[49][49]} \quad =5\cdot692. \\
\text{whence } a^{(1)\,(1)}_{[43\cdot11][43\cdot11]} & =16\cdot171 & a^{(1)\,(2)}_{[48\cdot90][48\cdot90]} =5\cdot718. \\
\text{also } a^{(2)\,(1)}_{[30][30]} & =10\cdot970 & a^{(2)\,(2)}_{[43][43]} \quad =4\cdot906. \\
a^{(2)\,(1)}_{[31][31]} & =10\cdot698 & a^{(2)\,(2)}_{[44][44]} \quad =4\cdot701. \\
a^{(2)\,(1)}_{[30\cdot75][30\cdot75]} & =10\cdot766 & a^{(2)\,(2)}_{[43\cdot11][43\cdot11]} =4\cdot883.
\end{array}$$

and, applying, as before, the select values of ρ and $(1-\rho)$,

$$\begin{array}{l}
a^{(f)\,(f)}_{[23][50]} = (\cdot59505\times\cdot85854\times16\cdot171)+(\cdot59505\times\cdot14146\times5\cdot718) \\
\quad +(\cdot85854\times\cdot40495\times10\cdot766)+(\cdot40495\times\cdot14146\times4\cdot883) \\
\quad =\qquad 8\cdot261+\ \ \cdot481\ +\ 3\cdot743\ +\cdot280\ \ =12\cdot765.
\end{array}$$

These examples were worked out by Mr. Lidstone on p. 232 of the volume of *Tables deduced.* The value at 3 per cent., as deduced by him by interpolation, is 14·195 (comparing with 14·194 given above), whilst at 4 per cent., the value by his approximate method comes out at 12·767 (comparing with 12·765 above).

The above examples show that the method suggested by Mr. G. F. Hardy, and illustrated in this Paper, for the calculation of joint annuity values, gives results which are practically identical

with those deduced by Mr. Lidstone's approximate method; and it will, I think, also be found that the results by both methods are practically exact. At the end of Mr. Lidstone's Memorandum, appended to the volume of *Tables deduced*, the results brought out by the use of his method are compared with the joint annuity values, as deduced by one of Mr. G. F. Hardy's summation formulas, given in chap. xxiv. of the *Institute of Actuaries Text-Book*, Part II. The differences, ·011 and ·030 in the two examples given, appear somewhat considerable, having regard to the much smaller differences arising between the values as deduced in this Paper, and Mr. Lidstone's approximate results; and it will, I think, be found that the formula of approximate summation employed in this comparison (probably Mr. G. F. Hardy's formula 39*a*) is not suitable for the calculation of *select* annuity values, because none of the terms of the formula (excepting u_0) falls within the period of selection. In order to test this, I have calculated the values, at 5 per cent. interest, of the three joint annuity values $a^{(m)(f)}_{[20][40]}$, $a^{(m)(f)}_{23:40}$ and $a^{(f)(f)}_{[30][22]}$ by Mr. G. F. Hardy's extended formula (38), employing in the first two examples 15 ordinates, at the points of duration 0, 3, 9, 15, 18 . . . 51, 54, 57, 63, and in the third example 17 ordinates, at the points of duration 0, 2, 6, 10, 12 . . . 38, 42, 46, 48. The resulting annuity values (which are no doubt practically exact) are 12·608, 12·293, and 11·561, showing differences of only ·005, 005, and ·004, as compared with those obtained by Mr. Lidstone's method. The values of these three joint annuities, deduced at 5 per cent. interest by the methods illustrated in this Paper (employing the values given in Tables XIX and XX) are respectively 12·606, 12·294, and 11·562.

It may be added that the formulas of approximate summation numbered (9) and (12), and possibly also (14), in Mr. Woolhouse's valuable paper in the *Journal of the Institute of Actuaries* (vol. xxvii, pp. 151-2) appear to be specially suitable for the calculation of select annuity values, having only five, seven, and seven terms respectively, of which one at least will usually fall within the first five years. Thus the true value of $a_{[20]}$ at 3 per cent, according to the $O^{[am]}$ Table, is 20·270. Mr. Woolhouse's formula (9) gives a value of 20·276; his formula (12) gives 20·275, whilst Mr. G. F. Hardy's formula (39*a*) (with six ordinates) gives 20·257; and his extended formula (38), with twenty-five ordinates, gives 20·270, an exact result.

APPENDIX.

On the relations between the Male Annuitants Table and the First and Second Series of the Female Table, as regards the calculation of identical annuity values by variations in the age of the life, and in the rate of interest.

1. Considering first the relations between the Male Table, and the Second Series of the Female Table :—

$$\text{Here } l_{[x]+t}^{(2)} = \kappa a^{x+24+t} l_{[x+24]+t}^{(m)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

where κ and a are constants, the values of which have to be determined.[1] We have

$$\log l_{[x]+t}^{(2)} = \log \kappa + (x+24+t) \log a + \log l_{[x+24]+t}^{(m)}$$

Setting out $\log l_{[x+24]+t}^{(m)}$ in terms of its constituent functions and constants, we have

$$\begin{aligned}\log l_{[x]+t}^{(2)} &= \log \kappa + (x+24+t) \log a + \log k_t^{(m)} + (x+24+t) \log s^{(m)} \\ &\quad + c^{x+24+t} \log g_t^{(m)} \\ &= [\log \kappa + 24 (\log a + \log s^{(m)}) + \log k_t^{(m)}] \\ &\quad + (x+t)(\log a + \log s^{(m)}) + c^{x+t}(c^{24} \log g_t^{(m)}) \\ &= \log k_t^{(2)} + (x+t) \log s^{(2)} + c^{x+t} \log g_t^{(2)}\end{aligned}$$

$$\therefore \log k_t^{(2)} = \log \kappa + 24 (\log a + \log s^{(m)}) + \log k_t^{(m)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\log s^{(2)} = \log a + \log s^{(m)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\log g_t^{(2)} = c^{24} \log g_t^{(m)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

From formula (3)

$$\begin{aligned}\log a &= \log s^{(2)} - \log s^{(m)}. \\ &= -\cdot 007870 + \cdot 00260 = -\cdot 00527.\end{aligned}$$

(the values of $\log s^{(2)}$ and $\log s^{(m)}$ being taken from the Table of Graduation Functions on p. 136 of *Account of Principles and Methods, etc.*)

$$\therefore a = \log^{-1}(-\cdot 00527) = \cdot 987939.$$

Inserting this value, and the values of $\log k_t^2$, $\log k_t^{(m)}$, and $\log s^{(m)}$, in formula (2) we have

$$\begin{aligned}\log \kappa &= \log k_t^2 - \log k_t^{(m)} - 24 (\log a + \log s^{(m)}) \\ &= -\cdot 238148 - (24 \times -\cdot 00787) \\ &= -\cdot 049268\end{aligned}$$

$$\therefore \kappa = \log^{-1}(-\cdot 049268) = \cdot 892754$$

Inserting the values of a and κ in formula (1).

$$l_{[x]+t}^{(2)} = \cdot 892754 \times (\cdot 987939)^{x+24+t} l_{[x+24]+t}^{(m)}$$

[1] This relation shows at once that the probabilities and annuity values can be equated, with variation both of the age and the rate of interest; and the value of the constant a could at once be deduced by equating the functions $p_{[x]+t}^{(2)}$ and $a p_{[x+24]+t}^{(m)}$. It has, however, been preferred to set out, for the benefit of students, the fuller demonstration of the relations between the male table, and each series of the female table, as throwing useful light on the functions and constants which enter into their construction respectively.

Similarly $l_{[x]+t+r}^{(2)} = \cdot 892754 \times (\cdot 987939)^{x+24+t+r} l_{[x+24]+t+r}^{(m)}$

$$\text{whence } {}_rp_{[x]+t}^{(2)} = (\cdot 987939)^r \, {}_rp_{[x+24]+t}^{(m)}$$

The modified rate of interest is thus such that

$$\frac{\cdot 987939}{1+i} = \frac{1}{1+i'}$$

$$\text{whence } i' = \frac{\cdot 012061 + i}{\cdot 987939}$$

The following Table shows the value of i' corresponding to particular values of i :—

Value of i	Value of i'
·025	·0375134
·03	·0425745
·035	·0476355
·04	·0526965
·045	·0577576
·05	·0628187

showing approximately an increase of $1\frac{1}{4}$ per cent. in the rate of interest.

The above results apply throughout to select and ultimate annuity values.

2. Considering now the relations between the Male Table, and the First Series of the Female Table :—

$$\text{Here } l_{[x]+t}^{(1)} = \kappa_t a^{x-1+t} l_{[x-1]+t}^{(m)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Analysing $\log l_{[x-1]+t}^{(m)}$ as before, we have

$$\log l_{[x]+t}^{(1)} = \log \kappa_t + (x-1+t) \log a + \log k_t^{(m)} + (x-1+t) \log s^{(m)}$$

$$+ c^{x-1+t} \log g_t^{(m)}$$

$$= [\log \kappa_t - \log a + \log k_t^{(m)} - \log s^{(m)}]$$

$$+ (x+t)(\log a + \log s^{(m)}) + c^{x+t}(c^{-1} \log g_t^{(m)})$$

$$\therefore \log k_t^{(1)} = \log \kappa_t - \log a + \log k_t^{(m)} - \log s^{(m)} \quad . \quad . \quad . \quad . \quad (6)$$

$$\log s^{(1)} = \log a + \log s^{(m)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\log g_t^{(1)} = c^{-1} \log g_t^{(m)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

From formula (7)

$$\log a = \log s^{(1)} - \log s^{(m)}$$
$$= \cdot 00300 + \cdot 00260 = \cdot 00560$$
$$a = \log^{-1}(\cdot 00560) = 1 \cdot 012978$$

and from formula (6)

$$\log \kappa_t = \log k_t^{(1)} + \log a + \log s^{(m)} - \log k_t^{(m)}$$

$$= (\log k_t^{(1)} - \log k_t^{(m)}) + \cdot 00560 - \cdot 00260$$

$$= (\log k_t^{(1)} - \log k_t^{(m)}) + \cdot 00300$$

Here the quantity $(\log k_t^{(1)} - \log k_t^{(m)})$ *varies with t*,

and, from the Table of graduation Functions, we have

$\log \kappa_0 = -\cdot 351540$	$\kappa_0 = \cdot 4451025$
$\log \kappa_1 = -\cdot 352440$	$\kappa_1 = \cdot 4441810$
$\log \kappa_2 = -\cdot 353160$	$\kappa_2 = \cdot 4434452$
$\log \kappa_3 = -\cdot 353640$	$\kappa_3 = \cdot 4429554$
$\log \kappa_4 = -\cdot 353880$	$\kappa_4 = \cdot 4427107$
$\log \kappa_5 = -\cdot 353940$	$\kappa_5 = \cdot 4426495$

Thus $\quad l_{[x]+t}^{(1)} = \kappa_t (1\cdot 012978)^{x-1+t} l_{[x-1]+t}^{(m)}$

and $\quad l_{[x]+t+r}^{(1)} = \kappa_{t+r} (1\cdot 012978)^{x-1+t+r} l_{[x-1]+t+r}^{(m)}$

$$\therefore \quad {}_rp_{[x]+t}^{(1)} = \frac{\kappa_{t+r}}{\kappa_t} (1\cdot 012978)^r \, {}_rp_{[x-1]+t}^{(m)}$$

As the ratio $\frac{\kappa_{t+r}}{\kappa_t}$ becomes equal to unity when $t = 5$ or upwards, we have, for *ultimate* probabilities and *ultimate* annuity values,

$${}_rp_x^{(1)} = (1\cdot 012978)^r \, {}_rp_{x-1}^{(m)}$$

The modified rate of interest is thus such that

$$\frac{1\cdot 012978}{1+i} = \frac{1}{1+i'}$$

$$\text{whence } i' = \frac{i - \cdot 012978}{1\cdot 012978}$$

The following Table shows the values of i' corresponding to particular values of i :—

Value of i	Value of i'
·025	·011868
·03	·016804
·035	·021740
·04	·026676
·045	·031612
·05	·036548

showing approximately a decrease of $1\frac{1}{3}$ per cent. in the rate of interest.

This result applies to *ultimate* annuity values only.

For *Select* annuity values, the factor $\frac{\kappa_{t+r}}{\kappa_t}$ enters into the equation. The values of this ratio, for different values of t and r, are as follow :—

t	$r=1$	$r=2$	$r=3$	$r=4$	$r=5$
0	·997931	·996278	·995177	·994628	·994490
1	·998343	·997241	·996690	·996552	...
2	·998896	·998344	·998206	...	...
3	·999448	·999310	...	...	...
4	·999861	...	...	...	...

An experimental calculation, combining the above values with the element of interest, suggests that, very roughly and approximately, it may be taken that $a_{[x]+t}^{(1)} = \cdot 995 \, a'_{[x-1]+t}^{(m)}$ nearly, the male select annuity value being of course computed at the modified rate of interest i'.

Comparison of Values of $e_{[x-5]+5}$ with Expectations of Life deduced from Select Data, and from Aggregate Data, after exclusion in each case of the Experience of the first 5 Years.

TABLE I.—MALE ANNUITANTS.

Groups of Ages attained.	Mean of 10 Values of Expectation of Life.						
	Aggregate Data, excluding first 5 years.	Select Data, excluding first 5 years.	Select Data, computed after 5 years.	Excess (+) or Defect (−) in values of $e_{[x-5]+5}$, as compared with Expectations deduced from			
	$e^{(5)}_x$	$e^{[5]}_x$	$e_{[x-5]+5}$	Aggregate Data. +	Aggregate Data. −	Select Data. +	Select Data. −
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
55—64	13·87	14·00	14·16	·29	...	·16	...
60—69	11·24	11·42	11·38	·14	...	...	·04
65—74	8·70	8·84	8·79	·09	...	...	·05
70—79	6·51	6·61	6·61	·10	...	...	...
75—84	4·77	4·80	5·17	·40	...	·37	...
80—89	3·39	3·39	3·53	·14	...	·14	...
Average	8·08	8·18	8·27	·19	...	·09	...

TABLE II.—FEMALE ANNUITANTS.

55—64	16·31	16·47	16·35	·04	...	...	·12
60—69	13·02	13·19	13·25	·23	...	·06	...
65—74	10·06	10·24	10·23	·17	...	...	·01
70—79	7·50	7·68	7·73	·23	...	·05	...
75—84	5·39	5·51	5·55	·16	...	·04	...
80—89	3·80	3·86	4·25	·45	...	·39	...
Average	9·35	9·49	9·56	·21	...	·07	...

Comparison of Values of $e_{[x-10]+10}$ with Expectations of Life deduced from Select Data, and from Aggregate Data, after exclusion in each case of the Experience of the first 10 *Years.*

TABLE III.—MALE ANNUITANTS.

Groups of Ages attained.	Mean of 10 Values of Expectations of Life.						
	Aggregate Data, excluding first 10 years.	Select Data, excluding first 10 years.	Select Data, computed after 10 years.	Excess (+) or Defect (−) in values of $e_{[x-10]+10}$, as compared with Expectations deduced from			
	$e_x^{(10)}$	$e_x^{[10]}$	$e_{[x-10]+10}$	Aggregate Data. +	Aggregate Data. −	Select Data. +	Select Data. −
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
55—64	13·22	13·51	13·75	·53	...	·24	...
60—69	10·94	11·18	11·30	·36	...	·12	...
65—74	8·52	8·73	8·55	·03	...	...	·18
70—79	6·29	6·46	6·43	·14	...	...	·03
75—84	4·60	4·68	4·64	·04	...	...	·04
80—89	3·35	3·37	3·38	·03	...	·01	...
Average	7·82	7·99	8·01	·19	...	·02	...

TABLE IV.—FEMALE ANNUITANTS.

55—64	16·04	16·15	15·86	...	·18	...	·29
60—69	12·82	12·92	12·93	·11	...	·01	...
65—74	9·93	10·05	10·21	·28	...	·16	...
70—79	7·42	7·52	7·53	·11	...	·01	...
75—84	5·36	5·41	5·47	·11	...	·06	...
80—89	3·77	3·82	4·00	·23	...	·18	...
Average	9·22	9·31	9·33	·11	...	·02	...

Table V.—Male Annuitants.

Excluding the first 5 Years' Experience.

Age (x)	Aggregate Data. $E_x^{(5)}$	$\theta_x^{(5)}$	$q_x^{(5)}$	$e_x^{(5)}$	Select Data. $E_x^{[5]}$	$\theta_x^{[5]}$	$q_x^{[5]}$	$e_x^{[5]}$	Age (x)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
50	154	3	·01948	19·225	166	3	·01807	19·375	50
51	188	6	·03191	18·607	199	6	·03015	18·732	51
52	208	4	·01923	18·220	220	4	·01818	18·314	52
53	237	3	·01266	17·577	252	3	·01190	17·654	53
54	271	10	·03690	16·803	294	10	·03401	16·866	54
55	316	6	·01899	16·446	350	7	·02000	16·460	55
56	364	8	·02198	15·765	404	9	·02228	15·796	56
57	421	7	·01663	15·119	472	11	·02331	15·156	57
58	481	19	·03950	14·375	543	23	·04236	14·518	58
59	533	27	·05066	13·966	609	31	·05090	14·160	59
60	567	17	·02998	13·711	651	18	·02765	13·919	60
61	634	24	·03785	13·135	742	23	·03774	13·315	61
62	662	19	·02870	12·652	781	23	·02945	12·838	62
63	723	27	·03734	12·025	858	29	·03380	12·227	63
64	770	31	·04026	11·492	920	38	·04130	11·655	64
65	866	28	·03233	10·974	1,026	35	·03411	11·157	65
66	932	42	·04506	10·341	1,115	50	·04484	10·551	66
67	1,002	56	·05589	9·828	1,214	57	·04695	10·046	67
68	1,078	49	·04545	9·410	1,330	60	·04511	9·541	68
69	1,127	60	·05324	8·858	1,398	70	·05007	8·992	69
70	1,207	82	·06794	8·356	1,511	97	·06420	8·466	70
71	1,239	83	·06699	7·966	1,568	104	·06633	8·046	71
72	1,290	94	·07287	7·537	1,640	125	·07622	7·618	72
73	1,269	82	·06462	7·130	1,619	106	·06547	7·247	73
74	1,266	108	·08531	6·622	1,620	132	·08148	6·754	74
75	1,244	113	·09084	6·240	1,603	145	·09046	6·353	75
76	1,203	114	·09476	5·864	1,546	141	·09120	5·985	76
77	1,125	115	·10222	5·477	1,456	142	·09753	5·586	77
78	1,059	130	·12276	5·101	1,388	164	·11816	5·190	78
79	975	131	·13436	4·815	1,283	155	·12081	4·885	79
80	883	121	·13703	4·562	1,183	152	·12849	4·556	80
81	789	113	·14322	4·287	1,055	156	·14787	4·228	81
82	704	115	·16335	4·003	933	153	·16399	3·962	82
83	587	104	·17717	3·785	776	138	·17784	3·739	83
84	485	74	·15258	3·600	625	99	·15840	3·548	84
85	427	80	·18735	3·248	549	108	·19672	3·215	85
86	350	69	·19714	2·997	438	89	·20320	3·003	86
87	284	52	·18310	2·733	353	65	·18414	2·769	87
88	228	66	·28947	2·345	282	87	·30851	2·394	88
89	165	50	·30303	2·301	201	62	·30846	2·461	89
90	116	40	·34483	2·301	142	45	·31690	2·559	90
91	77	24	·31169	2·512	95	25	·26316	2·747	91
92	51	10	·19608	2·649	67	14	·20896	2·728	92
93	39	10	·25641	2·296	51	15	·29412	2·448	93
94	28	10	·35714	2·087	35	11	·31429	2·469	94
95	19	4	·21053	2·247	25	7	·28000	2·600	95
96	13	4	·30769	1·846	16	4	·25000	2·611	96
97	9	4	·44444	1·667	12	4	·33333	2·481	97
98	6	2	·33333	2·000	9	2	·22222	2·722	98
99	2	...	·00000	2·000	4	...	...	2·500	99
100	2	1	·50000	1·000	4	1	·25000	1·500	100
101	1	...	·00000	1·000	3	...	...	1·000	101
102	1	1	1·00000	...	3	3	1·00000	...	102
Totals	28,677	2,452	...	...	35,639	3,066	...	...	Totals

TABLE VI.—MALE ANNUITANTS.

Excluding the first 10 *Years' Experience.*

Age	Aggregate Data.				Select Data.				Age
(x)	$E_x^{(10)}$	$\theta_x^{(10)}$	$q_x^{(10)}$	$e_x^{(10)}$	$E_x^{[10]}$	$\theta_x^{[10]}$	$q_x^{[10]}$	$e_x^{[10]}$	(x)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
50	70	...	·00000	19·185	72	...	·00000	19·498	50
51	81	2	·02469	18·185	83	2	·02410	18·498	51
52	90	3	·03333	17·646	93	3	·03226	17·954	52
53	104	2	·01923	17·254	108	2	·01852	17·553	53
54	113	1	·00885	16·592	119	1	·00840	16·884	54
55	126	1	·00794	15·740	132	1	·00758	16.027	55
56	161	4	·02484	14·866	168	4	·02381	15·150	56
57	180	3	·01667	14·245	189	4	·02116	14·519	57
58	204	11	·05392	13·487	216	12	·05556	13·833	58
59	231	12	·05195	13·255	248	12	·04839	13·647	59
60	248	10	·04032	12·982	271	11	·04059	13·341	60
61	275	13	·04727	12·527	303	14	·04620	12·801	61
62	300	13	·04333	12·149	336	13	·03869	12·421	62
63	323	14	·04334	11·699	369	14	·03794	11·921	63
64	371	23	·06199	11·229	430	27	·06279	11·391	64
65	401	10	·02494	10·971	465	14	·03011	11·154	65
66	433	20	·04619	10·251	508	24	·04724	10·500	66
67	473	24	·05074	9·748	552	24	·04348	10·021	67
68	502	18	·03586	9·269	602	21	·03488	9·476	68
69	545	31	·05688	8·614	646	32	·04954	8·819	69
70	612	41	·06699	8·133	728	46	·06319	8·279	70
71	636	41	·06447	7·717	780	55	·07052	7·837	71
72	668	52	·07784	7·249	822	67	·08150	7·432	72
73	682	53	·07771	6·861	842	64	·07601	7·091	73
74	686	60	·08746	6·439	858	65	·07576	6·674	74
75	694	66	·09510	6·056	876	86	·09817	6·221	75
76	706	67	·09490	5·693	896	79	·08817	5·899	76
77	707	68	·09618	5·290	902	79	·08758	5·469	77
78	690	89	·12899	4·852	895	111	·12402	4·994	78
79	648	94	·14506	4·571	841	108	·12842	4·701	79
80	578	91	·15744	4·346	786	112	·14249	4·394	80
81	513	78	·15205	4·159	699	110	·15737	4·124	81
82	467	75	·16060	3·904	620	100	·16129	3·894	82
83	409	74	·18093	3·651	539	101	·18738	3·643	83
84	355	64	·18028	3·458	452	79	·17478	3·482	84
85	316	63	·19937	3·218	402	81	·20149	3·220	85
86	264	55	·20833	3·020	327	71	·21713	3·033	86
87	221	43	·19457	2·815	268	51	·19030	2·874	87
88	184	49	·26630	2·495	224	62	·27679	2·549	88
89	138	42	·30435	2·400	169	53	·31361	2·524	89
90	98	33	·33673	2·450	118	37	·31356	2·678	90
91	63	19	·30159	2·694	78	20	·25641	2·901	91
92	44	8	·18182	2·857	57	10	·17544	2·902	92
93	35	9	·25714	2·492	47	14	·29787	2·519	93
94	25	8	·32000	2·355	32	9	·28125	2·588	94
95	18	4	·22222	2·463	25	7	·28000	2·600	95
96	12	3	·25000	2·167	16	4	·25000	2·611	96
97	9	4	·44444	1·889	12	4	·33333	2·482	97
98	5	1	·20000	2·400	9	2	·22222	2·722	98
99	2	...	·00000	2·000	4	...	·00000	2·500	99
100	2	1	·50000	1·000	4	1	·25000	1·500	100
101	1	...	·00000	1·000	2	...	·00000	1·000	101
102	1	1	1·00000	...	3	3	1·00000	...	102
Totals	15,720	1,571	...	...	19,243	1,926	...	...	Totals

Table VII.—Female Annuitants.

Excluding the first 5 Years' Experience.

Age (x)	Aggregate Data. $E_x^{(5)}$	$\theta_x^{(5)}$	$q_x^{(5)}$	$e_x^{(5)}$	Select Data. $E_x^{[5]}$	$\theta_x^{[5]}$	$q_x^{[5]}$	$e_x^{[5]}$	Age (x)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
50	553	17	·03074	22·634	583	19	·03259	22·677	50
51	615	7	·01138	22·352	651	7	·01075	22·441	51
52	704	9	·01278	21·609	745	9	·01208	21·685	52
53	792	12	·01515	20·889	841	13	·01546	20·950	53
54	872	6	·00688	20·210	924	6	·00649	20·279	54
55	1,026	22	·02144	19·350	1,093	23	·02104	19·411	55
56	1,151	18	·01564	18·774	1,232	27	·02208	18·828	56
57	1,306	17	·01302	18·073	1,406	20	·01422	18·253	57
58	1,484	26	·01752	17·311	1,597	26	·01628	17·517	58
59	1,669	29	·01738	16·620	1,796	32	·01782	16·807	59
60	1,873	39	·02082	15·914	2,029	40	·01971	16·112	60
61	2,088	47	·02251	15·252	2,282	49	·02147	15·435	61
62	2,301	54	·02347	14·603	2,533	58	·02290	14·774	62
63	2,575	57	·02214	13·954	2,852	62	·02174	14·120	63
64	2,808	69	·02457	13·270	3,129	79	·02525	13·434	64
65	3,120	96	·03077	12·604	3,497	109	·03117	12·782	65
66	3,433	119	·03466	12·004	3,883	125	·03219	12·193	66
67	3,742	136	·03634	11·436	4,265	154	·03611	11·599	67
68	3,972	141	·03550	10·867	4,581	165	·03602	11·034	68
69	4,180	186	·04450	10·267	4,849	211	·04351	10·446	69
70	4,401	201	·04567	9·745	5,151	234	·04543	9·921	70
71	4,545	215	·04730	9·212	5,349	248	·04636	9·393	71
72	4,643	239	·05148	8·669	5,519	276	·05001	8·850	72
73	4,643	284	·06117	8·139	5,586	327	·05854	8·316	73
74	4,574	278	·06078	7·670	5,559	337	·06062	7·833	74
75	4,483	313	·06982	7·166	5,488	366	·06669	7·338	75
76	4,324	352	·08141	6·204	5,352	425	·07941	6·863	76
77	4,066	359	·08829	6·198	5,062	454	·08969	6·455	77
78	3,816	353	·09251	5·908	4,747	429	·09037	6·091	78
79	3,483	334	·09589	5·510	4,359	409	·09383	5·696	79
80	3,177	376	·11835	5·095	3,985	458	·11493	5·175	80
81	2,843	351	·12346	4·779	3,577	442	·12357	4·847	81
82	2,534	342	·13496	4·452	3,182	411	·12916	4·531	82
83	2,182	313	·14345	4·146	2,751	378	·13740	4·203	83
84	1,832	295	·16103	3·841	2,328	386	·16151	3·872	84
85	1,509	261	·17296	3·578	1,909	327	·17129	3·618	85
86	1,239	236	·19048	3·326	1,574	302	·19187	3·366	86
87	991	212	·21393	3·109	1,255	261	·20797	3·165	87
88	763	155	·20315	2·955	973	204	·20966	2·996	88
89	590	134	·22712	2·708	747	164	·21954	2·791	89
90	459	114	·24837	2·504	590	142	·24068	2·577	90
91	333	84	·25225	2·332	430	103	·23958	2·393	91
92	243	65	·26749	2·118	323	82	·25387	2·147	92
93	168	62	·36905	1·892	224	84	·37500	1·878	93
94	103	36	·34951	1·998	137	49	·35766	2·004	94
95	66	20	·30303	2·072	86	22	·25581	2·120	95
96	45	14	·31111	1·973	59	20	·33898	1·849	96
97	27	8	·29630	1·863	34	9	·26471	1·797	97
98	18	8	·44444	1·648	24	11	·45833	1·445	98
99	10	2	·20000	1·967	13	3	·23077	1·667	99
100	8	3	·37500	1·458	10	5	·50000	1·167	100
101	3	1	·33333	1·333	3	1	·33333	1·333	101
102	2	1	·50000	1·000	2	1	·50000	1·000	102
103	1	...	·00000	1·000	1	...	·00000	1·000	103
104	1	1	1·00000	...	1	1	1·00000	..	104
Totals	102,389	7,120	...	...	121,158	8,605	...	...	Totals

Table VIII.—Female Annuitants.

Excluding the first 10 *Years' Experience.*

Age	Aggregate Data.				Select Data.				Age
(x)	$E_x^{(10)}$	$\theta_x^{(10)}$	$q_x^{(10)}$	$e_x^{(10)}$	$E_x^{[10]}$	$\theta_x^{[10]}$	$q_x^{[10]}$	$e_x^{[10]}$	(x)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
50	243	6	·02469	22·448	249	6	·02410	22·621	50
51	276	2	·00725	22·016	284	2	·00704	22·180	51
52	302	4	·01325	21·177	311	4	·01286	21·337	52
53	347	7	·02017	20·461	359	7	·01950	20·615	53
54	390	3	·00769	19·882	402	3	·00746	20·025	54
55	430	9	·02093	19·036	447	9	·02013	19·176	55
56	494	9	·01822	18·443	517	9	·01741	18·570	56
57	558	8	·01434	17·786	587	9	·01533	17·899	57
58	628	11	·01752	17·044	660	11	·01667	17·177	58
59	701	14	·01997	16·348	730	14	·01918	16·469	59
60	813	18	·02214	15·681	852	18	·02113	15·791	60
61	917	18	·01963	15·036	966	19	·01967	15·131	61
62	1,040	24	·02308	14·338	1,099	24	·02184	14·435	62
63	1,182	30	·02538	13·677	1,257	32	·02546	13·757	63
64	1,299	43	·03310	13·033	1,394	48	·03443	13·117	64
65	1,432	41	·02863	12·479	1,538	48	·03121	12·584	65
66	1,596	54	·03383	11·847	1,726	56	·03244	11·990	66
67	1,752	64	·03653	11·262	1,912	67	·03504	11·392	67
68	1,908	76	·03983	10·689	2,102	82	·03901	10·806	68
69	2,031	93	·04579	10·132	2,261	99	·04379	10·244	69
70	2,264	108	·04770	9·618	2,535	123	·04852	9·713	70
71	2,442	115	·04709	9·100	2,757	126	·04570	9·209	71
72	2,636	144	·05463	8·550	3,004	166	·05526	8·650	72
73	2,741	181	·06603	8·044	3,158	205	·06491	8·156	73
74	2,809	171	·06088	7·612	3,272	199	·06082	7·722	74
75	2,880	204	·07083	7·106	3,379	225	·06659	7·222	75
76	2,904	241	·08299	6·648	3,447	277	·08036	6·737	76
77	2,864	251	·08764	6·249	3,446	297	·08619	6·326	77
78	2,756	264	·09579	5·849	3,351	318	·09191	5·922	78
79	2,606	257	·09862	5·469	3,181	307	·09651	5·522	79
80	2,451	301	·12281	5·067	3,007	358	·11906	5·111	80
81	2,240	279	·12455	4·777	2,781	356	·12801	4·802	81
82	2,024	265	·13093	4·456	2,513	316	·12575	4·507	82
83	1,813	259	·14286	4·128	2,247	310	·13796	4·156	83
84	1,548	255	·16473	3·816	1,937	329	·16985	3·821	84
85	1,292	219	·16950	3·568	1,617	274	·16945	3·602	85
86	1,084	205	·18911	3·297	1,363	260	·19076	3·337	86
87	888	191	·21509	3·065	1,115	234	·20987	3·124	87
88	694	142	·20461	2·905	881	188	·21339	2·954	88
89	538	128	·23792	2·652	678	156	·23009	2·755	89
90	418	103	·24641	2·480	532	129	·24248	2·578	90
91	310	80	·25806	2·291	393	95	·24173	2·404	91
92	228	61	·26754	2·088	300	73	·24333	2·170	92
93	158	59	·37342	1·851	211	81	·38389	1·868	93
94	99	35	·35354	1·955	130	45	·34615	2·032	94
95	63	19	·30159	2·024	83	21	·25301	2·108	95
96	42	13	·30952	1·897	56	19	·33929	1·822	96
97	25	8	·32000	1·748	32	9	·28125	1·757	97
98	17	8	·47059	1·570	24	11	·45833	1·445	98
99	10	2	·20000	1·967	13	3	·23077	1·667	99
100	8	3	·37500	1·458	10	5	·50000	1·167	100
101	3	1	·33333	1·333	3	1	·33333	1·333	101
102	2	1	·50000	1·000	2	1	·50000	1·000	102
103	1	...	·00000	1·000	1	...	·00000	1·000	103
104	1	1	1·00000	...	1	1	1·00000	...	104
Totals	61,198	5,108	...	...	71,113	6,085	...	...	Totals

Table IX.—Male Annuitants. Select Data.

Comparison of Actual Deaths, in each of the first 5 years, and after 5 years, with the Expected Deaths computed (at central ages) by Mr. G. F. Hardy's graduated rates of mortality, in respect of each of the first 5 years, and of later durations.

Ages attained.	Curtate Duration :— 0		1		2		3		4		5 and upwards.		Ages attained.
	Actual Deaths.	Expected Deaths.	Actual Deaths.	Expected Deaths.	Actual Deaths.	Expected Deaths.	Actual Deaths.	Expected Deaths.	Actual Deaths.	Expected Deaths.	Actual Deaths.	Expected Deaths.	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
25—29	...	·1	1	·1	2	·1	1	·1	...	·1	...	·4	25—29
30—34	...	·2	...	·2	1	·2	1	·2	...	·2	3	·8	30—34
35—39	1	·4	3	·4	1	·4	...	·4	...	·4	2	1·8	35—39
40—44	1	·9	...	·9	...	·9	...	·9	...	·8	3	3·8	40—44
45—49	2	2·2	4	2·2	6	2·1	4	1·9	2	1·6	7	8·7	45—49
50—54	6	6·0	14	6·1	8	5·9	5	5·4	5	4·9	26	19·5	50—54
55—59	13	11·7	11	12·4	12	12·5	15	12·5	16	11·0	81	55·6	55—59
60—64	34	27·3	30	29·0	37	28·7	24	26·0	20	23·2	136	129·6	60—64
65—69	49	43·6	62	49·4	44	52·8	55	53·0	43	48·3	272	287·3	65—69
70—74	53	50·3	63	58·4	61	63·1	49	68·0	72	69·2	564	550·1	70—74
75—79	47	40·5	59	53·2	53	60·4	59	66·9	65	66·7	747	742·5	75—79
80—84	25	22·7	28	27·4	33	33·8	33	40·1	44	42·6	698	689·4	80—84
85—89	5	8·1	14	12·0	17	11·2	8	11·5	14	16·6	411	403·0	85—89
90—94	1	1·1	2	1·8	2	3·7	5	3·3	2	3·2	110	124·3	90—94
95—99	...	...	...	...	...	...	...	·9	1	·9	17	29·4	95—99
100—104	...	...	...	...	...	...	...	...	...	...	4	5·7	100—104
Totals.	237	215·1	291	253·5	277	275·8	259	291·1	284	289·7	3,081	3,051·9	Totals.

TABLE X.—FEMALE ANNUITANTS. SELECT DATA.

Comparison of Actual Deaths, in each of the first 5 years, and after 5 years, with the Expected Deaths computed (at central ages) by Mr. G. F. Hardy's graduated rates of mortality, in respect of each of the first 5 years, and of later durations.

Ages attained.	Curtate Duration :—												Ages attained.
	0		1		2		3		4		5 and upwards.		
	Actual Deaths.	Expected Deaths.	Actual Deaths.	Expected Deaths.	Actual Deaths.	Expected Deaths.	Actual Deaths.	Expected Deaths.	Actual Deaths.	Expected Deaths.	Actual Deaths.	Expected Deaths.	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
25—29	...	·2	...	·3	1	·3	...	·3	2	·3	1	1·4	25—29
30—34	1	·4	1	·5	...	·6	...	·6	1	·6	2	2·6	30—34
35—39	2	·9	1	1·1	3	1·3	2	1·3	2	1·1	5	4·6	35—39
40—44	5	2·2	6	2·5	3	2·5	...	2·4	1	2·3	14	12·8	40—44
45—49	7	5·0	7	6·1	2	6·6	7	6·3	1	5·6	32	27·9	45—49
50—54	20	12·2	20	14·7	12	15·2	15	14·7	9	12·8	54	61·5	50—54
55—59	23	23·2	24	29·4	32	31·4	29	30·7	20	27·3	128	138·1	55—59
60—64	60	45·3	68	57·6	47	63·7	62	62·3	62	56·9	288	308·3	60—64
65—69	82	65·8	107	86·5	105	102·2	118	111·1	102	109·3	764	710·2	65—69
70—74	78	73·6	110	97·9	105	117·7	111	136·1	129	144·0	1,422	1,419·6	70—74
75—79	63	54·8	71	73·3	87	95·6	114	112·1	114	123·2	2,083	2,065·7	75—79
80—84	26	28·2	58	37·9	45	47·4	56	60·9	74	69·7	2,075	2,030·9	80—84
85—89	15	12·9	24	17·1	23	19·4	24	21·6	20	23·9	1,258	1,257·1	85—89
90—94	2	2·8	6	4·3	6	5·6	3	5·8	8	7·1	460	490·1	90—94
95—99	2	·6	...	...	...	...	...	...	...	·4	65	88·7	95—99
100—104	...	...	...	...	...	...	...	...	...	...	8	9·5	100—104
Totals.	386	328·1	503	429·2	471	509·5	541	566·2	545	584·5	8,659	8,628·9	Totals.

TABLE XI.—MALE

Experimental Graduation of "Select" Ultimate

Age. (x)	Unadjusted Data.[1] Exposed to Risk at Age ($x+\frac{1}{2}$). $E_x-\frac{1}{2}\theta_x$	$\Sigma(2)$	Deaths. θ_x	$\Sigma(4)$	First Approximation. $A=\cdot00651642$; $B=\cdot0001179243$; $a=\cdot00283005$; $\beta=\cdot0000535211$. colog $p_x=a+\beta c^x$	$m_x=\frac{2q_x}{2-q_x}$	$\mu_{x+\frac{1}{2}}=A+Bc^{x+\frac{1}{2}}$	$\frac{\mu_{x+\frac{1}{2}}}{m_x}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
50	164·5	164·5	3	3	...	...	...	...
51	196	360·5	6	9	...	...	...	...
52	218	578·5	4	13	·007,894,5	·018,194	·018,174	·99890
53	250·5	829	3	16	...	...	...	...
54	289	1,118	10	26	...	...	...	...
55	346·5	1,464 5	7	33	...	...	...	...
56	399·5	1,864	9	42	...	...	...	...
57	466·5	2,330·5	11	53	·010,673,6	·024,568	·024,572	1·00016
58	531·5	2,862	23	76	...	...	...	...
59	593·5	3,455·5	31	107	...	...	...	...
60	642	4,097·5	18	125	...	...	...	...
61	728	4,825·5	28	153	...	...	...	...
62	769·5	5,595	23	176	·014,978,9	·034,485	·034,481	·99988
63	843·5	6,438·5	29	205	...	...	...	...
64	901	7,339·5	38	243	...	...	...	...
65	1,008·5	8,348	35	278	...	...	...	...
66	1,090	9,438	50	328	...	...	...	...
67	1,185·5	10,623·5	57	385	·021,646,0	·049,842	·049,828	·99971
68	1,300	11,923·5	60	445	...	...	...	...
69	1,363	13,286·5	70	515	...	...	...	...
70	1,462 5	14,749	97	612	...	...	...	...
71	1,516	16,265	104	716	...	...	...	...
72	1,577·5	17,842·5	125	841	·031,972,3	·073,603	·073,598	·99968
73	1,566	19,408·5	106	947	...	...	...	...
74	1,554	20,962·5	132	1,079	...	...	...	...
75	1,530·5	22,493	145	1,224	...	...	...	...
76	1,475·5	23,968·5	141	1,365	...	...	...	...
77	1,385	25,353·5	142	1,507	·047,966,0	·110,339	·110,413	1·00067
78	1,306	26,659·5	164	1,671	...	...	...	...
79	1,205·5	27,865	155	1,826	...	...	...	...
80	1,107	28,972	152	1,978	...	...	...	...
81	977	29,949	156	2,134	...	...	...	...
82	856·5	30,805·5	153	2,287	·072,739,4	·167,094	·167,436	1·00204
83	707	31,512·5	138	2,425	...	...	...	...
84	575·5	32,088	99	2,524	...	...	...	...
85	495	32,583	108	2,632	...	...	...	...
86	393·5	32,976·5	89	2,721	...	...	...	...
87	320·5	33,297	65	2,786	·111,103,3	·254,436	·255,749	1·00516
88	238·5	33,535·5	87	2,873	...	...	...	...
89	170	33,705·5	62	2,935	...	...	...	...
90	119·5	33,825	45	2,980	...	...	...	...
	33,825·0	665,759·0	2,980	43,294	...	...	...	...

[1] In order to save space, the quantities $(E_x-\frac{1}{2}\theta_x)c^{x+\frac{1}{2}}$ and $\Sigma(E_x-\frac{1}{2}\theta_x)c^{x+\frac{1}{2}}$, for former quantity, and of its continuous summation, between the limits $x=50$ and summation, and of continuous summation, are precisely as set out in columns (2)

ANNUITANTS.

Table, excluding the experience of the first 5 *Years.*

Modified Deaths. (4)×(9)	Difference in Deaths. (10)−(4)	Σ(11)	Second Approximation. $A = \cdot00632964$; $B = \cdot0001183640$; $\alpha = \cdot00274893$; $\beta = \cdot0000537208$				
			colog p_x $= \alpha + \beta c^x$	q_x	Estimated Deaths. $E_x \times (14)$	Δ (15)−(4)	ΣΔ
(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
3	·00	·00	·0070161	·01603	2·66	·34−	·34−
5·99	·01−	·01−	·0074063	·01692	3·37	2·63−	2·97−
4	·00	·01	·0078322	·01787	3·93	·07−	3·04−
3	·00	·01	·0082972	·01893	4·77	1·77+	1·27−
9·99	·01−	·02	·0088043	·02006	5·90	4·10−	5·37−
7	·00	·02	·0093582	·02132	7·46	·46+	4·91−
9	·00	·02	·0099625	·02267	9·16	·16+	4·75−
11	·00	·02	·0106217	·02416	11·40	·40+	4·35−
23	·00	·02	·0113421	·02577	13·99	9·01−	13·36−
31·01	·01+	·01	·0121275	·02754	16·77	14·23−	27·59−
18	·00	·01	·0129854	·02947	19·18	1·18+	26·41−
28	·00	·01	·0139212	·03154	23·40	4·60−	31·01−
23	·00	·01	·0149430	·03382	26·41	3·41+	27·60−
28·99	·01−	·02	·0160577	·03630	31·15	2·15+	25·45−
37·99	·01	·03	·0172750	·03901	35·89	2·11−	27·56−
34·99	·01	·04	·0186030	·04192	43·01	8·01+	19·55−
49·99	·01	·05	·0200529	·04512	50·31	·31+	19·24−
56·98	·02	·07	·0216350	·04861	59·01	2·01+	17·23−
59·98	·02	·09	·0233621	·05237	69·65	9·65+	7·58−
69·98	·02	·11	·0252466	·05648	78·96	8·96+	1·38+
96·97	·03	·14	·0273041	·06093	92·07	4·93−	3·55−
103·97	·03	·17	·0295491	·06578	103·14	·86−	4·41−
124·96	·04	·21	·0319999	·07103	116·49	8·51−	12·92−
105·97	·03	·24	·0346746	·07673	124·23	18·23+	5·31+
131·96	·04−	·28	·0375938	·08291	134·31	2·31+	7·62+
145·10	·10+	·18	·0407805	·08963	143·68	1·32−	6·30+
141·09	·09	·09−	·0442579	·09689	149·79	8·79+	15·09+
142·10	·10	·01+	·0480533	·10474	152·50	10·50+	25·59+
164·11	·11	·12	·0521962	·11325	157·19	6·81−	18·78+
155·10	·10	·22	·0567168	·12243	157·08	2·08+	20·86+
152·31	·31	·53	·0616538	·13234	156·56	4·56+	25·42+
156·32	·32	·85	·0670366	·14304	150·91	5·09−	20·33+
153·31	·31	1·16	·0729190	·15457	144·21	8·79−	11·54+
138·28	·28	1·44	·0793333	·16695	129·55	8·45−	3·09+
99·20	·20	1·64	·0863385	·18029	112·68	13·68+	16·77+
108·56	·56	2·20	·0939776	·19458	106·82	1·18−	15·59+
89·46	·46	2·66	·1023204	·20989	91·93	2·93+	18·52+
65·34	·34	3·00	·1114261	·22630	79·88	14·88+	33·40+
87·45	·45	3·45	·1213644	·24379	68·75	18·25−	15·15+
62·32	·32	3·77	·1322107	·26245	52·75	9·25−	5·90+
45·23	·23+	4·00+	·1440507	·28229	40·09	4·91−	0·99+
2,984·00	4·00+	21·43+	...	...	2,980·99	·99+	22·33−

each value of x, are not included in the above Table. The totals of the values of the $x=90$, come out at 23,401,530 and 330,348,534 respectively. The processes of and (3) above.

Table XII.

Female Annuitant Experience.

First Series Table. Logarithms of the graduated Numbers living O^{af^1}.

Age at Entry.	Years Elapsed since Date of Purchase:—						Age attained.
	0	1	2	3	4	5 or more	
$[x]$	$\log l_{[x]}$	$\log l_{[x]+1}$	$\log l_{[x]+2}$	$\log l_{[x]+3}$	$\log l_{[x]+4}$	$\log l_{x+5}$	$(x+5)$
20	4·757758	4·761334	4·764705	4·767820	4·770680	4·773343	25
21	4·760460	4·764020	4·767370	4·770459	4·773288	4·775916	26
22	4·763135	4·766667	4·770005	4·773065	4·775860	4·778449	27
23	4·765780	4·769303	4·772606	4·775635	4·778393	4·780940	28
24	4·768392	4·771895	4·775170	4·778165	4·780883	4·783385	29
25	4·770969	4·774449	4·777695	4·780653	4·783327	4·785778	30
26	4·773508	4·776963	4·780176	4·783094	4·785720	4·788116	31
27	4·776004	4·779432	4·782610	4·785483	4·788057	4·790393	32
28	4·778454	4·781852	4·784992	4·787817	4·790334	4·792605	33
29	4·780854	4·784220	4·787317	4·790090	4·792544	4·794744	34
30	4·783199	4·786530	4·789581	4·792297	4·794683	4·796805	35
31	4·785484	4·788777	4·791778	4·794431	4·796742	4·798779	36
32	4·787703	4·790954	4·793901	4·796485	4·798715	4·800660	37
33	4·789852	4·793057	4·795944	4·798454	4·800595	4·802439	38
34	4·791922	4·795078	4·797899	4·800328	4·802372	4·804105	39
35	4·793907	4·797009	4·799759	4·802099	4·804037	4·805650	40
36	4·795800	4·798842	4·801514	4·803758	4·805581	4·807062	41
37	4·797591	4·800569	4·803156	4·805294	4·806991	4·808329	42
38	4·799272	4·802179	4·804674	4·806696	4·808256	4·809437	43
39	4·800832	4·803662	4·806056	4·807952	4·809362	4·810372	44
40	4·802261	4·805006	4·807290	4·809048	4·810295	4·811118	45
41	4·803546	4·806199	4·808362	4·809971	4·811038	4·811659	46
42	4·804674	4·807227	4·809259	4·810703	4·811576	4·811974	47
43	4·805630	4·808074	4·809963	4·811228	4·811888	4·812044	48
44	4·806400	4·808725	4·810457	4·811527	4·811955	4·811846	49
45	4.806967	4·809161	4·810722	4·811579	4·811753	4·811357	50
46	4·807310	4·809362	4·810737	4·811361	4·811259	4·810546	51
47	4·807411	4·809307	4·810478	4·810850	4·810445	4·809388	52
48	4·807246	4·808973	4·809922	4·810017	4·809282	4·807850	53
49	4·806792	4·808334	4·809041	4·808833	4·807739	4·805897	54
50	4·806023	4·807363	4·807805	4·807267	4·805781	4·803491	55
51	4·804908	4·806028	4·806182	4·805283	4·803368	4·800591	56
52	4·803418	4·804297	4·804135	4·802844	4·800461	4·797151	57
53	4·801516	4·802133	4·801628	4·799907	4·797014	4·793122	58
54	4·799167	4·799497	4·798616	4·796428	4·792978	4·788451	59
55	4·796328	4·796346	4·795055	4·792356	4·788297	4·783078	60
56	4·792956	4·792632	4·790894	4·787637	4·782915	4·776939	61
57	4·789001	4·788305	4·786079	4·782213	4·776766	4·769965	62
58	4·784410	4·783307	4·780548	4·776018	4·769781	4·762079	63
59	4·779124	4·777578	4·774238	4·768982	4·761882	4·753197	64

Table XII (*continued*).

Female Annuitant Experience.

First Series Table. Logarithms of the graduated Numbers living.

Oar.

Age at Entry.	Years Elapsed since Date of Purchase:—						Age attained.
	0	1	2	3	4	5 or more	
$[x]$	$\log l_{[x]}$	$\log l_{[x]+1}$	$\log l_{[x]+2}$	$\log l_{[x]+3}$	$\log l_{[x]+4}$	$\log l_{x+5}$	$(x+5)$
60	4·773083	4·771051	4·767076	4·761029	4·752987	4·743229	65
61	4·766212	4·763653	4·758985	4·752074	4·743004	4·732075	66
62	4·758440	4·755304	4·749880	4·742026	4·731834	4·719626	67
63	4·749682	4·745917	4·739668	4·730785	4·719369	4·705766	68
64	4·739850	4·735397	4·728248	4·718242	4·705489	4·690363	69
65	4·728844	4·723641	4·715509	4·704277	4·690066	4·673278	70
66	4·716558	4·710536	4·701331	4·688762	4·672958	4·654356	71
67	4·702874	4·695959	4·685582	4·671553	4·654011	4·633429	72
68	4·687664	4·679774	4·668119	4·652496	4·633058	4·610315	73
69	4·670789	4·661834	4·648785	4·631422	4·609914	4·584813	74
70	4·652097	4·641980	4·627409	4·608147	4·584379	4·556704	75
71	4·631421	4·620036	4·603804	4·582470	4·556236	4·525751	76
72	4·608580	4·595812	4·577765	4·554170	4·525245	4·491692	77
73	4·583376	4·569098	4·549072	4·523008	4·491145	4·454247	78
74	4·555594	4·539666	4·517480	4·488722	4·453653	4·413102	79
75	4·524996	4·507270	4·482726	4·451027	4·412459	4·367921	80
76	4·491327	4·471636	4·444519	4·409610	4·367223	4·318334	81
77	4·454304	4·432470	4·402544	4·364133	4·317577	4·263938	82
78	4·413622	4·389448	4·356457	4·314222	4·263117	4·204294	83
79	4·368945	4·342219	4·305881	4·259473	4·203402	4·138923	84
80	4·319909	4·290396	4·250406	4·199444	4·137953	4·067298	85
81	4·266114	4·233559	4·189584	4·133651	4·066245	3·988851	86
82	4·207127	4·171252	4·122926	4·061568	3·987706	3·902955	87
83	4·142471	4·102973	4·049899	3·982619	3·901710	3·808932	88
84	4·071628	4·028176	3·969919	3·896177	3·807577	3·706035	89
85	3·994034	3·946265	3·882352	3·801556	3·704562	3·593456	90
86	3·909070	3·856590	3·786504	3·698008	3·591852	3·470309	91
87	3·816062	3·758441	3·681617	3·584719	3·468563	3·335627	92
88	3·714275	3·651042	3·566864	3·460795	3·333725	3·188354	93
89	3·602907	3·533549	3·441346	3·325265	3·186284	3·027341	94
90	3·481081	3·405038	3·304073	3·177068	3·025086	2·851331	95
91	3·347841	3·264501	3·153975	3·015046	2·848874	2·658951	96
92	3·202142	3·110839	2·989878	2·837934	2·656274	2·448706	97
93	3·042847	2·942852	2·810501	2·644352	2·445788	2·218962	98
94	2·868711	2·759231	2·614447	2·432795	2·215782	1·967935	99
95	2·678378	2·558544	2·400193	2·201619	1·964469	1·693680	100
96	2·470367	2·339232	2·166071	1·949030	1·689903	1·394074	101
97	2·243060	2·099593	1·910268	1·673069	1·389955	1·066797	102
98	1·994695	1·837764	1·630799	1·371600	1·062305	0·709318	103
99	1·723344	1·551722	1·325502	1·042291	0·704421	0·318857	104

Table XIII.

Female Annuitant Experience.

Second Series Table. Logarithms of the graduated Numbers living.

O^{af^2}.

Age at Entry. $[x]$	Years Elapsed since Date of Purchase:— 0 $\log l_{[x]}$	1 $\log l_{[x]+1}$	2 $\log l_{[x]+2}$	3 $\log l_{[x]+3}$	4 $\log l_{[x]+4}$	5 or more $\log l_{x+5}$	Age attained. $(x+5)$
20	4·630958	4·623182	4·614593	4·605061	4·594605	4·583398	25
21	4·620432	4·612514	4·603738	4·593973	4·583240	4·571718	26
22	4·609662	4·601589	4·592610	4·582592	4·571557	4·559690	27
23	4·598628	4·590385	4·581184	4·570888	4·559524	4·547282	28
24	4·587304	4·578876	4·569433	4·558834	4·547111	4·534459	29
25	4·575664	4·567034	4·557326	4·546398	4·534282	4·521182	30
26	4·563680	4·554830	4·544833	4·533545	4·521000	4·507412	31
27	4·551319	4·542229	4·531917	4·520235	4·507223	4·493103	32
28	4·538548	4·529195	4·518539	4·506429	4·492905	4·478203	33
29	4·525328	4·515689	4·504658	4·492079	4·477999	4·462662	34
30	4·511620	4·501668	4·490226	4·477137	4·462449	4·446419	35
31	4·497377	4·487084	4·475195	4·461548	4·446196	4·429410	36
32	4·482552	4·471886	4·459510	4·445254	4·429177	4·411566	37
33	4·467091	4·456018	4·443109	4·428189	4·411321	4·392809	38
34	4·450935	4·439419	4·425929	4·410283	4·392553	4·373058	39
35	4·434022	4·422022	4·407897	4·391460	4·372787	4·352219	40
36	4·416283	4·403754	4·388936	4·371635	4·351935	4·330195	41
37	4·397640	4·384535	4·368960	4·350717	4·329894	4·306877	42
38	4·378013	4·364277	4·347878	4·328605	4·306559	4·282146	43
39	4·357310	4·342888	4·325588	4·305192	4·281809	4·255873	44
40	4·335434	4·320262	4·301979	4·280357	4·255515	4·227918	45
41	4·312278	4·296287	4·276931	4·253971	4·227537	4·198125	46
42	4·287723	4·270839	4·250312	4.225892	4·197720	4·166329	47
43	4·261643	4·243783	4·221979	4·195965	4·165896	4·132344	48
44	4·233898	4·214974	4·191774	4·164021	4·131882	4·095972	49
45	4·204336	4·184250	4·159528	4·129876	4·095478	4·056993	50
46	4·172789	4·151436	4·125052	4·093328	4·056464	4·015170	51
47	4·139078	4·116341	4·088143	4·054157	4·014602	3·970241	52
48	4·103004	4·078756	4·048579	4·012125	3·969632	3·921925	53
49	4·064351	4·038455	4·006118	3·966969	3·921270	3·869910	54

TABLE XIII (*continued*).

FEMALE ANNUITANT EXPERIENCE.

Second Series Table. Logarithms of the graduated Numbers living.

O^{af^2}.

Age at Entry. [x]	Years Elapsed since Date of Purchase:— 0	1	2	3	4	5 or more.	Age attained. (x+5)
	$\log l_{[x]}$	$\log l_{[x]+1}$	$\log l_{[x]+2}$	$\log l_{[x]+3}$	$\log l_{[x]+4}$	$\log l_{x+5}$	
50	4·022884	3·995188	3·960492	3·918403	3·869205	3·813858	55
51	3·978344	3·948684	3·911415	3·866116	3·813099	3·753400	56
52	3·930450	3·898648	3·858570	3·809768	3·752582	3·688134	57
53	3·878898	3·844756	3·801612	3·748987	3·687251	3·617620	58
54	3·823350	3·786655	3·740165	3·683367	3·616665	3·541377	59
55	3·763444	3·723962	3·673820	3·612467	3·540346	3·458882	60
56	3·698778	3·656255	3·602127	3·535804	3·457767	3·369564	61
57	3·628920	3·583077	3·524598	3·452850	3·368356	3·272798	62
58	3·553393	3·503927	3·440700	3·363030	3·271490	3·167903	63
59	3·471680	3·418259	3·349850	3·265717	3·166485	3·054136	64
60	3·383215	3·325478	3·251412	3·160225	3·052599	2·930686	65
61	3·287380	3·224932	3·144692	3·045806	2·929018	2·796668	66
62	3·183500	3·115912	3·028934	2·921645	2·794858	2·651114	67
63	3·070843	2·997642	2·903310	2·786849	2·649148	2·492970	68
64	2·948603	2·869278	2·766919	2·640448	2·490835	2·321085	69
65	2·815906	2·729895	2·618776	2·481379	2·318765	2·134203	70
66	2·671794	2·578488	2·457806	2·308485	2·131681	1·930952	71
67	2·515224	2·413954	2·282736	2·120501	1·928209	1·709835	72
68	2·345057	2·235096	2·092588	1·916047	1·706851	1·469219	73
69	2·160049	2·040602	1·885662	1·693618	1·465972	1·207319	74
70	1·958844	1·829044	1·660534	1·451569	1·203786	0·922192	75
71	1·739960	1·598859	1·415540	1·188106	0·918346	0·611713	76
72	1·501781	1·348347	1·148864	0·901272	0·607525	0·273563	77
73	1·242543	1·075647	0·858522	0·588930	0·269002	$\bar{1}$·905210	78
74	0·960318	0·778731	0·542352	0·248747	$\bar{1}$·900243	$\bar{1}$·503897	79
75	0·653008	0·455386	0·197990	$\bar{1}$·878176	$\bar{1}$·498486	$\bar{1}$·066607	80
76	0·318216	0·103192	$\bar{1}$·822878	$\bar{1}$·474441	$\bar{1}$·060710	$\bar{2}$·590050	81
77	$\bar{1}$·953740	$\bar{1}$·719613	$\bar{1}$·414146	$\bar{1}$·034507	$\bar{2}$·583625	$\bar{2}$·070636	82
78	$\bar{1}$·556550	$\bar{1}$·301470	$\bar{2}$·968778	$\bar{2}$·555065	$\bar{2}$·063633	$\bar{3}$·504447	83
79	$\bar{1}$·123760	$\bar{2}$·845920	$\bar{2}$·483407	$\bar{2}$·032503	$\bar{3}$·496814	$\bar{4}$·887203	84

TABLE XIV.—UNIFORM SENIORITY. ANNUITY EXPERIENCE. *and $(x+h)$, experiencing respectively the mortality shown by the first, below, the number of years (t) to be added to the age of the younger life, respect of the same combination. Applicable at all rates of interest, and*

Value of h.	Male and Female.				Two Females.			Value of h.
	Male the Younger.		Male the Elder					
	$O^{am}f^{1}$	$O^{am}f^{2}$	$O^{af^{1}m}$	$O^{af^{2}m}$	$O^{af^{1}f^{1}}, O^{af^{2}f^{2}}$	$O^{af^{1}f^{2}}$	$O^{af^{2}f^{1}}$	
	Values of t.							
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0	0·00	0·00	·00	·00	0·00	0·00	0·00	0
1	0·49	0·90	·53	·11	0·51	0·90	0·10	1
2	1·00	1·80	1·09	·24	1·04	1·81	0·22	2
3	1·53	2·71	1·66	·37	1·60	2·73	0·34	3
4	2·09	3·63	2·26	·51	2·17	3·65	0·47	4
5	2·66	4·55	2·88	·66	2·77	4·58	0·62	5
6	3·26	5·48	3·52	·83	3·39	5·52	0·77	6
7	3·88	6·41	4·18	1·01	4·03	6·46	0·94	7
8	4·52	7·35	4·85	1·20	4·69	7·40	1·11	8
9	5·17	8·30	5·55	1·40	5·36	8·35	1·30	9
10	5·85	9·25	6·26	1·62	6·06	9·31	1·51	10
11	6·55	10·20	7·00	1·86	6·78	10·26	1·73	11
12	7·26	11·16	7·75	2·11	7·51	11·22	1·96	12
13	8·00	12·12	8·51	2·38	8·26	12·19	2·21	13
14	8·75	13·08	9·29	2·66	9·02	13·16	2·48	14
15	9·51	14·05	10·09	2·97	9·80	14·12	2·77	15
16	10·29	15·02	10·90	3·29	10·60	15·10	3·07	16
17	11·09	15·99	11·72	3·63	11·41	16·07	3·39	17
18	11·90	16·97	12·55	3·99	12·23	17·05	3·73	18
19	12·72	17·94	13·40	4·37	13·06	18·03	4·10	19
20	13·55	18·92	14·26	4·78	13·91	19·01	4·48	20
21	14·40	19·90	15·12	5·20	14·77	19·99	4·88	21
22	15·26	20·88	16·00	5·65	15·63	20·97	5·30	22
23	16·12	21·87	16·89	6·11	16·51	21·96	5·75	23
24	17·00	22·85	17·78	6·60	17·40	22·94	6·22	24
25	17·89	23·84	18·69	7·11	18·29	23·92	6·71	25
26	18·78	24·82	19·60	7·64	19·20	24·91	7·22	26
27	19·69	25·81	20·51	8·20	20·11	25·90	7·75	27
28	20·60	26·8[illegible]	21·44	8·78	21·02	26·90	8·30	28
29	21·51	27·79	22·37	9·37	21·95	27·89	8·88	29
30	22·44	28·78	23·30	9·99	22·88	28·88	9·48	30
31	23·37	29·77	24·24	10·63	23·81	29·87	10·10	31
32	24·30	30·76	25·19	11·29	24·75	30·86	10·73	32
33	25·24	31·76	26·14	11·97	25·70	31·86	11·39	33
34	26·19	32·75	27·09	12·66	26·65	32·85	12·07	34
35	27·13	33·74	28·05	13·38	27·60	33·84	12·77	35
36	28·09	34·74	29·01	14·11	28·56	34·84	13·48	36
37	29·05	35·73	29·97	14·85	29·52	35·84	14·21	37
38	30·01	36·73	30·94	15·62	30·48	36·83	14·96	38
39	30·97	37·73	31·91	16·40	31·45	37·83	15·73	39
40	31·94	38·72	32·88	17·20	32·42	38·82	16·51	40
41	32·91	39·72	33·85	18·01	33·39	39·82	17·30	41
42	33·88	40·72	34·83	18·83	34·36	40·82	18·11	42
43	34·85	41·71	35·81	19·66	35·34	41·81	18·94	43
44	35·83	42·71	36·79	20·51	36·32	42·81	19·77	44

TWO LIVES. *Showing, in the case of an annuity on two lives (x) and the second, Tables entering into either of the combinations specified to obtain the equal ages giving an equivalent joint annuity value, in in respect of all (identical) durations from date of purchase.*

Value of h.	Male and Female.				Two Females.			Value of h.
	Male the Younger.		Male the Elder					
	$Oam.f^1$	$Oam.f^2$	$Oaf^1m.$	$Oaf^2m.$	Oaf^1f^1, Oaf^2f^2	Oaf^1f^2	Oaf^2f^1	
	Values of t.							
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
45	36·81	43·71	37·77	21·37	37·30	43·81	20·62	45
46	37·79	44·70	38·75	22·24	38·28	44·81	21·47	46
47	38·77	45·70	39·74	23·11	39·26	45·81	22·34	47
48	39·75	46·70	40·72	24·00	40·25	46 80	23·22	48
49	40·74	47·70	41·71	24·90	41·23	47·80	24·10	49
50	41·72	48·70	42·70	25·80	42·22	48·80	25·00	50
51	42·71	49·70	43·69	26·71	43·21	49·80	25·90	51
52	43·70	50·69	44·68	27·63	44·20	50·80	26·81	52
53	44·69	51·69	45·67	28·55	45·19	51·80	27·73	53
54	45·68	52·69	46·66	29·48	46·18	52·80	28·63	54
55	46·67	53·69	47·65	30·41	47·17	53·80	29·58	55
56	47·66	54·69	48·64	31·35	48 16	54·79	30·52	56
57	48·65	55·69	49·64	32·30	49·16	55·79	31·46	57
58	49·64	56·69	50·63	33·25	50·15	56·79	32 40	58
59	50·64	57·69	51·63	34·20	51·15	57·79	33·35	59
60	51·63	58·69	52·62	35·16	52·14	58·79	34·31	60
61	52·63	59·69	53·62	36·12	53·13	59·79	35·26	61
62	53·62	60·69	54·61	37·08	54·13	60·79	36·22	62
63	54·62	61·69	55·61	38·05	55·12	61·79	37·19	63
64	55·61	62·68	56·61	39·02	56·12	62·79	38·16	64
65	56·61	63·68	57·60	39·99	57·12	63·79	39·12	65
66	57·61	64·68	58·60	40·97	58·11	64·79	40·10	66
67	58·60	65·68	59·60	41·94	59·11	65·79	41·07	67
68	59·60	66·68	60·59	42·92	60·11	66·79	42·05	68
69	60·60	67·68	61·59	43·90	61·11	67·79	43·03	69
70	61·59	68·68	62·59	44·88	62 10	68·79	44·01	70
71	62·59	69·68	63·59	45·87	63·10	69·79	44·99	71
72	63·59	70·68	64·59	46·85	64·10	70·79	45·97	72
73	64·59	71·68	65·58	47·84	65·10	71·79	46·95	73
74	65·59	72·68	66·58	48·82	66·10	72·79	47·94	74
75	66·58	73·68	67·58	49·81	67·09	73·79	48·93	75
76	67·58	74·68	68·58	50·80	68·09	74·79	49·92	76
77	68·58	75·68	69·58	51 79	69·09	75·79	50·90	77
78	69·58	76·68	70·58	52·78	70·09	76·79	51·89	78
79	70 58	77·68	71·58	53·77	71·09	77·79	52·89	79
80	71·58	78·68	72·58	54·76	72·09	78·79	53·88	80
81	72·58	79·68	73·58	55·76	73·09	79·79	54·87	81
82	73·58	80·68	74·58	56·75	74·09	80·79	55·86	82
83	74·58	81·68	75·57	57·74	75·09	81·79	56·86	83

Table XV.—Select Annuity Values.

Two Joint Lives of Equal Age.

$a_{[x][x]}$ 3 %

Age at Entry. [x]	Male and Female.		Two Females.			Age at Entry. [x]
	$a^{[am.f^1]}$	$a^{[am.f^2]}$	$a^{[af^1f^2]}$	$a^{[af^1f^1]}$	$a^{[af^2f^2]}$	
20	23·622	13·027	15·064	30·315	10·291	20
21	23·327	12·814	14·778	29·828	10·096	21
22	23·026	12·597	14·487	29·336	9·897	22
23	22·721	12·373	14·191	28·840	9·693	23
24	22·408	12·145	13·891	28·339	9·485	24
25	22·089	11·912	13·586	27·834	9·273	25
26	21·765	11·674	13·277	27·325	9·057	26
27	21·435	11·431	12·965	26·811	8·838	27
28	21·100	11·185	12·650	26·294	8·615	28
29	20·759	10·934	12·331	25·774	8·389	29
30	20·413	10·680	12·010	25·250	8·162	30
31	20·060	10·421	11·686	24·723	7·931	31
32	19·703	10·160	11·361	24·193	7·699	32
33	19·340	9·896	11·034	23·661	7·465	33
34	18·973	9·629	10·707	23·126	7·230	34
35	18·601	9·360	10·378	22·590	6·994	35
36	18·224	9·089	10·050	22·052	6·758	36
37	17·843	8·816	9·721	21·512	6·523	37
38	17·458	8·543	9·394	20·972	6·287	38
39	17·070	8·269	9·068	20·432	6·053	39
40	16·677	7·995	8·744	19·891	5·821	40
41	16·282	7·721	8·422	19·350	5·590	41
42	15·884	7·448	8·102	18·810	5·361	42
43	15·485	7·177	7·786	18·271	5·135	43
44	15·082	6·906	7·474	17·734	4·912	44
45	14·677	6·639	7·166	17·199	4·693	45
46	14·272	6 374	6·863	16·666	4·477	46
47	13·866	6·112	6·564	16·137	4·266	47
48	13·460	5·853	6·272	15·611	4·059	48
49	13·054	5·598	5·984	15·089	3·856	49
50	12·648	5·348	5·704	14·571	3·659	50
51	12·243	5·102	5·429	14·059	3·467	51
52	11·840	4·862	5·162	13·552	3·280	52
53	11·438	4·626	4·901	13·052	3·099	53
54	11·040	4·397	4·648	12·557	2·923	54
55	10·645	4·173	4·403	12·070	2·754	55
56	10·253	3·955	4·165	11·590	2·590	56
57	9·865	3·744	3·935	11·119	2·432	57
58	9·482	3·539	3·713	10·656	2·280	58
59	9·104	3·340	3·499	10·202	2·134	59
60	8·732	3·149	3·292	9·757	1·994	60
61	8·365	2·964	3·094	9·322	1·859	61
62	8·005	2·786	2·904	8 897	1·731	62
63	7·652	2·614	2·721	8·482	1·608	63
64	7·306	2·450	2·546	8·078	1·490	64

TABLE XV. (*continued*).—SELECT ANNUITY VALUES.

Two Joint Lives of Equal Age.

$a_{[x][x]}$ 3 %

Age at Entry. [x]	Male and Female.		Two Females.			Age at Entry. [x]
	$a^{[amf^1]}$	$a^{[amf^2]}$	$a^{[af^1f^2]}$	$a^{[af^1f^1]}$	$a^{[af^2f^2]}$	
65	6·969	2·292	2·379	7·686	1·378	65
66	6·638	2·141	2·219	7·304	1·272	66
67	6·317	1·996	2·067	6·934	1·171	67
68	6·004	1·858	1·922	6·575	1·075	68
69	5·699	1·726	1·784	6·228	·984	69
70	5·404	1·600	1·652	5·893	·898	70
71	5·118	1·480	1·528	5·571	·817	71
72	4·842	1·367	1·409	5·259	·740	72
73	4·575	1·259	1·297	4·960	·668	73
74	4·317	1·157	1·191	4·673	·600	74
75	4·069	1·060	1·090	4·397	·538	75
76	3·831	·968	·996	4·133	·479	76
77	3·602	·882	·907	3·880	·424	77
78	3·382	·800	·823	3·639	·373	78
79	3·172	·724	·744	3·408	·326	79
80	2·971	...	...	3·188	..	80
81	2·779	...	...	2·979	...	81
82	2·595	...	...	2·779	...	82
83	2·421	...	...	2·590	...	83
84	2·254	...	...	2·409	...	84
85	2·096	...	...	2·239	...	85
86	1·945	...	...	2·078	...	86
87	1·803	...	...	1·924	...	87
88	1·667	...	...	1·779	...	88
89	1·539	...	...	1·642	...	89
90	1·417	...	...	1·512	...	90
91	1·303	...	...	1·390	...	91
92	1·194	...	...	1·275	...	92
93	1·092	...	...	1·166	...	93
94	·996	...	...	1·064	...	94
95	·905	...	...	·968	...	95
96	·820	...	...	·878	...	96
97	·740	...	...	·793	...	97
98	·666	...	...	·714	...	98
99	·596	...	...	·641	...	99

Table XVI.—Ultimate Annuity Values.

Two Joint Lives of Equal Age.

$a_{x.x.}$ 3 %

Age. (x)	Male and Female.		Two Females.			Age. (x)
	O^{amf^1}	O^{amf^2}	$O^{af^1f^2}$	$O^{af^1f^1}$	$O^{af^2f^2}$	
25	21·744	11·601	13·296	27·543	8·981	25
26	21·422	11·361	12·985	27·035	8·761	26
27	21·094	11·117	12·670	26·523	8·538	27
28	20·760	10·869	12·352	26·007	8·312	28
29	20·420	10·615	12·030	25·487	8·082	29
30	20·076	10·358	11·705	24·964	7·849	30
31	19·725	10·097	11·378	24·438	7·613	31
32	19·369	9·833	11·048	23·908	7·376	32
33	19·007	9·565	10·718	23·376	7·137	33
34	18·641	9·295	10·385	22·841	6·897	34
35	18·268	9·021	10·051	22·304	6·655	35
36	17·893	8·747	9·718	21·765	6·413	36
37	17·512	8·470	9·384	21·225	6·171	37
38	17·127	8·192	9·051	20·683	5·930	38
39	16·738	7·913	8·719	20·141	5·689	39
40	16·345	7·634	8·388	19·599	5·450	40
41	15·949	7·355	8·060	19·055	5·213	41
42	15·550	7·076	7·734	18·512	4·978	42
43	15·148	6·799	7·411	17·971	4·746	43
44	14·744	6·523	7·092	17·431	4·517	44
45	14·339	6·250	6·777	16·892	4·292	45
46	13·931	5·979	6·467	16·356	4·071	46
47	13·522	5·711	6·162	15·822	3·854	47
48	13·113	5·446	5·862	15·291	3·642	48
49	12·703	5·186	5·569	14·764	3·435	49
50	12·293	4·930	5·281	14·241	3·234	50
51	11·885	4·679	5·001	13·724	3·039	51
52	11·477	4·433	4·728	13·211	2·850	52
53	11·072	4·193	4·461	12·704	2·667	53
54	10·669	3·959	4·204	12·203	2·490	54
55	10·268	3·731	3·954	11·710	2·320	55
56	9·871	3·510	3·712	11·223	2·157	56
57	9·478	3·295	3·478	10·744	2·000	57
58	9·089	3·088	3·254	10·274	1·851	58
59	8·705	2·888	3·038	9·812	1·708	59
60	8·326	2·696	2·830	9·360	1·572	60
61	7·954	2·511	2·632	8·917	1·443	61
62	7·588	2·334	2·442	8·484	1·321	62
63	7·228	2·164	2·261	8·062	1·205	63
64	6·876	2·002	2·089	7·651	1·097	64
65	6·532	1·848	1·926	7·250	·994	65
66	6·196	1·701	1·771	6·861	·898	66
67	5·868	1·562	1·624	6·484	·808	67
68	5·550	1·431	1·486	6·119	·724	68
69	5·240	1·306	1·355	5·766	·646	69

TABLE XVI. (*continued*).—ULTIMATE ANNUITY VALUES.

Two Joint Lives of Equal Age.

$a_{x.x.}$ 3 %

Age. (x)	MALE AND FEMALE.		TWO FEMALES.			Age. (x)
	O^{amf^1}	O^{amf^2}	$O^{af^1f^2}$	$O^{af^1f^1}$	$O^{af^2f^2}$	
70	4·940	1·189	1·233	5·425	·574	70
71	4·649	1·079	1·118	5·096	·507	71
72	4·369	·976	1·010	4·779	·445	72
73	4·098	·880	·910	4·476	·389	73
74	3·838	·790	·816	4·185	·337	74
75	3·587	·706	·729	3·906	·290	75
76	3·347	·628	·649	3·640	·247	76
77	3·118	·556	·574	3·386	·209	77
78	2·898	·490	·506	3·144	·175	78
79	2·689	·429	·443	2·914	·145	79
80	2·490	·373	·385	2·695	·118	80
81	2·300	·322	·333	2·488	·095	81
82	2·121	·276	·285	2·292	·076	82
83	1·951	·234	·242	2·107	·057	83
84	1·790	·195	·201	1·933	...	84
85	1·639	·145	·150	1·769	...	85
86	1·496	...	...	1·615	...	86
87	1·362	...	...	1·470	...	87
88	1·237	...	...	1·335	...	88
89	1·119	...	...	1·208	...	89
90	1·010	...	...	1·091	...	90
91	·908	...	...	·982	...	91
92	·813	...	...	·880	...	92
93	·725	...	...	·786	...	93
94	·644	...	...	·699	...	94
95	·569	...	...	·618	...	95
96	·500	...	...	·545	...	96
97	·437	..	...	·477	...	97
98	·380	...	...	·415	...	98
99	·327	...	...	·359	...	99
100	·280	...	...	·308	...	100
101	·237	...	...	·262	...	101
102	·196	...	...	·217	...	102
103	·146	...	...	·161	...	103

Table XVII.—Select Annuity Values.

Two Joint Lives of Equal Age.

$a_{[x][x]}$ 4 %

Age [x]	Male and Female.		Two Females.			Age. [x]
	$a_{[amf^1]}$	$a_{[amf^2]}$	$a_{[af^1f^2]}$	$a_{[af^1f^1]}$	$a_{[af^2f^2]}$	
20	20·016	11·749	13·460	25·092	9·450	20
21	19·811	11·577	13·233	24·763	9·287	21
22	19·601	11·401	13·000	24·427	9·119	22
23	19·386	11·220	12·762	24·087	8·946	23
24	19·164	11·034	12·520	23·740	8·769	24
25	18·937	10·842	12·272	23·387	8·587	25
26	18·703	10·645	12·020	23·029	8·402	26
27	18·464	10·444	11·763	22·665	8·213	27
28	18·219	10·239	11·503	22·296	8·020	28
29	17·970	10·027	11·238	21·921	7·824	29
30	17·713	9·814	10·970	21·541	7·625	30
31	17·451	9·595	10·698	21·156	7·422	31
32	17·182	9·373	10·424	20·767	7·218	32
33	16·908	9·147	10·147	20·372	7·011	33
34	16·628	8·918	9·867	19·973	6·802	34
35	16·345	8·686	9·586	19·569	6·592	35
36	16·053	8·451	9·303	19·162	6·381	36
37	15·759	8·214	9·019	18·750	6·169	37
38	15·459	7·975	8·735	18·335	5·957	38
39	15·153	7·734	8·450	17·918	5·745	39
40	14·843	7·492	8·165	17·496	5·533	40
41	14·528	7·249	7·881	17·072	5·323	41
42	14·210	7·007	7·599	16·646	5·114	42
43	13·887	6·764	7·318	16·218	4·906	43
44	13·561	6·522	7·039	15·789	4·701	44
45	13·232	6·281	6·762	15·358	4·498	45
46	12·899	6·041	6·489	14·927	4·298	46
47	12·564	5·803	6·220	14·495	4·101	47
48	12·227	5·568	5·954	14·064	3·908	48
49	11·888	5 335	5·692	13·633	3·718	49
50	11·548	5·105	5·435	13·204	3·533	50
51	11·207	4·879	5·183	12·776	3·352	51
52	10·865	4·657	4·937	12·351	3·176	52
53	10·523	4·439	4·696	11·928	3·004	53
54	10·182	4·225	4·461	11·509	2·838	54
55	9·841	4·016	4·232	11·093	2·676	55
56	9·502	3·812	4·010	10·681	2·520	56
57	9·164	3·614	3·795	10·274	2·369	57
58	8·829	3·421	3·586	9·872	2·223	58
59	8·498	3·234	3·384	9·476	2·083	59
60	8·169	3·052	3·189	9·086	1·948	60
61	7·844	2·877	3·001	8·703	1·818	61
62	7·523	2·707	2·820	8·327	1·694	62
63	7·207	2·544	2·646	7·958	1·575	63
64	6·897	2·386	2·479	7·598	1·461	64

TABLE XVII. (*continued*).—SELECT ANNUITY VALUES

Two Joint Lives of Equal Age.

$a_{[x][x]}$ 4 %

Age. [x]	Male and Female.		Two Females.			Age. [x]
	$O^{[amf^1]}$	$O^{[amf^2]}$	$O^{[af^1f^2]}$	$O^{[af^1f^1]}$	$O^{[af^2f^2]}$	
65	6·592	2·235	2·319	7·245	1·353	65
66	6·293	2·089	2·166	6·901	1·249	66
67	6·000	1·951	2·019	6·566	1·151	67
68	5·714	1·817	1·879	6·240	1·057	68
69	5·435	1·690	1·746	5·924	·968	69
70	5·164	1·568	1·619	5·617	·884	70
71	4·900	1·452	1·498	5·320	·805	71
72	4·643	1·342	1·383	5·032	·730	72
73	4·395	1·237	1·274	4·755	·659	73
74	4·155	1·137	1·170	4·488	·593	74
75	3·922	1·043	1·072	4·230	·530	75
76	3·698	·953	·980	3·983	·473	76
77	3·483	·869	·893	3·746	·419	77
78	3·275	·789	·810	3·518	·369	78
79	3·076	·713	·733	3·300	·322	79
80	2·885	...	...	3·092	...	80
81	2·702	...	...	2·892	...	81
82	2·526	...	...	2·703	...	82
83	2·359	...	...	2·522	...	83
84	2·199	...	...	2·349	...	84
85	2·047	...	...	2·185	...	85
86	1·902	...	...	2·030	...	86
87	1·764	...	...	1·882	...	87
88	1·633	...	...	1·742	...	88
89	1·509	...	...	1·609	...	89
90	1·391	...	...	1·483	...	90
91	1·279	...	...	1·365	...	91
92	1·174	...	...	1·252	...	92
93	1·074	...	...	1·146	...	93
94	·980	...	...	1·047	...	94
95	·891	...	...	·953	...	95
96	·808	...	...	·865	...	96
97	·730	...	...	·782	...	97
98	·657	...	...	·704	...	98
99	·588	...	...	·632	...	99

Table XVIII.—Ultimate Annuity Values.

Two Joint Lives of Equal Age.

$a_{x.x.}$ 4 %

Age. (x)	Male and Female.		Two Females.			Age. (x)
	O^{amf^1}	O^{amf^2}	$O^{af^1f^2}$	$O^{af^1f^1}$	$O^{af^2f^2}$	
25	18·643	10·560	12·012	23·144	8·319	25
26	18·411	10·363	11·757	22·786	8·130	26
27	18·173	10·159	11·498	22·423	7·937	27
28	17·929	9·951	11·234	22·053	7·739	28
29	17·678	9·738	10·965	21·679	7·539	29
30	17·423	9·521	10·694	21·299	7·335	30
31	17·160	9·299	10·418	20·914	7·127	31
32	16·893	9·073	10·139	20·523	6·917	32
33	16·618	8·844	9·857	20·128	6·705	33
34	16·340	8·611	9·573	19·728	6·491	34
35	16·054	8·374	9·286	19·324	6·274	35
36	15·764	8·135	8·998	18·915	6·057	36
37	15·467	7·893	8·708	18·501	5·839	37
38	15·167	7·649	8·417	18·085	5·620	38
39	14·860	7·403	8·126	17·664	5·402	39
40	14·550	7·156	7·835	17·241	5·183	40
41	14·233	6·908	7·545	16·814	4·966	41
42	13·913	6·659	7·255	16·385	4·751	42
43	13·588	6·411	6·967	15·953	4·536	43
44	13·260	6·163	6·681	15·520	4·325	44
45	12·928	5·915	6·398	15·086	4·116	45
46	12·593	5·669	6·117	14·650	3·910	46
47	12·255	5·425	5·840	14·214	3·707	47
48	11·914	5·183	5·567	13·778	3·509	48
49	11·571	4·944	5·299	13·342	3·315	49
50	11·227	4·709	5·035	12·907	3·125	50
51	10·881	4·477	4·776	12·473	2·940	51
52	10·535	4·248	4·524	12·042	2·761	52
53	10·188	4·025	4·277	11·612	2·587	53
54	9·841	3·806	4·037	11·186	2·419	54
55	9·495	3·593	3·803	10·763	2·257	55
56	9·150	3·385	3·576	10·344	2·100	56
57	8·807	3·184	3·357	9·930	1·950	57
58	8·466	2·988	3·145	9·521	1·806	58
59	8·127	2·798	2·940	9·116	1·669	59
60	7·792	2·615	2·744	8·719	1·538	60
61	7·460	2·439	2·555	8·327	1·413	61
62	7·133	2·270	2·374	7·943	1·295	62
63	6·810	2·108	2·201	7·566	1·182	63
64	6·493	1·952	2·036	7·198	1·077	64
65	6·181	1·804	1·879	6·837	·977	65
66	5·875	1·663	1·730	6·485	·883	66
67	5·577	1·528	1·588	6·143	·795	67
68	5·285	1·401	1·454	5·809	·713	68
69	5·000	1·281	1·328	5·486	·637	69

TABLE XVIII. (*continued*).—ULTIMATE ANNUITY VALUES.

Two Joint Lives of Equal Age.

$a_{x.x.}$ 4 %

Age. (x)	Male and Female.		Two Females.			Age. (x)
	O^{amf^1}	O^{amf^2}	$O^{af^1f^2}$	$O^{af^1f^1}$	$O^{af^2f^2}$	
70	4·722	1·167	1·209	5·172	·566	70
71	4·453	1·060	1·097	4·869	·500	71
72	4·192	·959	·993	4·576	·439	72
73	3·939	·865	·895	4·294	·384	73
74	3·695	·777	·803	4·022	·333	74
75	3·460	·695	·718	3·761	·286	75
76	3·234	·619	·639	3·511	·245	76
77	3·017	·548	·566	3·271	·207	77
78	2·809	·483	·499	3·042	·173	78
79	2·609	·423	·437	2·824	·143	79
80	2·420	·369	·380	2·616	·117	80
81	2·239	·319	·329	2·418	·094	81
82	2·066	·273	·282	2·231	·075	82
83	1·903	·232	·239	2·054	·056	83
84	1·749	·192	·199	1·886	...	84
85	1·602	·144	·148	1·728	...	85
86	1·465	...	...	1·579	...	86
87	1·335	...	...	1·440	...	87
88	1·213	...	...	1·309	...	88
89	1·099	...	...	1·186	...	89
90	·992	...	...	1·071	...	90
91	·893	...	...	·965	...	91
92	·800	...	...	·865	...	92
93	·714	...	...	·773	...	93
94	·634	...	...	·688	...	94
95	·561	...	...	·609	...	95
96	·493	...	..	·537	...	96
97	·431	...	...	·471	...	97
98	·375	...	...	·410	...	98
99	·323	...	...	·355	...	99
100	·277	...	...	·305	...	100
101	·234	...	...	·259	..	101
102	·194	...	...	·215	...	102
103	·145	...	...	·159	...	103

Table XIX.—Select Annuity Values.

Two Joint Lives of Equal Age.

$a_{[x][x]}$ 5 %

Age. [x]	Male and Female.		Two Females.			Age. [x]
	$O^{[am/f^1]}$	$O^{[am/f^2]}$	$O^{[af^1/f^2]}$	$O^{[af^1/f^1]}$	$O^{[af^2/f^2]}$	
20	17·233	10·671	12·122	21·158	8·722	20
21	17·089	10·532	11·940	20·931	8·584	21
22	16·939	10·388	11·751	20·698	8·441	22
23	16·785	10·239	11·559	20·459	8·294	23
24	16·625	10·085	11·361	20·215	8·142	24
25	16·459	9·927	11·158	19·964	7·986	25
26	16·289	9·763	10·950	19·708	7·825	26
27	16·112	9·594	10·737	19·445	7·661	27
28	15·931	9·421	10·520	19·178	7·493	28
29	15·743	9·244	10·299	18·904	7·322	29
30	15·551	9·062	10·073	18·624	7·147	30
31	15·352	8·875	9·843	18·339	6·968	31
32	15·148	8·685	9·610	18·048	6·787	32
33	14·939	8·491	9·373	17·751	6·603	33
34	14·723	8·292	9·133	17·449	6·417	34
35	14·502	8·091	8·891	17·141	6·229	35
36	14·276	7·886	8·646	16·829	6·039	36
37	14·044	7·678	8·398	16·511	5·848	37
38	13·807	7·468	8·150	16·189	5·656	38
39	13·564	7·256	7·900	15·861	5·463	39
40	13·316	7·042	7·649	15·529	5·270	40
41	13·063	6·826	7·397	15·194	5·078	41
42	12·806	6·609	7·146	14·853	4·886	42
43	12·543	6·391	6·895	14·510	4·694	43
44	12·276	6·173	6·645	14·163	4·505	44
45	12·006	5·955	6·396	13·814	4·317	45
46	11·731	5·738	6·150	13·461	4·131	46
47	11·452	5·521	5·905	13·106	3·947	47
48	11·170	5·306	5·663	12·750	3·767	48
49	10·886	5·093	5·424	12·391	3·589	49
50	10·598	4·881	5·188	12·033	3·415	50
51	10·308	4·673	4·956	11·674	3·244	51
52	10·016	4·467	4·728	11·313	3·077	52
53	9·723	4·264	4·505	10·954	2·915	53
54	9·429	4·065	4·287	10·596	2·756	54
55	9·133	3·870	4·073	10·238	2·602	55
56	8·838	3·679	3·865	9·883	2·453	56
57	8·543	3·492	3·663	9·530	2·309	57
58	8·249	3·310	3·467	9·179	2·169	58
59	7·956	3·133	3·276	8·832	2·034	59
60	7·665	2·961	3·091	8·489	1·904	60
61	7·375	2·794	2·913	8·150	1·779	61
62	7·089	2·633	2·741	7·816	1·659	62
63	6·805	2·477	2·575	7·487	1·544	63
64	6·525	2·326	2·415	7·163	1·434	64

TABLE XIX. (*continued*).—SELECT ANNUITY VALUES.

Two Joint Lives of Equal Age.

$a_{[x][x]}$ 5 %

Age. [x]	Male and Female.		Two Females.			Age. [x]
	$O^{[am f^1]}$	$O^{[am f^2]}$	$O^{[af^1 f^2]}$	$O^{[af^1 f^1]}$	$O^{[af^2 f^2]}$	
65	6·249	2·181	2·262	6·846	1·328	65
66	5·977	2·041	2·114	6·535	1·227	66
67	5·710	1·907	1·973	6·231	1·131	67
68	5·448	1·778	1·839	5·933	1·040	68
69	5·191	1·655	1·709	5·644	·953	69
70	4·941	1·537	1·586	5·362	·871	70
71	4·697	1·424	1·469	5·088	·793	71
72	4·458	1·317	1·357	4·822	·719	72
73	4·227	1·215	1·251	4·565	·650	73
74	4·002	1·118	1·150	4·315	·585	74
75	3·784	1·026	1·055	4·075	·524	75
76	3·574	·938	·965	3·843	·467	76
77	3·370	·856	·879	3·619	·414	77
78	3·174	·777	·799	3·404	·364	78
79	2·985	·704	·723	3·198	·319	79
80	2·803	...	...	3·000	...	80
81	2·628	...	...	2·811	...	81
82	2·461	...	...	2·630	...	82
83	2·300	...	...	2·456	...	83
84	2·147	...	...	2·291	...	84
85	2·001	...	...	2·134	...	85
86	1·861	...	...	1·984	...	86
87	1·727	...	...	1·841	...	87
88	1·600	...	...	1·706	...	88
89	1·480	...	...	1·577	...	89
90	1·365	...	...	1·455	...	90
91	1·257	...	...	1·340	...	91
92	1·154	...	...	1·230	...	92
93	1·056	...	...	1·127	...	93
94	·965	...	...	1·030	...	94
95	·878	...	...	·938	...	95
96	·796	...	...	·852	...	96
97	·720	...	...	·771	...	97
98	·648	...	...	·695	...	98
99	·580	...	...	·624	...	99

Table XX.—Ultimate Annuity Values.

Two Joint Lives of Equal Age.

a_{xx} 5 %

Age. (x)	Male and Female.		Two Females.			Age. (x)
	O^{amf^1}	O^{amf^2}	$O^{af^1f^2}$	$O^{af^1f^1}$	$O^{af^2f^2}$	
25	16·206	9·671	10·923	19·758	7·738	25
26	16·036	9·505	10·712	19·502	7·574	26
27	15·860	9·334	10·497	19·240	7·405	27
28	15·678	9·159	10·276	18·971	7·233	28
29	15·491	8·978	10·051	18·697	7·056	29
30	15·299	8·793	9·821	18·416	6·876	30
31	15·099	8·603	9·587	18·130	6·693	31
32	14·895	8·409	9·349	17·838	6·506	32
33	14·685	8·211	9·108	17·540	6·317	33
34	14·470	8·009	8·862	17·236	6·125	34
35	14·247	7·803	8·614	16 927	5·930	35
36	14·020	7·593	8·364	16·613	5·734	36
37	13·787	7·381	8·111	16·293	5·537	37
38	13·548	7·165	7·856	15·968	5·338	38
39	13·304	6·947	7·600	15·638	5·139	39
40	13·055	6·728	7·342	15·304	4·939	40
41	12·800	6·506	7·084	14·965	4·740	41
42	12·540	6·283	6·825	14·622	4·541	42
43	12·275	6·059	6·567	14·275	4·343	43
44	12·006	5·835	6·310	13·924	4·147	44
45	11·732	5·611	6·054	13·571	3·952	45
46	11·454	5·387	5·799	13·214	3·760	46
47	11·172	5·164	5·547	12·854	3·571	47
48	10·886	4·942	5·297	12·492	3·384	48
49	10·597	4·722	5·051	12·129	3·202	49
50	10·305	4·504	4·808	11·764	3·023	50
51	10·010	4·289	4·569	11·398	2·848	51
52	9·714	4·077	4·335	11·032	2·678	52
53	9·415	3·869	4·105	10·666	2·512	53
54	9·115	3·665	3·881	10·301	2·352	54
55	8·815	3·464	3·662	9·936	2·197	55
56	8·513	3·269	3·449	9·573	2·047	56
57	8·212	3·078	3·243	9·212	1·903	57
58	7·911	2·893	3·042	8·854	1·764	58
59	7·611	2·713	2·849	8·499	1·631	59
60	7·313	2·539	2·662	8·148	1·505	60
61	7·017	2·371	2·482	7·800	1·384	61
62	6·723	2·209	2·309	7·458	1·269	62
63	6·432	2·054	2·144	7·120	1·160	63
64	6·145	1·905	1·985	6·788	1·057	64
65	5·862	1·762	1·834	6·463	·960	65
66	5·583	1·626	1·690	6·143	·868	66
67	5·309	1·496	1·554	5·831	·783	67
68	5·041	1·373	1·424	5·526	·702	68
69	4·778	1·256	1·302	5·229	·627	69

TABLE XX. (*continued*).—ULTIMATE ANNUITY VALUES.

Two Joint Lives of Equal Age.

a_{xx} 5 %

Age. (x)	Male and Female.		Two Females.			Age. (x)
	O^{amf^1}	O^{amf^2}	$O^{af^1f^2}$	$O^{af^1f^1}$	$O^{af^2f^2}$	
70	4·521	1·145	1·186	4·940	·558	70
71	4·271	1·041	1·078	4·659	·493	71
72	4·028	·943	·975	4·387	·434	72
73	3·791	·851	·880	4·124	·379	73
74	3·562	·765	·790	3·870	·329	74
75	3·341	·685	·707	3·625	·283	75
76	3·127	·610	·630	3·389	·242	76
77	2·922	·541	·558	3·163	·205	77
78	2·724	·477	·492	2·946	·171	78
79	2·534	·418	·431	2·739	·142	79
80	2·353	·364	·376	2·541	·116	80
81	2·180	·315	·325	2·352	·094	81
82	2·015	·270	·279	2·173	·074	82
83	1·858	·229	·237	2·003	·056	83
84	1·709	·190	·197	1·842	...	84
85	1·568	·142	·147	1·689	...	85
86	1·434	...	...	1·546	...	86
87	1·309	...	...	1·410	...	87
88	1·190	...	...	1·283	...	88
89	1·079	...	...	1·164	...	89
90	·975	...	...	1·052	...	90
91	·878	...	...	·948	...	91
92	·787	...	...	·851	...	92
93	·703	...	...	·761	...	93
94	·625	...	...	·678	...	94
95	·553	...	...	·601	...	95
96	·487	...	...	·530	...	96
97	·426	...	...	·464	...	97
98	·370	...	...	·405	...	98
99	·319	...	...	·350	...	99
100	·273	...	...	·301	...	100
101	·232	...	...	·256	...	101
102	·192	...	...	·213	...	102
103	·143	...	...	·158	...	103

TABLE XXI.—FEMALE ANNUITANT EXPERIENCE.

Select and Ultimate Values of $\rho_x = \frac{l_x^{(1)}}{l_x^{(2)}}$ *and* $1-\rho_x = \frac{l_x^{(2)}}{l_x^{(1)}}$.

Age at Entry. [x]	Select Values. $\rho_{[x]}$	Select Values. $1-\rho_{[x]}$	Age. (x)	Ultimate Values. ρ_x	Ultimate Values. $1-\rho_x$
20	·57248	·42752	25	·60763	·39237
21	·57992	·42008	26	·61543	·38457
22	·58744	·41256	27	·62333	·37667
23	·59505	·40495	28	·63135	·36865
24	·60276	·39724	29	·63949	·36051
25	·61057	·38943	30	·64777	·35223
26	·61849	·38151	31	·65619	·34381
27	·62653	·37347	32	·66475	·33525
28	·63469	·36531	33	·67347	·32653
29	·64299	·35701	34	·68236	·31764
30	·65143	·34857	35	·69143	·30857
31	·66002	·33998	36	·70067	·29933
32	·66877	·33123	37	·71011	·28989
33	·67769	·32231	38	·71975	·28025
34	·68679	·31321	39	·72959	·27041
35	·69607	·30393	40	·73963	·26037
36	·70555	·29445	41	·74989	·25011
37	·71523	·28477	42	·76036	·23964
38	·72512	·27488	43	·77103	·22897
39	·73522	·26478	44	·78190	·21810
40	·74553	·25447	45	·79296	·20704
41	·75606	·24394	46	·80419	·19581
42	·76680	·23320	47	·81558	·18442
43	·77775	·22225	48	·82708	·17292
44	·78889	·21111	49	·83867	·16133
45	·80021	·19979	50	·85030	·14970
46	·81169	·18831	51	·86193	·13807
47	·82330	·17670	52	·87349	·12651
48	·83501	·16499	53	·88492	·11508
49	·84678	·15322	54	·89615	·10385
50	·85854	·14146	55	·90710	·09290
51	·87026	·12974	56	·91768	·08232
52	·88185	·11815	57	·92782	·07218
53	·89325	·10675	58	·93742	·06258
54	·90438	·09562	59	·94642	·05358
55	·91516	·08484	60	·95474	·04526
56	·92549	·07451	61	·96233	·03767
57	·93530	·06470	62	·96915	·03085
58	·94451	·05549	63	·97517	·02483
59	·95305	·04695	64	·98040	·01960
60	·96084	·03916	65	·98484	·01516
61	·96786	·03214	66	·98853	·01147
62	·97408	·02592	67	·99153	·00847
63	·97948	·02052	68	·99391	·00609
64	·98409	·01591	69	·99574	·00426
65	·98793	·01207	70	·99712	·00288
66	·99106	·00894	71	·99811	·00189
67	·99355	·00645	72	·99881	·00119
68	·99548	·00452	73	·99928	·00072
69	·99692	·00308	74	·99958	·00042
70	·99798	·00202	75	·99977	·00023
71	·99872	·00128	76	·99988	·00012
72	·99922	·00078	77	·99994	·00006
73	·99954	·00046	78	·99997	·00003
74	·99975	·00025	79	·99999	·00001
75	·99987	·00013	80	1·00000	·00000
76	·99993	·00007	81	..	..
77	·99997	·00003	82	..	..
78	·99998	·00002	83	..	..
79	1·00000	·00000	84	..	..

DISCUSSION

Dr. BUCHANAN.—I should like to express my obligations to Mr. Ackland for his valuable paper. He has illustrated very clearly the persistence of selection in the unadjusted data of the annuitant experience after five and even after ten years, and the consequent difficulty experienced by Mr. Hardy in joining the select portion of the graduated table on to the ultimate aggregate table. I would be glad, however, to have Mr. Ackland's opinion as to how far this persistence of selection is due to selection exercised as at entry, or to the improvement in vitality which has apparently been going on during the period covered by the experience. In an interesting and suggestive paper which was read before the Institute last session it was shown that when two experiences, showing different rates of mortality, are amalgamated, there may arise variations in the values of q, which have the appearance of selection, but which are due entirely to statistical processes. I have not been able to examine whether, or to what extent, the vitality of the "New" annuitants is superior to that of the "Old"; but a steady improvement in vitality appears to have been going on, and presumably it is the case. In the discussion which followed the reading of that paper, Mr. Ackland gave reasons for thinking that the effect in the experience of assured lives was not so great as had been suggested; but an annuitant experience seems to be essentially differentiated from one of assured lives by the fact that annuities are largely effected at the older ages, and as the measure of spurious selection depends on the difference $q' - q$, it seems possible that the effect in the annuitant experience may be more considerable. The point which I wish to suggest is that the elimination of cases of re-selection, when the general vitality had improved, might have the effect of lowering the vitality of the aggregate table, quite apart from the influence of re-selection. But while, if the object in view were simply to study the duration of selection exercised as at entry, it may be statistically desirable that the experience should be as nearly homogeneous as possible, it is perhaps a blessing in disguise that this persistence of selection has been retained, even though it may be in part what is technically spurious selection. It must be remembered that the annuitant experience is already past history, and, if the improvement in vitality is still going on, the question naturally suggests itself, what would be the effect on annuity values of a table accurately representing present-day mortality? I call to mind a remark which was made to me in another connection by a well-known English actuary, that we cannot regard even "select three per cent" as a final resting-place for our reserves. The object which Mr. Hardy set before him was "not merely to remove accidental irregularities in the ungraduated rates which are inherent in all statistical data, but to construct tables which, while adhering closely to all important features of the original experience, shall at the same time be both safe and convenient in practical use"; and again and again the purely theoretical was subordinated to the practical. For example, three preliminary graduations of the male aggregate table were made. The first was rejected because it gave too high rates of mortality at the younger ages; the third, though technically superior to the second, because it overstated the mortality

during the very important period of life from 65 to 75, while the constants brought out by the second graduation were modified in the transition to the hypothetical table so as to bring out expectations agreeing with the select expectations, and a final arbitrary reduction was made in the constant A, as otherwise the rates of mortality at the younger ages in the male annuitant table would have materially exceeded the corresponding rates in the assurance experience. Mr. Hardy's power of exposition is so great that I would echo the wish that he might be induced to explain more fully the processes followed in the transition to the hypothetical table. The agreement, as shown in Tables IX and X, between the actual deaths according to Mr. Ackland's select ultimate data and the expected deaths according to the graduated table, is remarkably close, and seems to show clearly that a good junction could have been effected with the select ultimate data after five years. The deviations for individual groups of ages occurring in the very important middle section of the table indicate a slightly superior vitality in Mr. Hardy's graduated table. Students of all ages are indebted to Mr. Ackland for the example given in Table XI of a Makeham graduation by Mr. Hardy's summation method. The only working figures which seem to be omitted are those giving the values of $(E_x - \frac{1}{2}\theta_x)c^{x+\frac{1}{2}}$, the sum and second sum of which are required for the determination of the constants A and B. The second part of the paper, dealing with the application of the law of uniform seniority to the calculation of joint-life annuities, has been rendered possible by the tenacity with which Mr. Hardy clung to that law in his graduation. The impossibilty of representing the female table by a single Makeham curve both complicates the formulæ and multiplies the number of tables necessary for the computation, when one or more female lives are involved, and unfortunately these are just the annuities with which we have to deal most frequently in practice. When not more than one female life is involved, Mr. Ackland's formula (21) is extremely simple; and the numerical application could be simplified if it were written in the form $a_{\overline{x}\,\overline{y}}^{(m)(f)} = a_{\overline{x}\,\overline{y}}^{(m)(1)} - \left(1 - \rho_y\right)\left(a_{\overline{x}\,\overline{y}}^{(m)(1)} - a_{\overline{x}\,\overline{y}}^{(m)(2)}\right)$. Only one multiplication instead of two would then be required. For annuities dependent on more than two lives it would probably be better to use a summation formula, such as suggested in the last paragraph of the paper. Theoretically, Woolhouse's formulæ are open to the objection that some of the differential co-efficients of the function to be integrated may be discontinuous, and then we should have a failing case of Taylor's theorem on which Woolhouse's formulæ are based. But practically, so far as annuity values are concerned, which do not involve the value of μ, this objection does not seem a serious one.

Mr. Lidstone has pointed out in the first paragraph of his memorandum that a first-difference interpolation between the values tabulated in the published volume may lead to an error of 2 in the second decimal place, so that to attain the same degree of accuracy as is given by Mr. Ackland's tables it would be necessary to make a second-difference interpolation. The extremely condensed form of the tables of this paper must also be borne in mind in comparing them, as regards the labour involved in their application, with those of the published volume. Much of the numerical work is connected with the interpolations necessary to find the equivalent-age annuities, but if the annuity values were tabulated to tenths of a year of age with a column of differences—as is done for certain annuities in the volume of *Tables deduced, etc.*—the equivalent-age annuities could be written down by inspection. The method seems to be specially adapted to the calculation of annuities involving one male and one female life, and these are probably the annuities which occur most frequently in practice. A set of extended

$O^{am/1}$ and $O^{am/2}$ tables, select and ultimate, would occupy about half the space of the corresponding tables in the published volume. The equivalent ages could be found by a simple inspection of the uniform seniority table; two inspections of the extended $O^{am/1}$ and $O^{am/2}$ tables would give the equivalent life annuities; and a simple multiplication would give the annuity required. That would be more accurate than a first-difference interpolation between the values of the published tables, and shorter than a second-difference interpolation. But the very fact that even in their present condensed state Mr. Ackland's tables are comparable in facility of application is the best tribute to his actuarial skill and to that of Mr. G. F. Hardy, and in thanking Mr Ackland for his valuable contribution to our *Transactions* I feel sure that I am simply voicing the feelings of every member of the Faculty.

Mr. Low.—This is essentially a student's paper—a paper not only for students preparing for the examinations, but for actuaries who are, as we ought all to be, students of actuarial science. As one somewhat out of touch with these abstruse modern calculations I cannot venture to offer any criticism of the paper. From the perusal I have been able to give it, however, I can form some idea of its extreme value and of the great debt of obligation under which we have been laid by having had such a paper submitted to us. It will be a conspicuous ornament to our *Transactions*, and will add greatly to their value and usefulness. I should like to join in the tribute of thanks and admiration which Dr. Buchanan has paid to the author, and to say what a great satisfaction and honour we feel it to be to have him among us.

Dr. A. E. Sprague.—Mr. Ackland's paper at first sight appeared to me rather formidable, but a closer examination removed that impression. It is so well put together, and the explanations are so clear, that no student who understands Makeham's Law should have much difficulty in following it. I quite agree with what is said as to the difficulties attending Mr. G. F. Hardy's explanations of his methods of graduating the Experience Tables, and Mr. Ackland's paper should be a real help towards removing some of these. In particular, I think the example of the summation method of determining the values of the constants in Makeham's formula will make the method very much clearer to many. I might point out, however, that Mr. Hardy's summation method and Professor Karl Pearson's method of moments are not quite the same thing, though there is a very close connection between them, as is shown in Mr. Elderton's recent book on *Frequency Curves and Correlation*, which is now in our Library. The only point as to which a little amplification seems to me to be desirable is the method of correcting the approximate values of the constants A and B in Makeham's formula by means of the ratio $\mu_{x+\frac{1}{2}} \div m_x$. This point is, I believe, a source of considerable difficulty to some students.* We are also indebted to Mr. Ackland for his demonstration of Mr. Hardy's formulas of relation between the Female Annuitants Table and the partial Tables of which it is composed. I think that the author's substitution of ρ_x and $(1-\rho_x)$ for Mr. Hardy's r_x functions is a decided practical improvement. Besides saving the labour of a division, it has the effect of making the formulas more symmetrical.

Mr. Chatham.—I think Mr. Ackland's paper is one of the most erudite we have had submitted to us. He has shown in it the difficulty which Mr. G. F. Hardy experienced in making a junction between the select and the ultimate portions of the Annuity Experience, and he has also shown the causes which contributed to it. I should like to ask Mr. Ackland whether

* Effect has been given to this suggestion (see p. 293).

it would not have been better, and have made matters simpler, if duplicate lives had either been excluded altogether, or had been included in both the Select and the Aggregate Tables. On the whole, I cannot help thinking it would have been better to have excluded them. As regards Mr. Hardy's method of graduation, I think those of us who have been unable to follow it may be excused in view of the difficulty experienced by so accomplished a mathematician as Mr. Ackland. The method is a very elaborate one, and two points have occurred to me in connection with it. The first is, that after all the elaboration, it does not reproduce faithfully the rates of mortality of the original Experience during the period of selection. There is often a considerable difference between the original and the graduated rates, the former giving a well-rounded curve, whereas Mr. Hardy's cut through that curve. Indeed his graduated rates resemble the original rates pretty much as the moon when full may be said to resemble the sun. The other point is that a very large proportion of our business is of a simple character, and only a small proportion requires the use of complicated formulæ. I do not wish for a moment to undervalue the law of uniform seniority, but the question arises whether it has not been carried a little too far in the method of graduation adopted. If anything could be done in future to simplify matters, it would be a great advantage to the profession generally. I should prefer if a method could be devised which would land us all on the other side instead of leaving most of us on this side. These remarks do not in any way detract from the value of Mr. Ackland's paper, but are all more or less applicable to the environment by which he finds himself surrounded. We are much indebted to him for his Tables of select and ultimate joint-life annuity values, and we should welcome such an extension of them as he offers. He refers to an interesting point in connection with the calculation of the values of select annuities by means of formulas of approximate summation, namely, that in a number of them no term except the first falls within the period of selection, and consequently they do not give to selection the weight it ought to have in determining the value. This is a difficulty I have often experienced, and I have generally been able to get over it by using a formula in which more than one term falls within the period of selection.

Mr. Gunn.—The Actuarial profession in general is under a debt of gratitude to Mr. Ackland for his valuable services in connection with the British Offices Experience, and I think it is a great compliment to us that he should take so much trouble in preparing for our *Transactions* the paper we have just heard, and still further, that he has put himself to the inconvenience of travelling all the way from London to Edinburgh to submit it in person. I am sure that the Members of the Faculty, and especially those who have not previously had an opportunity of becoming acquainted with his personality, are gratified at seeing him with us to-day.

There is one point: the paper suggests a line of thought which I should like to bring under the notice of the younger Members. There is undoubtedly a very large field for research in the British Offices Experience. Mr. Ackland has given us an object-lesson of the manner in which the statistics may be treated from one point of view. It would add to the strength of the Faculty if the younger Members would take example from him, and apply their energy to further research among the vast mass of material in the volumes published by the Faculty and the Institute.

Mr. McLauchlan.—As one who formerly gave a good deal of attention to the subject of joint-life annuities, I am extremely interested to see the great extension and development which the methods of calculating joint-life annuities depending upon Makeham's Law have now received. I used to think that, in calculating the values of annuities on several lives, it was

better to employ a formula of approximate summation than to adopt one of the methods depending upon Makeham's Law, but there is no doubt that the introduction of such tables as Mr. Ackland has given us greatly facilitates the application of these methods. I am sure the paper will be a most valuable contribution to our *Transactions*.

Mr. GORDON DOUGLAS.—We were already under a deep debt of gratitude to Mr. Ackland for the very important work he did in connection with the Joint Mortality Investigation—a debt which we felt ourselves proud to acknowledge even in a small degree by electing him an Honorary Fellow of the Faculty. From my association with Mr. Ackland in connection with the secretarial work relating to the Investigation, I have perhaps a better idea than most of the actual extent of his labours during the many years over which the Investigation extended, but I am afraid that even those most closely associated with him could form no real conception of the actual amount of work involved in his position as Supervisor, as in labours of this kind the bare results set out in printed form can give no indication of the trials and thought necessary before the proper and best methods of arriving at the results can be decided upon. Deep as our debt of gratitude to Mr. Ackland already was, it has been greatly increased by his present contribution to our *Transactions*. The subject is one of considerable interest. To myself, one of the most interesting parts of the paper is that in which Mr. Ackland proves the success, for all practical purposes, of Mr. George F. Hardy's method of dealing with the graduation of the Annuity Table after the first five years. My impression was that the Committee, when dealing with the Annuity Tables, felt that, even although it was proved that the effects of selection had not entirely passed off by the end of five years, it would be most inconvenient, in Annuity Valuations and such like, to have to deal with extended tables covering a wider range than five years, and it was not unnaturally suggested that if possible the aggregate data should be linked on to the select data at the end of a five year period. It was, however, pointed out that, for the reasons mentioned by Mr. Ackland, the two different sets of data could not be satisfactorily welded together at the point desired, but the Committee, recognising that Mr. Hardy's was a master mind, left it to him to find a way out of the difficulty. This he did, as Mr. Ackland reminds us, by means of a hypothetical table, which reproduced the expectations of life five years after entry, and which, when combined with the experience of the first five years, also reproduced the expectations of life from entry. No one, as Mr. Ackland has indicated, much likes hypothetical tables, if tables based on actual experience can be made use of instead, and I do not know if every member of the Committee grasped the full results of the method adopted as clearly as did Mr. Hardy himself. They had confidence, however, in his judgment, and it is most gratifying therefore to find that that confidence was not misplaced, and that Mr. Ackland has been able to prove that, while another method might have been adopted, Mr. Hardy's method has given us a table which we can rely upon as suitable for all practical purposes. When it came to the graduation of the $O^{[M]}$ Table it was found impossible to disregard the effects of selection up to ten years from entry, and consequently we had to be content with the slightly cumbrous ten years extended table which we have for the select assurance experience. I am sure we are all pleased, however, that, thanks to Mr. Hardy's ingenuity, we have got a select annuity table in the more convenient form of a five year period. As regards the second part of Mr. Ackland's paper, relating to the computation of joint annuities by special applications of the law of uniform seniority, when female lives are involved, this is, I think, somewhat of an academic nature only, when applied to rates of interest under 4 per cent,

as, for all practical purposes, one would no doubt be quite satisfied with the values which could be obtained from the published tables by simple methods of interpolation. I compared Mr. Ackland's 3 per cent. values with those obtained from the published tables by first-difference interpolation and the results are as follows :—

	Mr. Ackland	From the Tables
$a^{m\,f}_{[28:40]}$	15·888	15·871
$a^{m\,f}_{28:40}$	15·488	15·472
$a^{f\,f}_{[23:50]}$	14·194	14·187

The largest difference being ·017, I think that, for everyday practical work, the one value is really as good as the other. If, however, values at 4 per cent or 5 per cent were required, Mr. Ackland's Tables would prove very useful, as giving what cannot be had in any direct way from the Tables published, and as forming an alternative to Mr. Lidstone's method.

The PRESIDENT.—I beg to propose, as I have peculiar pleasure in doing, that we accord to Mr. Ackland a very grateful vote of thanks not only for the valuable contribution to our *Transactions* which he has just submitted, but also for his great kindness in coming from London to do so in person. Mr. Ackland, as we all know, has long held—and deservedly so—a distinguished position in the actuarial profession. A Fellow of the Institute of Actuaries of thirty years' standing, Mr. Ackland is at present one of its vice-presidents. His appearance here to-day affords us another opportunity, of which we are always glad to avail ourselves, of welcoming to our Hall men of light and leading from the Institute in London. Now that he has found his way to our Hall and has, in a sense, had the freedom of the Faculty conferred upon him, we hope that this, his first visit to our Sessional Meetings, will not be his last.

As you are aware, Mr. Ackland holds the responsible position of actuarial adviser to the Board of Trade. Many of us here would be glad to be advisers to that public body; for we believe we also could give the Board some excellent advice which might do them and the nation good. For one thing, we could tell them how to define the term "Actuary," and that to the great advantage of the backbone of our country—the provident classes, through their Friendly Societies and Life Assurance Institutions, whose affairs are best investigated by qualified actuaries.

Mr. ACKLAND.—Dr. Buchanan has asked whether the persistence of selection might not arise, to some extent, from the statistical processes of combining the data in respect of "Old" and "New" Annuities. I have not dealt with this aspect of the question, but no doubt it is one which deserves to be looked into, and I hope that some student of the Faculty may find time for the investigation. He also referred to the omission, in Table XI, of certain columns. I propose to add to the Table an explanatory note as to these columns, which were omitted from exigencies of space. As regards formula (21), no doubt his transformation of it is more concise, and would be a practical improvement, although perhaps the formula as it stands shows more clearly how the sectional annuity values enter into the final result. I quite agree, and had fully in mind, that the practical usefulness of the Annuity Tables would be increased if the values were tabulated for tenths of a year of age; but this would have involved a large extension of space.

With regard to Mr. Chatham's remark that duplicates might have been treated on some more consistent principle,—how far the course actually followed was the wisest and best it is very difficult to say. Perhaps, at that time, it was not fully in the mind of the Joint Committee that there would be a combination of the Select and Ultimate data at certain points.

The whole question of exclusion of duplicates seems to require further investigation. Mr. Chatham also criticised Mr. Hardy's graduation, as not accurately reproducing the original rates for the first five years, and he was unkind enough to say that the graduated figures were, to the original, as the full moon to the sun. We should, however, keep in view the great difficulty under which Mr. Hardy laboured, in attempting a graduation by Makeham's formula. And, after all, in the case of an Annuity Table, we need not, I think, attach too great weight to the correctness of the graduated rates in each of the first five years, so long as the Annuity values reproduce the ungraduated values at the end of the five years, and at selection. It may be that exceptional values, such as temporary or deferred annuities, would be affected, but, subject to this qualification, the graduation seems to me fully to meet all practical requirements. Mr. Chatham thinks there was too much regard for mathematical theory in Mr. Hardy's graduation. It may be that in this we can go too far; but there are two points to be kept in view: (1) by following a mathematical law, such as Makeham's, and completing a successful graduation, one has a Table which is absolutely smooth, and not, like the Carlisle and other Tables, subject to irregularities; and (2) we retain the Law of Uniform Seniority. I think therefore, personally, that Mr. Chatham's criticism is somewhat too drastic. In connection with Mr. Chatham's remarks as to formulas of approximate summation, I should like to refer to Dr. Buchanan's valuable paper published in the *Transactions* of the Faculty (vol. ii. p. 281), in which he suggests some useful formulas for deducing the values of select annuities and assurances. I might add, with reference to Mr. M'Lauchlan's remarks, that I have found his paper on Joint-Life Annuities, published in 1879 by the Actuarial Society of Edinburgh, most useful, as bringing together in a convenient form the different methods of approximation that had been suggested up to that time.

With reference to Mr. Gordon Douglas's remarks—it is probable, as he suggested, that the Joint Committee might be responsible for the restriction of the Select Annuity Tables to five years of duration. At that early stage, however, Mr. Hardy was not, I think, prepared to consider an extension beyond five years; when, however, he came to the later graduation of the $O^{[M]}$ Tables, he extended the duration of the Select Tables to ten years, and the Table in this extended form, when once it has been computed, is just as easy to use as a more restricted one, and the extension is to be preferred, if we get better results thereby. Mr. Douglas's comparisons of annuity values at 3 per cent, interpolated by first differences, with those deduced by the formulas in the paper are interesting, as showing how nearly identical both results are. Any difference is, of course, due to the interpolation, as Mr. Hardy's formulas are rigidly accurate. In conclusion, I have to thank you for your reception of the paper, and to express my appreciation of the laudatory remarks of the President and other speakers.

CORRESPONDENCE

DAMAGED LIVES

To the Editor of the Transactions of the Faculty of Actuaries,

SIR,—Referring to Mr. Low's remarks on the number of damaged lives in the "ultimate" column of a Select Mortality Table (p. 275 of this volume) I have to explain that the figures given in my paper (p. 204) are obtained according to the principle laid down by Dr. T. B. Sprague (*J. I. A.* xxii. 422), namely that the l_x mixed lives aged x in the ultimate column of the H^M select table consist of $l_{[x]}$ lives who are still select and $l_x - l_{[x]}$ damaged lives who will all die in 5 years. Stated in more general terms the principle is that in a table where selection wears off in n years the l_x mixed lives in the ultimate column consist of $l_{[x]}$ lives who are still select and $l_x - l_{[x]}$ damaged lives who will all die in n years. As Dr. T. B. Sprague's proof of this proposition seems to present some difficulties, a further investigation of the problem may be of interest, and I therefore venture to submit the following considerations in the hope that they may help to make the matter clearer.

Considering the $l_{[x]}$ select lives aged x, the table tells us that at the end of n years l_{x+n} of these will survive. It also tells us that out of l_x mixed lives l_{x+n} will survive at the end of n years. The table may therefore be considered to exhibit two groups, each containing the same number of lives, namely l_{x+n}, who will experience identical rates of mortality in every subsequent year down to the extreme limit of human life. It follows that these two groups must be similarly constituted; that is, each must contain the same number of select lives and the same number of damaged lives. For if the proportion of damaged lives were greater in one group than in the other, the rate of mortality in that group would be higher than in the other.

Now let σ_{x+n} and δ_{x+n} be respectively the number of select lives and of damaged lives in each of the groups; and let σ_x and δ_x denote respectively the number of select lives and of damaged lives among the l_x mixed lives, so that $\sigma_x + \delta_x = l_x$ and $\sigma_{x+n} + \delta_{x+n} = l_{x+n}$. Then out of the σ_x there will remain after n years σ_{x+n} who will be still select, and a number (as yet unknown) who will be damaged. Some of the damaged lives may also survive (the number being as yet unknown) and the sum of these two unknown numbers will be δ_{x+n}. Hence the probability of a select life aged x being still select at the end of n years is $\frac{\sigma_{x+n}}{\sigma_x}$. But from a consideration of the $l_{[x]}$ select lives we see that this probability is also $\frac{\sigma_{x+n}}{l_{[x]}}$ and therefore $\frac{\sigma_{x+n}}{l_{[x]}} = \frac{\sigma_{x+n}}{\sigma_x}$. It follows that either $\sigma_{x+n} = 0$, or $l_{[x]} = \sigma_x$.

The first of these equations would imply that after n years of assurance every life must become damaged; but the number of duplicate policies in the "experience" proves that this is impossible. There remains, therefore, only the latter solution, namely, that $l_{[x]}$ is the number of select lives in the "ultimate" column of the table, all the others being therefore damaged.

It may be objected that this reasoning involves the tacit assumption that none of the δ_x damaged lives will become select, and survive as such until the end of the n years. This assumption seems to me not unreasonable, having regard to the fact that damaged lives are necessarily worse than

select lives, and throughout their existence must be expected to exhibit heavier rates of mortality. As, however, there is room for difference of opinion here, the point may be investigated further. Assume, therefore, that some of the σ_{x+n} lives were included in the δ_x damaged lives in existence n years earlier. By considering the $l_{[x]}$ select lives and their survivors after n years we find, as before, that the probability of a select life surviving as "select" $=\frac{\sigma_{x+n}}{l_{[x]}}$. Similarly the probability of a select life surviving as damaged $=\frac{\delta_{x+n}}{l_{[x]}}$. Hence out of the σ_x select lives there will survive after n years $\sigma_x \times \frac{\sigma_{x+n}}{l_{[x]}}$ select, and $\sigma_x \times \frac{\delta_{x+n}}{l_{[x]}}$ damaged. But we know from the ultimate column of the table that out of $\sigma_x + \delta_x$ lives there will survive after n years σ_{x+n} select and δ_{x+n} damaged. Therefore, by subtraction, we see that out of δ_x damaged lives there will survive after n years $\sigma_{x+n}\left(1-\frac{\sigma_x}{l_{[x]}}\right)$ select and $\delta_{x+n}\left(1-\frac{\sigma_x}{l_{[x]}}\right)$ damaged; that is to say, the survivors of the damaged lives will consist of select lives and damaged lives mixed in the same proportions as are the survivors of a body of select lives, and therefore they will thereafter experience identically the same rates of deterioration and mortality as the survivors of the select lives. This result seems to me quite inconsistent with the fact that the damaged lives are undoubtedly inferior to the select; for if any of the damaged do become select during the n years we should expect them to be a much smaller proportion of the survivors than if we were dealing with select lives. Looking at this point more closely, we see that the only difference in the subsequent histories of the $l_{[x]}$ select lives and the l_x mixed lives is during the first n years, when the mixed lives experience an excess of $l_x - l_{[x]}$ deaths. If we leave these $l_x - l_{[x]}$ lives out of account, the remainder of the l_x mixed lives are found to die, or to survive as select lives, or to survive as damaged lives, in identically the same manner as an equal body of select lives. Now if a body of lives is found to possess identically the same rates of survivance, of deterioration, and of death, as select lives, that body must necessarily be composed entirely of select lives. Hence the l_x mixed lives are composed of two groups, namely, one numbering $l_x - l_{[x]}$ who all die in n years and must be regarded as damaged lives, and the other numbering $l_{[x]}$, who are all select.

In applying this principle it should be remembered that our mortality tables are only approximate, and that selection can in fact be traced (though to a very small extent) for a period longer than is shown by those tables. Further, if the figures in those tables be regarded as approximations of the first order of accuracy, the differences between them will only be approximations of the second order; and as the numbers of damaged lives are found by these differences we cannot place so much reliance upon them as upon the figures relating to select lives.

I am, etc.,

A. E. SPRAGUE.

22 George Street,
Edinburgh, *6th March* 1907.

To the Editor of the Transactions of the Faculty of Actuaries,

Sir,—Dr. Ernest Sprague has been kind enough to show me his letter on this subject. I am much obliged to him for replying so fully to the observations I made, and as the subject is not devoid of interest I should

like, with your permission, to pursue it a little further, in the hope that we may be able to arrive at a clearer understanding with regard to it.

The question at issue is whether, by means of a Select Mortality Table, we can determine the number of "select" lives contained in a body of "mixed" lives at any given age; and in particular whether the number of select lives among the mixed lives shown in the "ultimate" column of the table is the same as the number of select lives entering at that age. Dr. Ernest Sprague, founding on the principle laid down by Dr. T. B. Sprague in a well-known paper, holds this to be true, while I, on the other hand, venture to call in question its accuracy, and to hold, on the contrary, that we cannot by means of a Select Mortality Table determine at all the number of select lives contained in a body of mixed lives.

There are two senses in which a theory or a proposition may be held to be true—(1) it may be demonstrably true in fact, or (2) it may be true only as a deduction or corollary from a hypothesis which, although not demonstrably true, is held to be so for practical purposes. From the language employed by Dr. Ernest Sprague in his paper, I understood him to affirm his proposition as true in the former sense, but it is manifestly not so. It depends not only on the fundamental hypothesis employed by Dr. T. B. Sprague, that the benefit of selection wears off in a definite short period of years, but also on a conclusion drawn by him (whether warrantably or not is now in question) that all "damaged" or non-select lives will die within a period similar to that in which the effect of selection wears off. That this deduction is not true in fact scarcely needs to be proved, as it is contradicted by everyday observation. Formal proof, however, may be had by referring to the statistics of "diseased" or damaged lives, collected at the same time as the material of the H^M Table, where the history of such lives is traced for 20, 30, 40, 50 and more years, and up to practically the same limit of extreme old age as in the case of "healthy" or originally select lives.

Dr. Ernest Sprague's proposition therefore is not true in the first sense above indicated. Whether it is true in the second sense depends on whether Dr. T. B. Sprague's principle is correctly deduced from his fundamental hypothesis. This is what that gentleman himself says on the subject (*J. I. A.*, xxii, 422):—

"Turning to Table No. 2, let us in the first instance consider the 862,820 select lives of 30. We see that out of these there survive at the end of five years 829,800 who are then of the age 35. Some of these will be still select lives and some will be damaged, and the rates of mortality prevailing among these two classes will of course be widely different, but the 829,800 will as a whole be subject to the mortality of the $H^{M(5)}$ Table. (It is true that according to the table the $H^{M(5)}$ mortality is reached at the age of 33, but this does not in any way affect our argument.) After the lapse of five more years, or at the age of 40, the number of survivors will be 786,500. Now, according to the construction of the table, this is also the number of survivors out of 820,928 select lives of 35, that is to say, the 820,928 select lives and the 829,800 mixed lives of 35 will give us after five years the same number of survivors who will thenceforward be subject to the $H^{M(5)}$ mortality. *It is therefore clear* that the difference, 8,872, between these numbers, must be the number of damaged lives included among the latter; and consistently with our supposition that the effect of selection wears off in five years at most, these damaged lives must all be dead before the end of five years, or none of them will attain the age of 40."

The words which I have put in italics indicate that the writer is content to base his conclusion on what precedes. Now, with all deference, I submit that the conclusion does not necessarily follow from the premises. The problem is of this nature:—We have *A* lives entering at the age of 30,

becoming reduced by death to B at the age of 35, and further reduced to D at the age of 40. We have also C lives entering at the age of 35 and becoming reduced by death to the same number D at the age of 40. From that age onwards both sets of lives are subject to the same rate of mortality. Dr. T. B. Sprague says it is clear that the difference between B and C must be the number of damaged lives included in the former. But is this so? Let B' be the number of select lives and B'' be the number of damaged lives among the B mixed lives of 35, and let p', p'', and p be the probabilities respectively of a select life, a damaged life, and one of the mixed or ultimate lives—whether select or damaged being unknown—surviving to the end of five years. Then

$$B'p' + B''p'' = Bp = D.$$

But Cp' also $= D$, hence

$$B'p' + B''p'' = Cp'.$$

This is an indeterminate equation, in which B', B'', and p'' are unknown. Dr. Sprague's solution makes $p'' = 0$ and hence $B' = C$; but, as we have already seen, 0 is an impossible value for p'' in point of fact. Any value of p'' within the limits of 0 and p would satisfy the equation, and if any conclusion may properly be drawn it would rather seem to be that the value of p'' must lie between those limits, the extreme values 0 and p being both inadmissible, and that the value of B' must lie between 0 and C.

The danger inherent in Dr. Sprague's line of reasoning will be seen if we apply it to a similar problem where selection operates in another direction. Suppose that a Life Assurance Office opened its membership to all persons of a particular class who might choose to come in, without inquiry being made as to their fitness for assurance. At the outset the quality of the lives admitted would naturally be below the standard of even "ultimate" lives, but it is reasonable to suppose that after five years the worst of those "sub-standard" lives would be eliminated by death, and the rate of mortality would thenceforward be similar to that of the $H^{M(5)}$ table. Suppose we have a table of such lives constructed on a plan similar to that of a Select Table, and we endeavour by means of it to ascertain how many of the ultimate lives of a given age are still of "sub-standard" quality. In this case the rate of mortality for ultimate lives of 35 will be lower than that of sub-standard lives entering at the same age; and hence, in correspondence with a given number of ultimate lives at the age of 40, the number of sub-standard lives at 35 will be larger than the number of ultimate lives at that age. Reasoning, then, by Dr. Sprague's method, the ultimate lives of 35 will consist of C sub-standard lives and *minus* $(C - B)$ others; a result to which we can attach no intelligible meaning.

The truth is, it seems to me, that a Select Mortality Table, constructed on the principle now familiar to us, and based on the assumption that the effects of selection wear off in a definite number of years, has said its last word on those effects when it has traced the lives from entry into the "ultimate" column. If we try to make it say more—to tell us, for instance, how many of the ultimate lives are "select" and how many are "damaged"—we must perforce introduce some new element or assumption not involved in the construction of the table, and we must be prepared to accept any anomaly or incongruity that may be elicited by our effort. We cannot, however, claim that our result is taught by the table, or that it follows from the hypothesis on which the table is based. We can only ascribe it to our requiring from the table more than it professes to teach.

Concerning the lives in the "ultimate" column, the table only tells us that they are no longer select, but are henceforth subject to the ultimate rate of mortality. When Dr. Sprague says of them that some will still be select and some will be damaged, he goes beyond anything that we find in the table, and draws upon experience and observation altogether outside of it.

But if experience and observation are to be introduced as factors in the problem, we cannot admit their validity in one direction and deny it in another. Now, as we have seen, they emphatically contradict the conclusion that all lives which are not insurable at the ordinary rate of premium must die within five years. But this is a necessary conclusion from the principle that the number of select lives contained among the l_{x+5} lives in the ultimate column of a Select Mortality Table is the same as the number of lives entering the table as select at $x+5$. We must therefore conclude that that principle is itself erroneous. Dr. Sprague has not proved that it necessarily follows from the fundamental supposition as to the effect of selection wearing off in five years, and I do not think this is capable of proof. If it were, the reasonable view would seem to be, not that we must follow out that supposition to all extremes, but that we should carefully limit its application, and not be led by it to a manifestly impossible conclusion.

Dr. Ernest Sprague in his letter seeks to carry the demonstration further than Dr. T. B. Sprague has done. He dwells more on the identity of the ultimate rate of mortality of the entrants at age x with that of the entrants at age $x+n$, and argues that after the ultimate column is reached, every group of the same attained age—whatever the age at entry—must be held to consist in similar proportions of select and damaged lives. Here again he must be reminded that the presumption that there are select and damaged lives, and not merely a homogeneous body of ultimate lives, after the period of selection has expired, does not arise from the table but is inferred from independent observation and experience which did not enter into its construction. It is read into and not evolved from the table, and therefore neither the table itself nor the fundamental assumption upon which it is constructed can be held responsible for any inadmissible conclusion that may result. Such a conclusion does in fact result, for Dr. Ernest Sprague finds himself immediately at issue with the undoubted fact that damaged lives, so far from all dying off in the period of n years, within which selection is assumed to operate, do in many instances become select lives, and gain admission at the ordinary rate of premium on applying a second time for assurance. Here again the facts of observation, being brought in evidence on one point, must be permitted to bear testimony to another on which they are equally emphatic, and they vitiate the otherwise able argument adduced in Dr. Ernest Sprague's letter.

On the whole matter I submit, that however convenient the principle laid down by Dr. T. B. Sprague may be as a working hypothesis for certain purposes of calculation, it is by no means a proved conclusion—does not necessarily follow from the fundamental assumption of a Select Life Table—and in practice should be employed with caution, and only after careful circumspection as to its tendency in the particular case which is the subject of calculation.

I am, etc.,

GEO. M. LOW.

28 St. Andrew Square,
Edinburgh, 14*th March* 1907.

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THE FACULTY OF ACTUARIES.

EXAMINATION PAPERS, 25 AND 26 APRIL 1907

FIRST EXAMINATION.

Examiners—MR. W. A. SIM AND DR. J. BUCHANAN

FIRST DAY—PART I.

1. Express £23, 10s. $7\frac{3}{4}$d. in pounds, and 17 tons 15 cwts. 3 qrs. 12 lbs. 4 ozs. in tons, each to four decimal places, taking in the fourth place the digit which is nearest to the true result.

Employ these figures to find the value of 17 tons 15 cwts. 3 qrs. 12 lbs. 4 ozs. at £23, 10s. $7\frac{3}{4}$d. per ton, and show that the result differs from the true value by a quantity which is greater than $\frac{1}{10}$d. and less than $\frac{3}{10}$d.

2. A, B, and C start to go round a circular course in the same direction. There are 20 laps to the mile. A and B do a lap in 15 and 20 seconds respectively. C lost $5\frac{1}{2}$ yards on B each lap. What distance will A have gone when all three are again together?

3. Show that $2^{\frac{1}{2}}.4^{\frac{1}{4}}.8^{\frac{1}{8}}.16^{\frac{1}{16}}.\ldots\ldots\ldots=2$.

4. Solve the equations—

$$(1)\ \frac{x-a}{b}+\frac{x-b}{a}=\frac{b}{x-a}+\frac{a}{x-b}.$$

$$(2)\ x^4-5x^3+6x^2-5x+1=0.$$

5. Find the relation that subsists among the coefficients of the equations—

$$ax^3+bx^2+cx+d=0$$

and

$$a'x^2+b'x+c'=0,$$

if the equations have one root in common.

6. A purchaser is to take a plot of land fronting a street. The plot is to be rectangular, and three times its frontage added to twice its depth is to be 96 yards. What is the greatest number of square yards he may take?

7. Assuming the truth of the binomial theorem for positive integers, show that it holds for fractional and negative indices also.

FIRST DAY—PART II.

8. What is the *Exponential Theorem*? Derive it from the binomial expansion. Find the value of $\log_e 2$ correct to five decimal places.

9. Show that the expression $\dfrac{x^2-mn}{\{2x-(m+n)\}}$ has no real value between m and n, if x be real.

10. A system of any number of binomial factors, each the sum of two squares, being supposed multiplied together, show that their product is the sum of two squares.

11. Given

$$x+y+z=a,$$
$$x^2+y^2+z^2=b^2,$$
$$x^3+y^3+z^3=c^3,$$

find the product xyz in terms of a, b, c.

12. Two persons, A and B, divide equally a sum of money consisting of half-crowns, shillings, and sixpences; the values of the several parts being respectively in the proportion of 15, 4, and 1. It is found that each has 60 coins, A having two half-crowns more than B. Determine the sum and the coins each had.

13. What are the operations which are facilitated by the use of logarithms? Mention the simpler operations which the use of logarithms enables us to substitute for them. Calculate, by the aid of the logarithms given below, the value of

$$\frac{\sqrt[5]{23\cdot0009}\times(24)^5\times(43092)^{\frac{3}{2}}}{(80935)^2\div512}$$

log ·023	$=\bar{2}\cdot3617278$	log ·5	$=\bar{1}\cdot6989700$
log 2·3001	$=\cdot3617467$	log 3	$=\cdot4771213$
log 21546	$=4\cdot3333667$	log 8·0935	$=\cdot9081364$

log 143·21 = 2·1559733

log 1432·2 = 3·1560037

14. Show that the integral part of $(3+\sqrt{5})^n$ is an odd number, and that the next greater integer is divisible by 2^n, if n be a positive integer.

SECOND DAY—PART I.

15. Insert n Arithmetic and n Harmonic means between a and b; and show that, if corresponding terms be multiplied together, the geometric mean of the series of terms so obtained is the square of the geometric mean between a and b.

16. Show that the sum of the squares of any n consecutive odd integers is divisible by n, unless n is a multiple of 3.

17. Sum to n terms the series

$$1.2.3x+2.3.5x^2+3.4.7x^3+\ldots\ldots$$

18. Find the nth term and the sum of n terms of the series

$$6+14+34+90+254+\ldots\ldots$$

19. Show that the number of combinations of n things r at a time increases with r, so long as r is less than $\frac{1}{2}(n+1)$.

20. In how many ways may three numbers be selected from the first 20 numbers, so that their sum shall be odd and greater than 20?

21. If repetitions be allowed, show that the number of arrangements, four at a time, which can be made of n pairs of letters, like one another but unlike all the rest, is

$$n^2(n^2-1)-3n(n-1).$$

SECOND DAY—PART II.

22. A bag contains three balls, each of which may be red, black, or white. One is drawn and found to be white. What is the chance that there was one of each colour?

23. Two persons throw simultaneously with two pairs of dice. Show that it is about 8 to 1 against them making the same total throw.

24. A box contains three shillings, one half-crown, and one sovereign. What is the value of my expectation, if I am allowed to draw until I draw the sovereign?

25. Show that the sum of n terms of the series

$$\frac{2}{3}+\frac{2.4}{3.5}+\frac{2.4.6}{3.5.7}+\ldots\ldots$$

is

$$\frac{2.4.6\ \ldots\ldots\ (2n+2)}{3.5.7\ \ldots\ldots\ (2n+1)}-2.$$

26. If $u_1, u_2, \ldots\ldots u_n$ form a series of equidistant terms of which one is missing, show how to supply the missing term.

Supply the missing term of the following series—

$u_1 = 81436.$ $u_4 = 78830.$

$u_2 = 79717.$ $u_5 = 77919.$

27. Given a series of equidistant values u_0, u_5, u_{10}, ..., show how to interpolate the intermediate values, assuming third differences to be constant.

Complete the series from u_0 to u_5, having given

$u_0 = 20{\cdot}047$ $u_{10} = 17{\cdot}379$

$u_5 = 18{\cdot}788$ $u_{15} = 15{\cdot}829.$

28. What are central differences, and what advantages do central difference formulae possess over those involving ordinary advancing differences?

SECOND EXAMINATION.

Examiners—Mr. Hugh W. Brown and Mr. A. Graham Donald.

FIRST DAY—PART I.

1. In how many years will a sum of money double itself at 5 per cent. Compound Interest, given log 2 = ·30103, log 3 = ·47712, log 7 = ·84510? Compare your result with the approximation obtained by dividing 69 by the rate per cent. State carefully the justification of this rule.

2. Define the Force of Interest and the Force of Discount, and show how they are related.

Express δ in terms of a series of powers (*a*) of d, (*b*) of i; use one of these series to find correct to seven decimal places the value of δ corresponding to 2 per cent. interest.

3. Deduce Makeham's formula for finding the value of a loan when the capital repayable differs from that invested and the rate of interest named in the security differs from the actual return obtained. Explain how the formula can be used to determine the rate yielded when the value of the loan is known.

4. What is the usual approximation to the value of a complete life annuity? Explain the nature and direction of the error.

Which is the greater for a fraction of a year, Simple or Compound Interest, and why?

Calculate the value of £1 due six months hence at $3\frac{3}{4}$ per cent. for (*a*) Simple Interest, (*b*) Compound Interest.

5. State briefly the meanings attached to the following small letters, distinguishing cases where the same letter has more than one use, occasioned say, by the addition of a subscript letter—

a c d e g i j k l m p q r s t v w x y z.

6. The following is a quotation from a recent arithmetic: "In the case of life annuities the most probable number of years for which the annuity is likely to be paid is calculated, and the annuity becomes an annuity certain for that period." Discuss this statement by general reasoning, supporting your views by a mathematical demonstration.

7. Find the value of ${}_nV_x^{(m)}$ and explain your result.

If $P'_x = P_x + \phi_x$ and ${}_tP'_x = {}_tP_x + {}_t\phi_x$, what must be added to ${}_nU_x$ if each policy is to have the same reserve for future expenses, bonuses, etc.?

FIRST DAY—PART II.

8. Show that the error involved in the common approximation for equated time of payment is always in favour of the debtor.

9. Show that

$$\ddot{a}_x^{(m)} = \left(1 - \frac{\delta}{2m}\right) \ddot{a}_x + \frac{\delta}{12m^2}.$$

10. If B is the value of a benefit of unity payable at the end of the year in which the event may happen, and (IB) is the value of a similar benefit increasing by unity each successive year, prove that

$$(IB) = -(1+i)\frac{dB}{di}.$$

11. Find the formula for a premium payable m times a year in terms of the annual premium.

12. State what you know of the materials from which the O^[NM] Mortality Table is constructed, and how far it is applicable to the calculations of Life Assurance Offices.

13. If a population be constituted with l_x persons of age x, l_{x+1} of age $x+1$, and so on down to the extremity of life, show that the total fund to be raised to provide an assurance of unity on the death of each person is equal to $l_x(a_x + A_x)$.

SECOND DAY—PART I.

14. Discuss the formula for ascertaining the value of an annuity payable so long as at least two out of three lives shall be in existence.

15. Find the value of an annuity-certain for n years, payable m times a year, interest being convertible at indefinitely small intervals.

16. Prove the following—

(*a*) $({}_{\frac{1}{2}}p_x + {}_{\frac{3}{2}}p_x + {}_{\frac{5}{2}}p_x + \ldots \text{ etc.}) - ({}_1p_x + {}_2p_x + {}_3p_x + \ldots \text{ etc.}) = \frac{1}{2}.$

(*b*) $\dfrac{e_x \cdot e_{x+1} \cdot e_{x+2} \cdot \cdots \cdot e_{x+n-1}}{(1+e_{x+1})(1+e_{x+2}) \cdots (1+e_{x+n})} = {}_np_x.$

17. What modification should be made upon the value of the following annuities, payable yearly, in order to provide for payment half-yearly :—

(1) Immediate Annuity.
(2) Deferred Annuity.
(3) Temporary Annuity.
(4) Reversionary Annuity to x after y.

18. How would you calculate the annual premium payable during the joint lives of a child aged 8 next birthday and his father aged 40 next birthday to secure a Pure Endowment on the child's attaining the age of 15 years, all premiums received to be returned with 4 per cent. simple interest in the event of the death of the child before reaching 15?

19. Given $P_x = \cdot 01240$ and $a_{x+n} = 13\cdot 900$ at 4 per cent. interest, what Paid-up Policy could be given in respect of a Whole Life Policy effected at age x which has been n years in force?

20. Prove that

$$\int_0^1 u_x dx = \tfrac{1}{12}(5u_0 + 8u_1 - u_2),$$

and find the value of ${}_{n-1}/q^1_{xy}$, having given that

$$\begin{aligned} {}_{n-1}p_{xy}\,\mu_{x+n-1} &= \cdot 0056482 \\ {}_np_{xy}\,\mu_{x+n} &= \cdot 0056331 \\ {}_{n+1}p_{xy}\,\mu_{x+n+1} &= \cdot 0056173. \end{aligned}$$

SECOND DAY—PART II.

21. Given $l_{71} = 3809$ $l_{72} = 3553$ $l_{73} = 3295$ $l_{74} = 3035$ $l_{75} = 2775$, obtain graphically a rough approximation to $\dfrac{dl_x}{dx}$ when $x = 73$. Compare the value of μ_x thus indicated with that obtained by a formula taking the above five values into account.

22. Given $l_{97} = 58$ $l_{98} = 30$ $l_{99} = 15$ $l_{100} = 7$ $l_{101} = 3$ $l_{102} = 1$ $l_{103} = 0$, construct at 3 per cent. a table for these ages of the following functions :—

$$C_x\ D_x\ L_x\ M_x\ \mathbb{N}_x\ R_x\ S_x\ T_x\ Y_x;$$

the highest available value of v^n being $v^{60} = \cdot 0520$.

23. Evaluate $\int_0^1 tv^t dt$.

Show that for a Makeham table

$$\frac{d}{dx}\bar{a}_{\overline{x|n|}} = -Bc^x\int_0^n v^t(c^t-1)\,{}_tp_x dt.$$

24. Show that $Du_x = e^{\frac{d}{dx}}u_x$.

Hence deduce the fourth and fifth terms in the value of $\Sigma_0^\infty u_x$, which begins $\int_0^\infty u_x dx + \frac{1}{2}u_0 - \frac{1}{12}\frac{du_0}{dx}$.

25. Given tables of annuities on single lives and on two joint lives, how can we get an approximate value for an annuity on three joint lives

(*a*) if Gompertz's law holds;
(*b*) if Makeham's law holds?

Mention two other methods suggested for obtaining an approximation.

26. Answer *two* of the following—

(*a*) Tell what you know of a table which gives $1-\frac{x}{y}$ when x and y are known. Obtain a formula for the value of a paid-up policy available for use with such a table.

(*b*) Having a table with argument $\log x$ and result $\log(1+x)$, show how to construct a table of a_x at a given rate of interest, given a table of p_x.

(*c*) Tell what you know of a table where argument and result are related as $\frac{1}{x+d}-1$ and x (where d is discount). What will be the result of entering with $a_{\overline{x|}}$?

FINAL EXAMINATION.

Examiners—Dr. A. E. Sprague and Mr. Alex. Fraser.

FIRST DAY—PART I.

1. Show that an assurance on the life of a bachelor is equal to $(bA)_x + (bmA)_x$, and deduce a formula expressing this function in terms of $(ba)_x$, $(bma)_x$, and $(bmE)_x$.

2. Give a short account of the method by which Dr. T. B. Sprague constructed his select mortality table from the H^M experience.

3. Give a short account of the method of graduation of the O^M table.

4. Prove Lubbock's formula of approximate summation, and show how the demonstration can be simplified by means of the Calculus of Operations.

Deduce from it a formula expressing a continuous benefit in terms of an ordinary benefit.

5. For ages 10-50 the O^M complete expectation of life can be roughly represented by $\frac{4}{5}(75-x)$.

Show that if this relation were exact, the difference between the reciprocals of μ_x and q_x within these limits of age would be approximately 1·5. Find an expression giving the exact value of this difference.

6. Explain the alternative methods of finding the value of a joint life annuity according to a table which follows Makeham's law.

Give the formulae (for two lives) expressing the value of a joint life annuity in terms of annuities according to the First and Second partial series of the British Offices female annuitants.

FIRST DAY—PART II.

7.* Calculate the value of $\bar{A}_{60:40}^{\;2}$ at O^M 3%. The following are the logarithms of the coefficients of Hardy's summation formula :—

u_0	$\bar{1}$·44716
u_n	0·20952
u_{3n}	0·34242
u_{5n}	0·20952
u_{6n}	$\bar{1}$·74819
u_{7n}	0·20952

8. Discuss the various published tables of premiums for survivorship assurances which have been derived from the recent British Offices experience. How would you frame a table of office annual premiums for such assurances?

9. State how the results of a valuation by the O^M and $O^{M(5)}$ tables may be expected to compare with one by the H^M and $H^{M(5)}$ tables. If the results of both were obtained, what considerations would influence you in deciding which to adopt?

10. Describe Mr. Lidstone's method of obtaining the average age in valuing endowment assurances in groups, and prove that Mr. Manly's is a particular case of the same method.

11. Give an account of the objections which have been taken

* Copies of the O^M Tables and Erskine Scott's logarithms will be supplied.

to the net premium method of valuation. If an Office, valuing on this system, lowers the rate of interest employed in its valuation, how far does the change increase the strength of its reserves, and to what extent is the apparent increase illusory?

12. State the general principles which would guide you in fixing a scale of surrender values for whole life assurances, and mention various methods which have been recommended at different times, giving your opinion of each.

SECOND DAY—PART I.

13. An office has a scheme of endowment assurances where the bonus is given as a reversionary addition payable only on surviving the term of assurance. How would you deal with applications for Paid-up Policies, Surrender Values, and Loan Values?

14. Explain the different methods of treating reversions in the bookkeeping of an office, and state the advantages and disadvantages of each.

15. State your opinion as to the suitability of the 5% short dated securities which have been recently issued by American Railways as an investment for the funds of a progressive life office, distinguishing between (*a*) 3 year notes, (*b*) serial equipment bonds. If accepted, would you deal with them in any special manner in the office books?

16. State the terms of the recent Japanese Loan, and of the loans which it is to replace, and discuss its suitability as an investment for a life office.

17. An office making a net H^M valuation changes to the O^M table and thereby increases its reserves. The results are given in the annual report, together with a statement that the expected claims are less than they would have been under the H^M table. Draft a letter in reply to one from a policyholder stating that these results are inconsistent.

18. State your opinion as to discounted bonus schemes of assurance, with particular reference to their effect on office valuations.

SECOND DAY—PART II.

19. What are the stamp duties on the following documents:—Policy of assurance, whole life, endowment assurance, double endowment; pure endowment with and without return of

premiums; reassurance of each of these; life annuity bond, reversionary life annuity bonds, by annual and by single premiums, deferred annuity bond; agreement; appointment of new trustee; appointment of a trustee of a policy under the Married Women's Acts.

20. Name the various documents which may be required to prove the title to an ordinary British life insurance policy which has never been assigned, giving a short description of each.

21. Draw up a form of statement to be completed in connection with proposals for purchase or loans on the security of reversionary interests. Would you make any distinction between the forms for the two classes of transaction?

22.* What price would you recommend an insurance office to give for a well-secured reversion to £10,000, invested to yield 3½%, on the death of a female aged 65 contingent on a male aged 40 being then alive, together with the income from one half of the estate payable to the male during the joint lifetime of himself and the female.

23. Give short particulars of the more important investigations which have been made during the last 50 years into the sickness and mortality experience of Friendly Societies in Great Britain, and state generally the main trend of the results.

24. Membership of a Widows' Fund was open to Members of a Friendly Society, but owing to various circumstances no new members joined the Fund for many years, and ultimately all existing members and their widows died. A considerable balance of funds remained, as the annuity had been fixed at too low a rate and the rules gave no power to increase it. It is proposed to reconstitute the Fund with power to vary the annuity. State fully the information you would require to enable you to estimate the annuity which could probably be allowed in future.

* Copies of Jones on Annuities (vol. i.), Erskine Scott's logarithms, and an Office Prospectus will be supplied.

REPORT BY THE COUNCIL, 1907.

THE Council have pleasure in submitting their Annual Report.

The following 11 Associates have been admitted Fellows in terms of Rule 3, viz.:—Messrs. G. G. BLACKWOOD, H. N. BOYD, S. D'E. COLAM, W. DAVIDSON, D. L. S. DOUGLAS, W. FENWICK, T. FRAZER, Jr., C. K. GRANGER, F. M. HOPE, J. G. KYD, and W. L. RAMSAY.

The Council have to report with regret the death of one Honorary Fellow, Mr. DAVID CLUNIE GREGOR, an original member of the Faculty; and of two Fellows, Mr. HUGH BLAIR, C.A., and Mr. PHILIP R. D. MACLAGAN.

The total Membership is now 197, viz.:—6 Honorary Fellows, 188 Fellows, and 3 Associates.

28 Students were admitted, 10 became Associates, 4 resigned, and the names of 32 were removed from the roll—27 in terms of Bye-Law 6, and 5 in terms of Bye-Law 16. The number now on the roll of Students is 121.

The Annual Examinations were held on 25th and 26th April last. Seventy-three Candidates entered for examination, viz.:—32 for the First Examination, 19 for the Second, and 22 for the Final. The following is a list of the successful Candidates arranged in order of merit:—

FIRST EXAMINATION.

1. D. S. FRASER.
2. J. B. MACLEAN.

3 and 4 (*Equal*) { C. H. ADAM. / G. O. GUNN.

5 and 6 (*Equal*) { A. G. R. BROWN. / W. M. MORRIS.

7, 8, and 9 (*Equal*) { G. V. GIBSON. / H. E. JONES. / H. C. MATHIESON.

10 and 11 (*Equal*) { T. D. GOVAN / K. SMELLIE.

12. G. RANKEN.

13 and 14 (*Equal*) { C. M'RAE. / W. E. WYSE.

SECOND EXAMINATION.

1. A. COCKBURN.
2 and 3 (*Equal*) R. O. BLYTH. / A. M. HOGG.
4. A. T. MACLEAN.
5. A. A. FYFE.
6. W. MACFARLANE.
7. D. G. YOUNG.
8. R. JEFFREY, Jr.
9. W. H. MEIKLEM.
10. G. H. CLARK.
11. A. W. BEWS.
12. A. J. GREEN.
13. J. M. MACFARLANE.
14. D. M. ROBERTSON.
15. J. B. JACKSON.
16. M. MORHAM.
17. A. D. MACPHAIL.

FINAL EXAMINATION.

1 and 2 (*Equal*) DAVID L. S. DOUGLAS. / CHARLES K. GRANGER.
3. HENRY N. BOYD.
4. STANLEY D'E. COLAM.
5 and 6 (*Equal*) JAMES G. KYD. / JOHN T. SMITH.
7. FRANCIS M. HOPE.
8 and 9 (*Equal*) GEORGE G. BLACKWOOD. / WILLIAM DAVIDSON.

The above 9 gentlemen who passed the Final Examination were admitted Associates of the Faculty in terms of Rule 6, as was also Mr. C. H. L. CATCHLOVE, who passed in 1906.

The Honorary Treasurer's Accounts for the year ended 31st December 1906, with relative Statement of Funds, audited by Mr. A. W. Robertson-Durham, C.A., are herewith submitted. From these it will be seen that the Income of the year was £692, 5s. 11d., the Outgo £572, 5s. 1d., and the Funds at the close of the year £2099, 10s.

Donations of books for the Library have been received from Mr. C. J. Lewis and Mr. J. N. Lewis, Mr. M. M. Dawson, Dr. T. B. Sprague, Mr. James Pringle, Mr Henry Moir, Mr. William Young, Mr. David Heron, and others, to whom the thanks of the Council have been conveyed. A number of books have been purchased. Particulars of these donations and purchases will be found in the book which is kept in the Library for the purpose.

Intimation has already been made of the recent publication, under the joint authority of the Institute of Actuaries and the Faculty, of an inclusive volume of Select Tables, based upon the Male Assurance Experience of the British Offices ($O^{[M]}$ and $O^{[NM]}$ Tables). It having been thought that it would be for the convenience of the

Actuarial profession if the two volumes of Select Tables already published—that issued by the Joint Mortality Committee in June 1903 and that published by Messrs. Baker and Raisin in July 1904—were incorporated in one volume, arrangements were made for the necessary transfer of the rights in the latter volume to the Institute and the Faculty jointly, with a view to the issue of a single inclusive volume of Tables. Upon publication of the new volume the sale of the two existing volumes of Tables was discontinued. The opportunity has been taken to include in the new volume important Additional Tables based upon the $O^{[M]}$ and $O^{[NM]}$ Experiences, including Whole-Life ($O^{[M]}$) Policy-Values, for all entry ages and durations, at six different rates of interest; further complete Tables of term annuities, according to the $O^{[NM]}$ Table; and certain Annuity Tables on Two Lives.

During the Session 1906-1907 the following Papers were read:—

5th November 1906. "The Actuary in Scotland." Inaugural Address by the President, Mr. Archibald Hewat.

3rd December 1906. "Some Notes on the Death Duties from an Actuarial Point of View." By Mr. John Charles Denmead, M.A., F.F.A., F.I.A., Barrister-at-Law.

14th January 1907. "Paid-up Policies and Surrender Values." By Mr. Alfred Ernest Sprague, M.A., D.Sc., F.F.A., F.I.A.

18th February 1907. "Friendly Societies." By Mr. W. A. Sim, F.F.A., F.I.A.

11th March 1907. "Notes on the British Offices Life Annuity Tables (1893) with special reference (*a*) to the persistence of Selection in relation to the Graduated Tables; (*b*) to the computation of Joint Annuities by special applications of the Law of Uniform Seniority." By Mr. Thomas G. Ackland, F.I.A., Honorary F.F.A.

Four of these have been published in the Transactions.

The Council reappointed Mr. John L. Wark, Advocate, as Reporter to the Faculty on law cases connected with actuarial questions or the business of life assurance.

The Council accepted an offer by Mr. Archibald Hewat to present to the Faculty a gold official Badge, suitable to be worn by the President for the time being when officially attending full dress public functions.

The Fifth International Congress of Actuaries was held in Berlin in September last. It was attended by the President, the Honorary Treasurer, and five Fellows, as Delegates of the Faculty, and by several other Fellows. Of the Papers submitted to the Congress, six were by Fellows of the Faculty. Mr. Hewat, President, and Mr. Chatham, Honorary Treasurer, as representing Scotland, were elected a Vice-President and a Secretary respectively of the Congress, and they were also elected members of the Permanent Committee. A gratifying feature was the State recognition accorded to the Congress. The Council record with regret the death, in December last, of Dr. Ferdinand Hahn, Magdeburg, who discharged with much acceptance the duties of President of the Congress.

At the request of the Select Committee of the House of Lords, appointed "to inquire and report what steps should be taken, by deposit of funds or otherwise, to provide adequate security for British policy-holders in Life Insurance Companies which have their chief office outside the United Kingdom, but which carry on business in this Country," the President attended and gave evidence before that Committee on 6th July last. In compliance with an invitation from the Board of Trade for observations or suggestions to assist the Board in considering the question of the amendment of the Life Assurance Companies Acts, a communication was addressed to the Board of Trade on 25th January last. Recently the Board of Trade has invited the views of the Faculty on the provisions of the Employers' Liability Insurance Companies Bill now before Parliament, and the matter is under the consideration of a Special Committee.

In connection with the sale by the Edinburgh Life Assurance Company, for the use of the Royal Society of Edinburgh, of the Company's Building at Nos. 22-24 George Street, the Faculty entered into an Agreement with the Company to vacate, at Whitsunday 1908, the premises at present leased to the Faculty. Under this Agreement it was arranged that the Company, in making their plans, would confer

with the Council and endeavour to provide the Faculty with equally suitable and commodious premises in the Company's new Building, which they propose to erect on the site west of and immediately adjoining the present Building, failing which certain allowances would fall to be made by the Company in connection with the Faculty's removal elsewhere. After careful consideration of the plans of the new Building, the Council came to the conclusion that the accommodation which the Company were able to offer in their new Building would not in their opinion be suitable for the requirements of the Faculty, and accordingly, this having been intimated to the Company, it will be necessary to arrange for other premises for the Faculty elsewhere after Whitsunday 1908.

ARCH: HEWAT,
President.

EDINBURGH, *5th June* 1907.

RECEIPTS AND EXPENDITURE FOR YEAR 1906.

I.—General Fund.

FUND at 31st December 1905							£1,937	19	2			
ENTRANCE FEES—												
Fellows				£16	16	0						
Associates				14	14	0						
Students				18	18	0						
Do. (1907)				2	2	0						
				£52	10	0						
ANNUAL SUBSCRIPTIONS—												
Fellows	£330	15	0									
Do. (1907)	3	3	0									
Associates	12	12	0									
Students	17	5	0									
				363	15	0						
Examination Fees				42	0	0						
Interest and Dividends				60	12	5						
Rents received				15	0	0						
Donation to Funds				21	0	0						
Publications sold				137	8	6						
							692	5	11			
							£2,630	5	1			

COST OF *Transactions*							£128	14	7
LIBRARY—									
Purchase and Binding of Books, etc.							7	19	8
FACULTY'S HALL—									
Rent, less Taxes	£143	13	9						
Rates, Taxes, and Fire Insurance	37	13	5						
Furniture (written off for depreciation)	11	1	3						
Sundries	35	13	9						
							228	2	2
EXAMINATION, MANAGEMENT, AND MISCELLANEOUS EXPENSES—									
Examination Expenses	£43	10	0						
Stationery, Printing, etc.	5	15	0						
Secretary's and Librarian's Allowances	39	7	6						
Miscellaneous Expenses, Postages, etc.	20	5	7						
							108	18	1
JUBILEE DINNER—Expenditure	£162	5	7						
Receipts	63	15	0						
							98	10	7
							£572	5	1
FUND at 31st December 1906							2,058	0	0
							£2,630	5	1

II.—Commutation Fund.

FUND at 31st December 1905	£28	7	0	FUND at 31st December 1906	£41	10	0
Subscription, one Member	13	3	0				
	£41	10	0		£41	10	0

EDINBURGH, 14th *May* 1907.—As instructed by the Council of the Faculty of Actuaries in Scotland, I have examined and audited the foregoing Account of the Intromissions of the Honorary Treasurer of the Faculty for the year ended 31st December 1906, and have found the same to be correctly stated and sufficiently vouched and instructed. I have also examined the Securities and other Documents representing the invested funds at 31st December 1906, and have found them in order.

A. W. ROBERTSON-DURHAM, C.A.

STATEMENT OF FUNDS, 31st DECEMBER 1906.

£750 Caledonian Railway 4 per cent Consolidated Preference Stock, No. 1 (Cost),				£892	17	9
£300 Highland Railway 4¼ per cent Debenture Stock (Cost),				389	2	9
£220 Victoria Government 3½ per cent Inscribed Stock (Cost),				196	13	0
£200 North British Railway 3 per cent Debenture Stock (Cost),				208	7	2
£150 Great Central Railway 4½ per cent Preference Stock (Cost),				195	12	9
Value of Furniture in Faculty's Rooms,				99	11	6
				£1,982	4	11
Sum on Account Current with the Royal Bank,	£98	17	2			
In hands of Hon. Treasurer,	18	7	11	117	5	1
				£2,099	10	0
General Fund,	£2,058	0	0			
Commutation Fund,	41	10	0	£2,099	10	0

NOTE.—In addition to the above Assets the Faculty owns (1) the Library, which contains a large and valuable collection of Books and Pamphlets; (2) one-half share of the rights in the New Experience Tables, published jointly by the Institute and the Faculty; and (3) a considerable stock in hand of the Faculty's *Transactions*.

ANNUAL MEETING, 1907.

The Annual General Meeting of the Faculty of Actuaries in Scotland was held on Monday, 17 June 1907, in the Faculty's Hall, Edinburgh,—Mr. ARCHIBALD HEWAT, President, in the chair. The Report by the Council (see pp. 365-371), which had been circulated among the members, was held as read.

The PRESIDENT said :—In submitting, as I have now the honour of doing, the Report by the Council for the past year—with relative Accounts—I suppose I may assume that, it having been printed and circulated among the members, it will be your pleasure to hold it as read.

In moving the adoption of the Report and Accounts, perhaps you will allow me to make a few observations upon some of the items referred to therein. I think you will agree with me that the Report is one on which we have good cause to congratulate ourselves. Our membership is now all but 200—the net increase in the year being 7. While there are 9 more Fellows, there are an Honorary Fellow and an Associate fewer than we had a year ago. We do not swell our numbers by including students in our membership, nor do we allow our roll of students—now numbering 121—to be unduly inflated by keeping thereon those who have obviously ceased to be students in the sense of having now any intention of presenting themselves for the qualifying examinations. Perhaps the application of the Rule, under which the student renders himself liable to be removed from that Roll if he does not present himself for examination at least once in three years, may have had something to do with the large number who came up for examination in April last. That number—73—is above the average of recent years.

The Faculty holds that when a student has passed our three examinations, and thus becomes possessed of our Certificate of Competency, he is fairly entitled to the status of a qualified Actuary—whether he is content to remain an Associate, which he becomes on passing the Final Examination, or applies to be admitted a Fellow, which he may do at any time thereafter.

Of the 40 students who have this year passed our several

examinations, 21 are in Edinburgh, 13 in Glasgow, 2 in Aberdeen, 1 in Perth, and 3 in England. We are glad to see this widening of our connections. In my Inaugural Address in November last you may remember that I indicated with satisfaction the wide geographical distribution of our membership—which extends far beyond the limits of Scotland.

While the Council desire to give every encouragement to students to come forward for our examinations, they are not disposed to lower their standard of qualification—a standard to which it ought not to be difficult to attain, seeing that nowadays our students have text-books, literature, teachers, special training, and means of observation and practical experience which were not available to the Actuaries of a now past generation. At the same time the Council, through their Examination Committee, desire to make the examinations so fair and reasonable that no really earnest, hard-working, intelligent student can find fault with them. The tendency is I think to broaden—if not to deepen—and never to lose sight of the practical by any undue prominence being given to the theoretical and the perhaps too highly technical. We must remember that ours is a profession having a wide outlook, and one in which a narrowing tendency must be guarded against.

Our membership has all but doubled in number during the past twenty years, while our usefulness, with a corresponding public recognition, has been steadily growing year by year.

We have had five Sessional Meetings during the past year, at which more or less interesting and valuable Papers have been submitted and discussed. The attendances at those Meetings have been above the average, and they encourage us to look for still greater interest being taken in them. At the opening Meeting of the Session I referred to the Fifth International Congress of Actuaries, which was held in Berlin in September last, at which the Faculty had an honoured place. For the first time in the history of these Congresses, Scotland, through the Faculty, then received its rightful independent position—your President and and your Honorary Treasurer having been elected respectively a Vice-President and a Secretary, along with the same representing

each of fifteen other countries, including England, the British Colonies, United States and Canada, Germany, France, Russia, Holland, etc. It is perhaps not sufficiently well known that at that Congress we enjoyed the hospitality of His Majesty the Emperor, who in many ways showed his interest in us and in our work. Much of the success of that Congress was due to the tact, courtesy, and administrative ability of Professor Alfred Manes, Ph.D. (*phil. et jur.*), the General Secretary. Dr. Manes paid his first visit to Scotland a week ago, when he spent four days in this city. One of the "sights" which interested him was this Hall and Library. He now knows more of the Faculty than he knew before, and will keep in touch with us and we with him.

We are kept in view by the Government, through the Board of Trade and otherwise, when it is thought we can be helpful in contemplated legislation and in other ways. As we believe we can render assistance in giving advice and guidance in this and many other directions, we shall always esteem it an honour and a privilege when our services are requisitioned by Government and other public bodies. We are ourselves an increasingly important public body, and in our corporate as well as in our individual capacity we are concerned with vast interests which deeply affect the welfare of the nation and which place heavy responsibilities upon us, to which we have always endeavoured to respond.

While it is no part of our business as a Faculty to pile up Funds, it is gratifying to find that such Funds as we do possess increased during the year by £133. The increase would have exceeded £223 but for an item—or the balance thereof—which appears in our Accounts for the first time in our history, and entered as "Jubilee Dinner." While we "cannot eat our cake and have it" that comparatively small net outlay—once in half a century—was an excellent investment, and a most enjoyable way of preventing the Funds growing too rapidly. At the same time it is well for us to have a substantial sum at our command for emergencies and possible extensions. At present our Funds amount to the modest sum of £2100—the book value at 31st December last being about £77 (nearly double that now) above

the exceptionally depreciated market value, which difference need not distress us. Besides these Funds we have other valuable Assets on which we do not place any actual money value. The Fees, Income from Subscriptions, Interest, Dividends, and Rent received during the past year amount to much the same as in the former year. The outgo on the Library for books, etc., is greater by £39, while that on the Faculty's Hall is practically the same as before. The cost of our *Transactions* was £115 less, and the sales brought in £28 less. In the sales we have included during the last few years the Faculty's one half share of the proceeds of sales of British Offices Life Tables. We must be careful in our selection of Papers for the *Transactions*—as the printing of many of these is, from their nature, very costly. In this, however, we shall avoid doing anything that might discourage those from whom we are entitled to expect valuable contributions to Actuarial Science and Practice suitable for publication.

The closing paragraph of the Report refers to a matter which cannot fail to cause a feeling of regret to all of us. We have been comfortably housed in this convenient and commodious Hall for eight years, and had fully expected to have remained here for many years to come, and no doubt our landlords as confidently hoped we would. But an unexpected opportunity having arisen for our landlords to develop the adjoining corner site, through the Royal Society of Edinburgh requiring just such a building as that of which this Hall forms a part, we resolved to vacate at or soon after Whitsunday of next year—terms mutually agreed upon having been arranged. You may rely upon the Council doing their best for the Faculty in the way of arranging for new and possibly even more convenient accommodation.

I conclude by formally moving that the Report and Accounts now submitted be approved and adopted.

Mr. JOHN LAMB:—I have very much pleasure in seconding the motion for the adoption of the Report, which we are all glad to see shows many signs of the Faculty's continued life and progress. You, Sir, have referred to the more salient points and I need not detain the meeting by adding much to what has been already so well said by you. The only matter for regret is that mentioned

in the last paragraph, and to which you have already referred. We shall soon have to vacate these rooms, where, I am sure you will all agree with me, we have had many pleasant and profitable meetings—not the least successful being the various social functions which have been held here. When originally we took possession we thought we were sure at least of what we have at times heard a good deal of, viz., fixity of tenure. The Fates, however, have decreed otherwise, and we must now make way for our elder sister and good friend, the Royal Society of Edinburgh. I hope, however, the Council will have no great difficulty in finding suitable accommodation elsewhere, and that the new arrangements will be such as will conduce to the comfort and convenience of our Members and Students.

The Report was unanimously adopted.

On the motion of Mr. J. J. M'Lauchlan, the services of Mr. Gordon Douglas and Mr. James Chatham, the Honorary Secretary and Honorary Treasurer respectively, and of Mr. A. W. Robertson-Durham, by whom the Accounts had been audited, were heartily acknowledged.

The usual ballot was then taken for Office-bearers and Members of Council, and the following gentlemen were elected for the year 1907-8, as recommended by the Council:—

PRESIDENT

ARCHIBALD HEWAT.

VICE-PRESIDENTS

NIEL BALLINGAL GUNN.

GEORGE MACRITCHIE LOW.

ANDREW HUGH TURNBULL.

HONORARY SECRETARY	HONORARY TREASURER
GORDON DOUGLAS.	JAMES CHATHAM.

ORDINARY MEMBERS OF COUNCIL

ALEXANDER FRASER, JUN.	WILLIAM HUTTON
ROBERT M. M. RODDICK.	JAMES J. M'LAUCHLAN.
ROBERT MURRIE.	*JOHN EDGAR.
VYVYAN MARR.	*GEORGE C. STENHOUSE.
R. HILL STEWART.	*JOHN LAMB.
CHARLES F. WHIGHAM.	*HUGH W. BROWN.

* New Members.

The PRESIDENT:—I thank you on behalf of myself and my colleagues for our re-election, accepting that as a renewed token of your confidence in us. While we regret to part with Members of Council who have been useful and agreeable we welcome the four new Members—two having been on the Council before, and two entering now for the first time, one of whom I am glad to see is Mr. Edgar, who has done good service as Acting Secretary during the past two years. In his place we are glad to welcome Mr. Thomson. We shall all endeavour to do our best for the Faculty in the discharge of our duties. But I would remind you that as it is you who have elected us it is to you we shall look for the support and encouragement which we must have if all is to go well and the Faculty is to continue to make good progress. I appeal specially to the younger Members for Papers for the ensuing Session. They are not only fresh from their studies, but they also have, presumably, spare time at their disposal. I know no better way—examinations having been passed and a solid foundation having been laid—for widening the horizon, perfecting knowledge, and making a good reputation than by an active participation in the work of the Faculty, especially at the Sessional Meetings. The Acting Secretary will be glad to have early intimation of Papers; and I am sure it will add much to the enjoyment of the summer holidays if those who will thus favour us will make up their mind as to their subject before going away, so that by river side, on mountain top or placid sea, in continental city or on lonely isle, they may have something really interesting to think about, get into form, and bring forth at our Meetings next Session—to their own credit and to the advantage of the Faculty.

Mr. GEORGE M. LOW:—We shall all wish, before we separate, to express to Mr. Hewat our thanks, not only for his conduct as chairman this afternoon, but also for what he has done for the Faculty as President during the past year. We have first to thank him for his very handsome gift of a gold badge to be worn by the President on ceremonial occasions. Then we have to thank him for the way in which he upheld and promoted the interests of the Faculty at the International Congress of Actuaries at Berlin last

year, and for the recognition he has been the means of securing to us as one of the representative bodies of the profession in the world. It has also happened this year that the President has had to deal with several important questions, and it has been most fortunate for us that we have had the advantage of Mr. Hewat's great ability and energy in dealing with these matters. It is peculiarly gratifying that the Board of Trade have so far recognised our position as to refer to us on questions of legislation. Mr. Hewat has been exceedingly useful in giving expression to our views, and we can only hope they will have due influence, and help to guide the decision of the Government as to what it should propose to Parliament. Again, in the matter of our accommodation here, Mr. Hewat as our President was placed in a somewhat difficult position, and I am glad to have the opportunity of giving testimony to the extremely impartial way in which he handled this question. As matters now stand it will be an advantage to us, as it must be a relief to him, to feel that he has now no interests to consider but those of the Faculty, and we shall hope to be able, under his guidance, to arrive at a satisfactory solution of the difficulty that still remains—that of securing new premises. I beg to propose a hearty vote of thanks to Mr. Hewat for his conduct in the chair and for his services to us as President during the past year.

The motion was carried with acclamation, and was suitably acknowledged by Mr. Hewat, after which the proceedings terminated.

CORRESPONDENCE

DAMAGED LIVES

To the Editor of the Transactions of the Faculty of Actuaries,

SIR,—I learn from the correspondence which you have printed in the *Transactions*, that Mr. Low is of opinion that "we cannot by means of a Select-Mortality Table determine at all the number of select lives contained in a body of mixed lives"; whereas this, as I showed in my paper on Select-Mortality Tables, is one of the simplest problems that such Tables can solve. I see also that my son, Dr. Ernest Sprague, has written to you defending my theory, of which he had made use in a paper lately submitted to the Faculty. In these circumstances I think it is right that I should myself say something in defence of the proposition that Mr. Low has attacked, and not leave the whole burden of the defence to fall on others, however willing they may be to support me. At the same time I cannot refrain from mentioning, that I find it extremely irksome to take up again and reconsider a subject that I laid aside so long ago; and that, therefore, whatever reply Mr. Low may make to my present remarks, I shall not be tempted to pursue the matter any further.

It is now more than twenty-six years since my Select-Mortality Tables were published (*J. I. A.* for January 1879 and January 1881). The principles on which they were based were entirely novel, and it was not surprising that a good many years elapsed before there was any evidence that they were received with favour by my professional brethren; but in the year 1896 such evidence was furnished by the resolution of the Council of the Institute of Actuaries, to publish a volume containing my tables, along with others calculated on the same principles. At the request of the Council of the Institute I wrote an Introduction to that volume, in which I described briefly the construction and properties of my tables; and I think that Mr. Low would do well to study that Introduction, if he has not already done so. Further and still more conclusive evidence of the esteem in which the principles of my tables were held by those most competent to judge, was the publication in the year 1903 of two volumes of "Select Tables, computed and published on the authority and under the superintendence of the Institute of Actuaries and the Faculty of Actuaries in Scotland." No writer has hitherto expressed any doubt as to the soundness of the principles on which my tables were based, or of the truth of the conclusions I drew from them. Has Mr. Low, then, discovered a flaw in my reasoning, which has been overlooked by all other actuaries for so many years? I think I shall be able to show that this is not the case, and that the proposition which Mr. Low calls in question, is perfectly true.

The fundamental assumption on which a Select-Mortality table is based, is that the effect of selection wears off after a limited term of years; that is to say, that if the effect of selection lasts for n years and no more, a body of persons who have been insured for n years, will be subject to the same rate of mortality as persons of the same age who have been insured for $n+1$ or more years. I think that in the passage which Mr. Low has quoted from my paper, I have satisfactorily proved that it necessarily follows from the above assumption, that all the damaged lives in a body of mixed lives that have been insured for n years, die off in

the following n years. As Mr. Low does not admit this conclusion, I will endeavour to make the point clearer by expanding the reasoning. Adopting Mr. Low's notation, we have A, or $l_{[30]}$, lives entering at age 30; these after five years are reduced by death to B, or l_{35}, mixed lives of 35; and after five years more they are reduced to D, or l_{40}, mixed lives of 40. Mr. Low now proceeds: "We have also C (or $l_{[35]}$) lives entering at the age of 35." But this is not correct; for these lives do not enter at 35, but are a section of the A select lives that entered at 30, and also of the B mixed lives that were still insured at the age of 35; and their number, C, is calculated from the known number D, by means of the probabilities of living a year, as found from the probabilities of dying in a year contained in my table of the values of q_x, for different ages at entry and different periods since entry (*J. I. A.*, xxii, 394). We thus get successively from D (or l_{40}) the values of

$$l_{[35]+4},\ l_{[35]+3},\ l_{[35]+2},\ l_{[35]+1},\ l_{[35]}, \text{ or C.}$$

This number C, or $l_{[35]}$, that we thus obtain, is the number of select lives of 35 which, after being insured for five years, will give us D, or l_{40}, survivors. Mr. Low appears to ignore altogether the process by which we have obtained C from D; but if he will consider what is the effect of it, I think he must admit that the C select lives of 35 correspond to the D survivors of 40, just in the same way as the A select lives of 30 correspond to the B survivors of 35. From A select lives we calculate, by means of the above-mentioned probabilities, the number of survivors B; and from the D survivors we calculate, by an inverse process, the corresponding number of select lives, C. It follows that C is the number of select lives contained in B; or in other words, $C = B'$. Also $B'' = B - B' = B - C$. Making these substitutions in Mr. Low's equation $B'p' + B''p'' = Cp'$, we get $(B - C)p'' = 0$; and since $B - C$ is not $= 0$, we have $p'' = 0$; and we learn that none of the $B - C$ lives (or $l_{35} - l_{[35]}$ lives) will survive for five years. Since C is the number of select lives in B, $B - C$ is the number of the damaged lives; and we see that it is a necessary consequence of our fundamental assumption, that all the damaged lives die off within five years.

It is quite true that, as Mr. Low points out, experience shows that all damaged lives do not die off in five years; but the proper conclusion from this is that the effect of selection does not wear off in five years. This is a fact which I have repeatedly mentioned in my paper, and which has long been well known. In the Select Tables published by the Institute of Actuaries and the Faculty of Actuaries, the effect of selection is traced for ten years; and it probably lasts, though in a greatly diminished degree, for many years longer. It must therefore be admitted that the numerical conclusions drawn from tables in which selection is traced for five or ten years, are only approximately true; and I agree with Mr. Low in thinking that they should be employed with caution.

Mr. Low professes to apply my "line of reasoning" to a supposed case, in which, he says, selection operates in another direction; but in which there is really no selection at all. He supposes that a Life Office invites persons of a certain class to become members, without any inquiry being made as to their fitness for insurance. In these circumstances the lives admitted, whom he calls sub-standard lives, would be subject to a heavy rate of mortality at first; but "it is reasonable to suppose" that after five years the worst of them would have died and the survivors would be subject to the $H^{M(5)}$ mortality. He proceeds:—"Suppose we have a table "constructed on a plan similar to that of a Select Table, and we endeavour "by means of it to ascertain how many of the ultimate lives of a given age are "still of sub-standard quality. In this case the rate of mortality of ultimate

"lives of 35 will be lower than that of sub-standard lives entering at the "same age, and hence in correspondence with a given number of ultimate "lives of the age of 40, the number of sub-standard lives at 35 will be larger "than the number of ultimate lives at that age"; that is, C is greater than B. There is, I think, a good deal of confusion here. It is, in fact, impossible to make out what is the exact problem which Mr. Low here proposes for solution. What does he mean by "still of sub-standard quality"? Does he mean at the age of 40? Apparently not, for he presently speaks of the ultimate lives of 35. By his supposition the entrants, taken as a whole, are of sub-standard quality; but they are a body of mixed lives, containing a certain number of what he calls the "worst" lives, who all die off in five years; and others, whom he calls ultimate lives. It is therefore incorrect to speak of sub-standard lives of any age as different from the entrants at that age. Nevertheless, Mr. Low says that, reasoning by my method, the ultimate lives of 35 will consist of C sub-standard lives and *minus* (C – B) others. (It is to be noticed that he deals here with only one age—35, or the age at entry; whereas my problem deals with the number of survivors at 35, out of entrants at the age of 30.) The proper conclusion from the facts supposed by Mr. Low, is that in C, the number of sub-standard lives who enter at 35, there are included B ultimate lives, and C – B "worst" lives who all die within five years; and the absurd conclusion drawn by Mr. Low, is not got by any reasoning at all analogous to mine, as explained above. The "danger" is not due to my method of reasoning, but to the application of my formula to a totally different problem, to which it is not applicable.

To sum up:—the assumption that the benefit of selection wears off after n years, is a convenient one to make in the investigation of the problems of selection; but, when we suppose that n is equal to 5 or even 10, this is only approximately, and not strictly true. If it were strictly true, then all the damaged lives would, as I have proved above, die off in the five or ten years after entry; but this, as Mr. Low points out, is not the case. I think, however, that Mr. Low greatly exaggerates when he says that "many" of the damaged lives become select, and are admitted at the ordinary rate of premium on applying a second time for insurance. In forming a Select Table and in drawing conclusions from it, we are entitled to use any facts that observation and experience furnish us with; and one of the most important of these, to the use of which Mr. Low, strangely enough, objects, is the fact that, after the lapse of any number of years, a body of select lives will cease to be select, not in consequence of any deterioration that affects them all, but because a comparatively small number of them have been attacked by various diseases and infirmities which lessen their chances of life, while the majority are still select lives. In conclusion, although a select table cannot be trusted to give us the exact number of damaged lives after the lapse of any number of years, the number given by the table is approximately correct; and this is all that can be said about any conclusion deduced from even the best mortality tables.

I am, etc.,

T. B. SPRAGUE.

EDINBURGH, 26*th May* 1907.

To the Editor of the Transactions of the Faculty of Actuaries,

SIR,—Mr. Low, in his letter of 14th March, refers to the D^{MF} experience as affording formal proof that damaged lives do not all die in five years, and

lest this reference should give rise to some misunderstanding I think it well to point out that the lives forming that experience are not a fair sample of the class of damaged lives. The whole business of insurance is based upon averages; and when speaking of the rate of mortality among a body of select lives of a given age we mean the rate of mortality that will on the average be experienced by such a body. The individuals composing that body do not all possess the same vitality, but range from the life who has barely been accepted without an extra premium, to the most perfect possible specimen of mankind; and it is probably correct to say that no two individuals have exactly the same prospects of life. For the purpose of our business, however, we group all these different lives together under the heading of select lives and treat them as if their prospects of life were identical. Similarly, the class of damaged lives includes some who are very nearly select and are accepted with a small extra; some who are charged a large extra; some who are actually suffering from diseases more or less acute, and are therefore uninsurable; and all who are on their deathbeds. It will therefore be seen that the class of damaged lives comprises a vastly greater variety than the class of select lives; and the differences between special sections of it, and the general average of damaged lives, may therefore be very wide. In particular the section entering into the D^{xr} experience resembles select lives much more closely than it resembles the average of the damaged lives. Those who are on their deathbeds, or who are known to be seriously ill, are not proposed for assurance (save in rare cases of fraud), and the worst of the damaged lives are thus weeded out. Many others are eliminated by the medical examinations and other inquiries made by the Offices, and there is thus a double process of selection, the result being that only the very best of the damaged lives are accepted for assurance and enter into the D^{xr} experience. We are therefore not entitled to draw from that experience any conclusion as to the rates of mortality that prevail in the class of damaged lives as a whole.

Considering now Mr. Low's mathematical investigation, it must be remembered that the problem of finding n unknown quantities from n conditions connecting them, is determinate or indeterminate according as those n conditions are independent or not; but if one of the conditions be discarded and an attempt made to find the n quantities from the remaining $n-1$ conditions, an indeterminate equation is the result whether the problem be in fact soluble or indeterminate. The problem at present in question is to find the two unknown quantities B'' and p'' (using Mr. Low's notation) from two conditions, namely, (1) that the survivors of the two groups B and C after 5 years are equal in number, and (2) that those survivors are subject to the same rates of mortality. But Mr. Low has in his investigation discarded the second condition, and accordingly his resulting equation must of necessity be indeterminate whether the problem be soluble or not. Therefore no conclusion whatever can be drawn from the fact that he has arrived at an indeterminate equation. It is true that he has introduced into his formulas a third quantity, B'; but as it is by definition equal to $B - B''$, there are really only the two above-mentioned unknown quantities to be determined from the two conditions. These conditions are easily seen to be independent, for it is obviously possible to have two groups containing equal numbers but subject to different rates of mortality, or containing different numbers but subject to the same rates of mortality; and it follows that the problem is not indeterminate, but is capable of a definite solution. Dr. T. B. Sprague's solution is therefore not a mere particular case of an infinite number of possible solutions, but represents the complete solution of the problem.

It does not appear to me that Mr. Low has applied Dr. T. B. Sprague's reasoning in a correct manner to the case where selection is supposed to operate in a negative direction. The lines of that reasoning, stated in general terms, are somewhat as follows :—If there are two groups of lives containing different numbers B and C and subject to different rates of mortality, and if after n years these two groups are found to contain precisely the same number of survivors who are thereafter subject to precisely the same rates of mortality, then it follows that the larger of the two original groups is made up of (1) a body numerically equal to the smaller group and subject to the same rates of mortality as that group, and (2) a body numerically equal to the difference between the two groups, and subject to such rates of mortality that they will all die in n years.

Applying this to the hypothetical case put by Mr. Low, we see that Dr. T. B. Sprague's reasoning leads to the conclusion that the entrants aged 35 will consist of B lives subject to the same rates of mortality as the "ultimate" lives, and C – B "damaged" lives who will all die in 5 years. This part of Mr. Low's argument, therefore, seems to be based upon a misapprehension.

With regard to the constitution of the lives in the "ultimate" column of the Select tables, it must be remembered that select lives differ from the population owing to the rejection of a minority of bad lives. It is a fundamental fact, therefore, that the population consists of mixed lives, and this must also (after the outset) be true of assured lives, whose rates of mortality tend to approximate to those of the population as the effect of selection gradually wears off. The proposition that some of the lives in the ultimate column of the Select tables are "select" and others "damaged" is, therefore, not read into the tables from an extraneous source ; but is one of the fundamental facts embodied in the data upon which the tables are based.

I do not know what authority Mr. Low has for his statement that damaged lives "do in many instances become select lives and subsequently gain admission at the ordinary rate of premium," and my own impression is that such instances are rare. Even if the statement be literally correct, the use of the word "many" is likely to mislead some readers by conveying the impression that a substantial *proportion* of damaged lives become select. The whole theory of life contingencies is based on relative numbers or proportions, not upon absolute magnitude ; and even if a considerable *number* of damaged lives were proved to become select, the conclusions to be drawn therefrom would depend entirely upon the *proportion* that number bears to the total number of damaged lives. This proportion might be (and I think it probably is) so small, that it would not seriously disturb the general conclusions to be drawn regarding the damaged lives. Further, it is probable that some of these cases would, under modern conditions, have originally been passed as select lives, and that their apparent improvement merely represents an error in the original classification. But admitting that some damaged lives survive for more than five years, it does not follow that Dr. T. B. Sprague's proposition is wrongly deduced from the assumption that selection wears off in five years—for the error may lie in the original assumption and not in the process by which the proposition was deduced therefrom. In point of fact, that assumption is known to be erroneous, and the effect of selection has been shown to persist for more than ten years, though after that time it is only faintly traceable ; and this is, I submit, the true explanation of any discrepancy between Dr. T. B. Sprague's proposition and the observed facts.

It is to be observed that Mr. Low bases his criticisms largely on the ground that the proposition leads to a conclusion greatly at variance with the results of experience and observation, and I will now show that this

variation is much less than Mr. Low supposes, by putting the proposition to the test of actual facts regarding the number of damaged lives.

We know on the high authority of Dr. Farr (*J. I. A.*, xix, 413) that "it may be broadly stated that 27 in 1000 men of the population of the age of 20 and under 60 are suffering from some kind of disease or other." Now, applying Dr. T. B. Sprague's proposition to the $O^{[M]}$ tables between the ages mentioned, we find that there are 3,214,435 "ultimate" and 3,135,690 "select" lives, and the difference between these numbers, namely, 78,745, is the number of "damaged" lives. The proportion of damaged lives is therefore 24·5 to every 1000 "ultimate" lives, which is about 10 per cent. lower than the proportion of damaged lives in the male population as estimated by Dr. Farr. The class from which ordinary insured lives are drawn is, however, not a fair sample of the whole male population, but represents a higher standard of living and healthier surroundings, so that we should expect it to exhibit a lower proportion of damaged lives. Allowing for this, and for the fact that selection has not entirely worn off in ten years, we see that Dr. T. B. Sprague's principle gives a fair approximation to the true number of damaged lives, and this furnishes independent evidence of the strongest possible nature, of the truth of that principle.

From the foregoing considerations I submit that the following conclusions may be drawn:—

I. If the effect of selection wears off in a definite period of n years, it necessarily follows that all the damaged lives must die in n years.

II. The fact that the effect of selection is traceable for more than the 10 years shown by the $O^{[M]}$ table accounts for the cases where the damaged lives may not all die in 10 years.

III. As the effect of selection is only faintly traceable after 10 years, the number of damaged lives who survive that period must be small compared with the total number of damaged lives.

IV. As the $O^{[M]}$ table is believed to give very close approximations to the true rates of mortality, the conclusions drawn from it regarding damaged lives should on a broad average be in fair accordance with the facts.

I am, etc.,

A. E. SPRAGUE.

22 George Street,
Edinburgh, 31*st May* 1907.

To the Editor of the Transactions of the Faculty of Actuaries,

Sir,—I was lately favoured with a perusal of Dr. T. B. Sprague's letter on this subject, dated 26th May, and I had previously seen Dr. Ernest Sprague's second letter, dated 31st May.

When I took exception to a portion of the paper read by the last named gentleman in January, it did not occur to me that I was raising a question on which there could be serious controversy. I considered that the author of the paper had by certain unqualified expressions elevated to the position of an acknowledged principle what at the best was a conclusion based upon mere assumption, and my hope was that he would so modify those expressions as to bring them into accord with reality. To my surprise it has appeared that he maintains all that the language seemed to imply, and to my great regret I find myself at seeming variance not only with the author of the paper, but with his honoured and distinguished father. I had not

supposed the theory now in question to be regarded by its author in any higher light than as a mere working hypothesis, divorced indeed from actual facts, but convenient as a practical basis for certain calculations. Even now, I can scarcely believe that on the main issue my views are seriously divergent from his, for I observe that Dr. Sprague agrees with me as to the necessity for caution in employing the numerical conclusions resulting from his theory. There would be no need for such caution if, as Dr. Ernest Sprague claims, the problem dealt with is capable of definite solution, and has in fact been completely solved.

Your readers would in other circumstances have welcomed with unalloyed satisfaction a contribution to this correspondence by our esteemed Honorary Fellow, but I am sure they will share my regret for any annoyance he may have been caused by the reopening of a question which he doubtless regarded as long settled. As Dr. Sprague intimates that he does not intend to write again on the subject, it will I think be more suitable, and more consistent with my high regard for him, if I do not take up any controversy with Dr. Sprague himself, but continue the discussion with Dr. Ernest Sprague, whose letters cover all the points that are necessary for the elucidation of the subject.

Until we reach the "conclusions" at the end of that gentleman's second letter, it seems to me difficult to apprehend what is his view on the main question at issue, namely, whether we can or cannot, by means of a select mortality table such as we are familiar with, determine the number of "select" in a body of "mixed" lives. In one place he speaks of the problem as capable of a definite solution, and as having been completely solved, but elsewhere he speaks of this "complete" solution as a "fair approximation," and as being deduced from an "erroneous" assumption. Obviously a solution cannot be definite and complete and at the same time be merely approximate and founded on error. I suppose the meaning to be drawn from those apparently conflicting statements is that if the fundamental hypothesis were correct, the solution would be definite and complete, but the hypothesis being inexact, the solution is necessarily only approximate. This accords with his conclusion No. 1:—"If the effect of selection wears off in a definite period of n years, it necessarily follows that all the damaged lives must die in n years."

Now in spite of all the argument that has been adduced, and with the utmost respect for the high authority invoked, I submit that this conclusion is untenable, and further, that even if it were a proved conclusion, it would by no means follow that we had found a means of determining even approximately the number of select lives in a body of mixed lives. On the contrary, it would follow that we had ascertained the impossibility of solving this problem on the assumption that the effect of selection wears off in a definite short period of years, the attempt to solve it having led to such a conclusion. For consider what "select lives" and "damaged lives" respectively are. Select lives, strictly speaking, are lives which have just been ascertained, after examination and inquiry, to be lives eligible for insurance at the ordinary rate of premium. In this strict sense there are no select lives after n years, because there is no fresh selection exercised by the Insurance Office, but for convenience we employ the term "select lives" to denote lives of such quality that they would be entitled to admission as select if they again applied for insurance and again underwent the usual scrutiny. "Select lives," then, may be briefly defined as lives which are eligible for insurance at the ordinary rate of premium. "Damaged lives," on the other hand (since those two kinds compose the whole body of "ultimate" lives), are lives which, although at one time select, are not now eligible for insurance at the ordinary rate of premium.

Within this description must be embraced all those persons of the "mixed" or "ultimate" class :—

(1) Who are actually the subjects of mortal sickness, or have become the victims of discoverable disease of such a type as to render them unfit for insurance ;
(2) Who have fallen into bad habits ;
(3) Who have contracted or have developed a tendency to any disease or infirmity which increases the risk of insurance, but does not prevent acceptance at an extra premium, large or small according to circumstances ;
(4) Whose family history has developed taints which would either prevent insurance or require the imposition of an extra premium, large or small ;
(5) Who are suffering from illness of a more or less serious nature, rendering them for the time unfit for insurance, or fit only at an extra premium, but who may ultimately recover.

The catalogue could no doubt be extended, but these examples will suffice. To say of those various classes of persons that they must of necessity all die within a limited period, during which the effect of selection is assumed to persist, is so entirely opposed to our knowledge and experience that any argument employed in support of such a proposition may be dismissed as unworthy of serious consideration. I adduced the D^{MF} table as evidence to the contrary, and Dr. Ernest Sprague has thought it worth while to call attention to the fact that the lives embraced in that table differ from the ultimate lives of a select mortality table in respect that the latter include persons in a dying condition, while the former (at the time of entry) do not. Needless to say I had not overlooked so obvious a consideration, but it in no way invalidates the evidence, as my point is established if only a section of the $l_{[x]+n}$ lives resemble those of the D^{MF} table, and this undoubtedly is the case.

Dr. Ernest Sprague, however, cites a passage from Dr. Farr as affording some support to his view. The passage occurs in Dr. Farr's letter to the Registrar-General, introducing the English Life Table No. 2 :—"It may be broadly stated that 27 in 1000 men of the population of the age of 20 and under 60 are suffering from some kind of disease or other." For a writer like Dr. Farr this is a somewhat loose statement, and the precise meaning is not quite apparent. It may, however, be gathered from the context. Dr. Farr gives 100 illustrative cases showing the causes of death and durations of fatal illness as furnished to the Registrar. Extracting from among these the cases of men between the ages of 20 and 60, we find that the average duration of illness was about two years. Now according to the mortality table which Dr. Farr was then introducing, the number of men who die in a year between the ages of 20 and 60 is almost exactly 13·5 per 1000. It may therefore be reasonably inferred that Dr. Farr derived his figure of 27 per 1000 by multiplying the number of deaths by the average duration of fatal illness, and that what he really meant was that at any given time about 27 per 1000 of the male population, within the ages mentioned, were already attacked by their mortal illness. So far therefore as Dr. Farr's evidence goes, it tends to show, not that Dr. Ernest Sprague's figure of 24·5 per 1000 is a "fair approximation" to the true number of damaged lives in a body of mixed lives, but that it is probably wide of the mark, inasmuch as it corresponds with only one of the five classes of damaged lives above detailed, and that possibly not the most numerous, and surely less numerous than all the other classes combined.

On the whole therefore, we may conclude that we cannot arrive at the

number of damaged lives of any age $x+n$ by the simple process of subtracting $l_{[x+n]}$ from $l_{[x]+n}$; and, as no other means of solving the problem presents itself, we may further conclude that we cannot arrive at any true solution by means of a select life table based on the usual hypothesis that selection wears out in a definite short period of years. This establishes my main proposition, and (I venture to submit) justifies the criticism I offered upon Dr. Ernest Sprague's paper.

There remains the question whether, if the hypothesis that selection wears off in n years were true, it would necessarily entail the conclusion that all damaged lives must die in n years. Supposing this were the case, it does not follow that we may proceed to build upon the conclusion even if for practical purposes we adopt the hypothesis. Dr. Ernest Sprague himself says in his first letter—"If the figures in those tables be regarded as approximations of the first order of accuracy, the differences between them will only be approximations of the second order." Now an approximation of the second order may be (as in fact we have seen) very far indeed from accuracy. If, starting from a given point, we wish to reach approximately another point not far distant, it may suit quite well to take a short and easy path that will bring us somewhere near our object; but if, having attained with sufficient nearness our first object, we wish to reach a second that is further distant, we may find the same path lead us far astray, for any divergence from the true path increases with every step we take. Hence the conclusion in question, being based upon a hypothesis admittedly "erroneous," must be received with extreme caution, and cannot be admitted to have even the same degree of validity as the hypothesis itself. In short, if for our convenience we adopt the hypothesis, we are by no means bound to adopt the conclusion, and are not entitled to maintain it as approximately true even if it seems to follow strictly from the hypothesis.

But, in this case, does the conclusion strictly follow? In my previous letter I endeavoured to show that the problem of finding the number of damaged lives on the above hypothesis is an indeterminate one. Dr. Ernest Sprague complains that I left out one of the conditions of the problem, and so rendered it indeterminate, but I think he is mistaken in this, and I observe that Dr. T. B. Sprague (if I may for a moment refer to that gentleman's letter) does not adopt Dr. Ernest Sprague's contention, but on the contrary reaffirms what I had pointed out, that the whole demonstration is contained in the passage quoted in full in my letter, and upon which I founded. We do not transform an indeterminate problem into a determinate one by introducing a new factor or condition unless that serves to give a definite value to one of the quantities hitherto unknown, and Dr. Ernest Sprague does not show that the condition on which he lays stress—the identity of "ultimate" mortality for all ages at entry—has this effect. I consider that in point of fact it has not, because I do not see how by means of it my equation is to be solved, nor has Dr. Ernest Sprague assisted me in this. He did in his previous letter found largely on the condition in question, but if his argument be closely examined, it will be seen that it requires, as an assumption, the very thing that is to be proved—namely, that the increased mortality of mixed lives is due to the presence among them of a small minority of bad lives, so small and so bad that they are all doomed to disappear within the next n years—whereas the condition itself is equally consistent with the contrary assumption, that the "ultimate" lives have all alike become deteriorated. The reasoning, in fact, proceeds in a circle, and ignores some obvious considerations. For example, it leaves no room for the patent fact that damaged lives, so far from all disappearing within a short time, do sometimes regain their place as select lives. Exception is taken to my statement that this happens in "many" cases—my

own experience suggests that it does—but if it happens even in "some" the assumption is vitiated and the claim to an exact solution fails.

We may look at the matter in another light, which yet is the same light as that in which it is presented in my indeterminate equation. There are two opposite ways in which the increased mortality among mixed or ultimate lives might be accounted for—(1) it might be due to a general deterioration in quality, so that the surviving lives present individually less resisting power to the forces which make for death, or (2) it might be due to the presence of a small proportion of lives which have already broken down irretrievably and are doomed to early extinction. These are the two extremes, for the one assumes all the l_{x+n} lives to be of a quality inferior to that of select lives—in other words to be "damaged"—while the other assumes the number of "select" lives to be the greatest that is possible. The latter is the alternative we are asked to adopt, but the former is equally consistent with the material afforded by a select life table, and has, moreover, this in its favour, that it is also consistent with the assumption implied in the use we make of every mortality table, that the lives of a given age in the l column are equally exposed to the risk of death. But neither the one alternative nor the other is tenable, because neither is consistent with known facts. The only possible conclusion is that the truth as to the numbers of "select" and "damaged" lives respectively must lie somewhere between the two extremes, but where precisely it does lie the select life table affords no means of ascertaining.

Two other points remain to be noticed. I had said that the proposition which assumes some of the lives in the ultimate column to be select and some to be damaged, is read into and not evolved from the select life table. Dr. Ernest Sprague says this is not so, because the population consists of select and damaged lives, and the rates of mortality of assured lives tend to approximate to those of the population as the effect of selection wears off. But does not this very statement confirm what I have said, and indicate the extraneous source from which (in Dr. Ernest Sprague's view) the proposition is derived? Then he says that I have not applied Dr. T. B. Sprague's reasoning in a correct manner to the case in which selection is supposed to operate in a negative direction. He restates the reasoning in language of his own, and brings out a different though not necessarily a contradictory conclusion. This does not disprove what I have said, and it still remains that the reasoning which I quoted would, if applied in the same manner to the other case, bring out the impossible result to which I pointed.

I submit, with all deference, that the line of reasoning in question is fallacious. It takes the number of lives in the ultimate column, l_{x+n}, finds by proportion how many lives, selected n years before, would be now represented by l_{x+n} lives, and then assumes that the rate of mortality $(l_x - l_{x+n}) \div l_{[x]}$ must have been in operation among $l_{[x]}$ of the $l_{[x-n]+n}$ lives appearing in the ultimate column as l_x. The table does not warrant this. On the contrary it only teaches us that the l_x lives as a whole become l_{x+n} by being subject to the rate of mortality $(l_x - l_{x+n}) \div l_x$. As well might we take the proportionate number of lives at age x by any other table—for in reality a select life table gives a different mortality table for the n years following each age at entry—and then assume that the rate of mortality shown by that table must have been in operation among so many of the lives.

I am sorry that through my having challenged what appeared to me an unwarranted use of language, it has fallen to me to point out what I conceive to be mistaken in the theory we have been discussing, and I should regret still more if it should be thought that I have unjustifiably attacked the theory in question. To my mind that theory is by no means necessarily bound up with the principle of select life tables, in the development of

which Dr. T. B. Sprague has rendered such distinguished service, and even if the theory were set aside, that would not detract from the great obligation under which he has laid the actuarial profession in regard to all the legitimate uses of such tables.

I am, etc.,

GEO. M. LOW.

28 ST. ANDREW SQUARE,
EDINBURGH, *7th August* 1907.

NOTES ON THE BRITISH OFFICES LIFE ANNUITY TABLES (1893).

To the Editor of the Transactions of the Faculty of Actuaries,

SIR,—My attention has been called to the fact that the values of the ratio $\mu_{x+\frac{1}{2}} \div m_x$, as tabulated in column (9) of Table XI. of my Paper upon the above subject (p. 323 of the present volume), show irregularities in passing through successive quinquennial values of x, which are not in accordance with the theoretical relations of the quantities m_x and $\mu_{x+\frac{1}{2}}$, when deduced on the basis of Makeham's first modification of Gompertz's formula. These irregularities in the Table appear to arise from the fact that I have not computed the values of colog p_x (from which m_x is deduced) and of $\mu_{x+\frac{1}{2}}$, as given in columns (6) and (8) respectively, to a sufficient number of places. Mr. D. C. Fraser, M.A., F.I.A., very kindly sends me the following Table, in which the values are more fully computed :—

x	colog $_{10}p_x$	m_x	$\mu_{x+\frac{1}{2}}$	$\frac{\mu_{x+\frac{1}{2}}}{m_x}$
52	·007,894,424	·018,177,082	·018,173,863	·999,823
57	·010,673,839	·024,576,185	·024,571,662	·999,816
62	·014,978,643	·034,486,181	·034,480,679	·999,840
67	·021,645,995	·049,831,430	·049,827,928	·999,930
72	·031,972,501	·073,586,172	·073,598,004	1·000,161
77	·047,966,365	·110,334,501	·110,413,492	1·000,716
82	·072,737,928	·167,094,849	·167,433,931	1·002,029
87	·111,104,536	·254,441,440	·255,748,136	1·005,136

It will be seen that the ratios, given in the last column above, fall to a minimum at age 57, and then steadily increase to the end of life, whilst the values in column (9) of my Table XI. show great irregularities at successive ages. The correction of the values in column (9) necessarily involves consequential alterations in the following columns (10) to (17) of my Table, but the changes in the final values of the constants A, B, α, β are very slight, and need not now be followed out, as the Table was only intended to illustrate a particular method of experimental graduation.

I am indebted to Mr. John Spencer, F.I.A., for the following interesting and useful demonstration of the relations of m_x and $\mu_{x+\frac{1}{2}}$, when Makeham's law applies :—

Taking the usual approximation for m_x, we have

$$m_x = \frac{q_x}{1-\frac{1}{2}q_x} = 2\,\frac{1-p_x}{1+p_x} = 2\,\frac{1-e^{-\text{colog}_e p_x}}{1+e^{-\text{colog}_e p_x}}$$

$$= \text{colog}_e\, p_x - \frac{(\text{colog}_e\, p_x)^3}{12} + \frac{(\text{colog}_e\, p_x)^5}{120} -$$

$$= \frac{\text{colog}_e\, p_x}{1+\frac{(\text{colog}_e\, p_x)^2}{12}} \text{ approximately.}$$

Again—

$$\mu_{x+\frac{1}{2}} = A + Bc^{x+\frac{1}{2}} = A + B'c^x + B'c^x\left\{\frac{\log_e c}{c-1}.c^{\frac{1}{2}} - 1\right\}$$

where B′ is written for $-\log_e g(c-1)$

$$= \text{colog}_e\, p_x + B'c^x\left\{\frac{\log_e c}{c^{\frac{1}{2}} - c^{-\frac{1}{2}}} - 1\right\}$$

Now $c^{\frac{1}{2}} - c^{-\frac{1}{2}} = e^{\frac{1}{2}\log_e c} - e^{-\frac{1}{2}\log_e c}$

which, after expansion, $= \log_e c + \frac{1}{24}(\log_e c)^3 + \ldots$

$$\therefore \frac{\log_e c}{c^{\frac{1}{2}} - c^{-\frac{1}{2}}} - 1 = \frac{\log_e c}{\log_e c + \frac{1}{24}(\log_e c)^3 + \ldots} - 1$$

$$= -\frac{(\log_e c)^2}{24} \text{ approximately;}$$

and $\mu_{x+\frac{1}{2}} = \text{colog}_e\, p_x - \frac{(\log_e c)^2}{24}B'c^x$ very nearly.

It follows from the above that

$$\frac{\mu_{x+\frac{1}{2}}}{m_x} = \left[1 - \frac{(\log_e c)^2}{24}.\frac{B'c^x}{\text{colog}_e\, p_x}\right]\left[1 + \frac{(\text{colog}_e\, p_x)^2}{12}\right] \text{ approximately,}$$

$$= 1 - \frac{(\log_e c)^2 B'c^x}{24\,\text{colog}_e\, p_x} + \frac{(\text{colog}_e\, p_x)^2}{12}, \text{ neglecting the minute final term;}$$

$$= 1 - \frac{(\log_e c)^2}{24} + \frac{A(\log_e c)^2}{24\,\text{colog}_e\, p_x} + \frac{(\text{colog}_e\, p_x)^2}{12} \quad . \quad . \quad . \quad . \quad (1),$$

after inserting for $B'c^x$ its equivalent $\text{colog}_e\, p_x - A$. The above applies to all tables based on Makeham's hypothesis.

Dealing now with the Male Annuitants Ultimate Table and substituting for A and $\log_e c$ their respective values ·00651642 and ·0874982, we have

$$\frac{\mu_{x+\frac{1}{2}}}{m_x} = \cdot 999681 + \frac{\cdot 0000020787}{\text{colog}_e\, p_x} + \frac{(\text{colog}_e\, p_x)^2}{12} \quad . \quad . \quad . \quad . \quad (2).$$

At the early ages in the Table, where $\text{colog}_e\, p$ only very slightly exceeds A, the ratio is practically equal to unity. As the age advances, the increase in the third term of (2) is for a time less than the decrease in the second term, and the ratio consequently diminishes. Afterwards, however, it increases steadily, until at extreme old ages it appreciably exceeds unity. By differentiation we find that the ratio is a *minimum* when $(\text{colog}_e\, p_x)^3 = \cdot 000012472$, *i.e.* when $\text{colog}_e\, p_x$, and therefore $\mu_{x+\frac{1}{2}}$, approximately $= \cdot 0232$, at which point $\frac{\mu_{x+\frac{1}{2}}}{m_x} = \cdot 999815$.

Mr. Spencer further remarks that the approximation given by (1) up to age 90 is so close that this expression may safely be used in practice in computing values of $\frac{\mu_{x+\frac{1}{2}}}{m_x}$, the labour attending the accurate determination of m_x being thus obviated. Thus, writing (2) in the slightly altered form

$$\frac{\mu_{x+\frac{1}{2}}}{m_x} = \cdot 999681 + \frac{\cdot 0000009028}{\text{colog}_{10}\, p_x} + \cdot 4418\,(\text{colog}_{10}\, p_x)^2$$

and using the values of $\text{colog}_{10}\, p_x$ given above, it will be found that up to age 77 the numerical values of the ratio, as shown in the final column above, are exactly reproduced, while at ages 82 and 87 the approximation exceeds the true value by merely 2 and 7 respectively in the sixth decimal place.

I am, etc.,

THOMAS G. ACKLAND.

26th August 1907.

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VYVYAN MARR.
RALPH HILL STEWART.
CHARLES F. WHIGHAM.
WILLIAM HUTTON.
JAMES JOHN M'LAUCHLAN.
JOHN EDGAR.
GEORGE C. STENHOUSE.
JOHN LAMB.
HUGH W. BROWN.

HALL AND LIBRARY COMMITTEE
THE PRESIDENT AND HON. SECRETARY (*ex officio*)
AND

NIEL BALLINGAL GUNN.
GEORGE MACRITCHIE LOW.
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COMMITTEE ON EXAMINATIONS
THE PRESIDENT AND HONORARY SECRETARY (*ex officio*)
AND

JAMES JOHN M'LAUCHLAN.
JAMES CHATHAM.
WILLIAM HUTTON.
ROBERT MURRIE.
*ALFRED ERNEST SPRAGUE.
*ALEXANDER FRASER, JUNIOR.
*HUGH WYLIE BROWN.
*ALEX. GRAHAM DONALD.
*WILLIAM ABERNETHY SIM
*JAMES BUCHANAN.

* *Examiners*

COMMITTEE ON SESSIONAL MEETINGS AND TRANSACTIONS
THE PRESIDENT AND HONORARY SECRETARY (*ex officio*)
AND

RALPH HILL STEWART.
ALEXANDER FRASER, JUNIOR.
VYVYAN MARR.
†STUART F. M. CUMMING.
†JOHN W. WINK.
†JAMES BUCHANAN.

† *Nominated by Council under Rule 24.*

HONORARY EDITOR OF THE TRANSACTIONS
JOHN ROBERT ARMSTRONG.

SECRETARY
JAMES ALLAN THOMSON, SCOTTISH WIDOWS' FUND LIFE ASSURANCE SOCIETY,
9 ST. ANDREW SQUARE, EDINBURGH.

LIST OF MEMBERS AND STUDENTS

August 1907

HONORARY FELLOWS

Date of Admission.

1905 Ackland, Thomas Gans, F.I.A., F.S.S., F.A.S., *5 & 6 Clement's Inn, Strand, London, W.C.*

1868 Blackburn, Hugh, M.A. (*Late Professor of Mathematics, University of Glasgow*), *Roshven, Moidart, Inverness-shire.*

1868 Fuller, Frederick, M.A., LL.D. (*Late Professor of Mathematics, University of Aberdeen*), *9 Palace Road, Surbiton, Kingston-on-Thames.*

Date of Admission.

1905 Hardy, Ralph Price, F.I.A., *61 Addison Road, London, W.*

1902 Sprague, Thomas Bond, M.A. (Cantab.), LL.D., F.I.A., F.R.S.E. (*Late Manager and Actuary, Scottish Equitable Life Assurance Society*), *29 Buckingham Terrace, Edinburgh.*

1904 Thomson, Spencer Campbell, B.A. (Cantab.), F.R.S.E. (*Late Manager and Actuary, Standard Life Assurance Company*), *10 Eglinton Crescent, Edinburgh.*

FELLOWS

*Those marked thus * have passed the Faculty's Examinations.*

Date of Admission.

1902* Adamson, William, A.I.A., *Scottish Accident Life and General Insurance Company, Ltd., 115 George Street, Edinburgh.*

1901* Adamson, William Livingstone, *Standard Life Assurance Company, 3 George Street, Edinburgh.*

1896* Anderson, David Leslie, C.A., *Secretary, The Linen Thread Company, Ltd., 180 Hope Street, Glasgow.*

1864 Anderson, James John Philip (*Late Secretary, Scottish Widows' Fund and Life Assurance Society*), *51 Melville Street, Edinburgh.*

Date of Admission.

1903* Anderson, Thomas Frederic, F.I.A., *Royal Exchange Assurance Corporation, Royal Exchange, London, E.C.*

1898* Armstrong, John Robert (Honorary Editor), *Scottish Provident Institution, 6 St. Andrew Square, Edinburgh.*

1902* Bannatyne, William, *Scottish Temperance Life Assurance Company, Ltd., 105 St. Vincent Street, Glasgow.*

FELLOWS

*Those marked thus * have passed the Faculty's Examinations.*

Date of Admission.

1907* Blackwood, George Glendinning, *Edinburgh Life Assurance Company, 22 George Street, Edinburgh.*

1898* Blount, Edward Thomas Joseph, A.I.A., F.S.S., *Standard Life Assurance Company, Shanghai.*

1878 Bloxsom, William Gibson, *Managing Director, The Profits and Income Insurance Company, Ltd., 9 Fleet Street, London, E.C.*

1893 Blyth, Robert, C.A., *General Manager, Union Bank of Scotland, Limited, Glasgow.*

1902* Bowie, John, *Scottish Temperance Life Assurance Company, Ltd., 105 St. Vincent Street, Glasgow.*

1907* Boyd, Henry Norris, *Secretary, City of Glasgow Life Assurance Company, 21 St. Andrew Square, Edinburgh.*

1905* Boyd, James Colville, *North British and Mercantile Insurance Company, 64 Princes Street, Edinburgh.*

1886* Bremner, Thomas William, A.I.A., *Mutual Life of New York Building, Martin Place, Sydney.*

1904* Brodie, Robert Raynal, A.I.A., *Scottish Provident Institution, 6 St. Andrew Square, Edinburgh.*

1897* Brown, Hugh Wylie, F.I.A., *Actuary, Scottish Union and National Insurance Company, 35 St. Andrew Square, Edinburgh.*

1902* Buchanan, James, F.I.A., D.Sc. (Glasgow), M.A. (Cantab.), *Scottish Widows' Fund and Life Assurance Society, 9 St. Andrew Square, Edinburgh.*

1904* Cameron, Finlay James, A.I.A., *Friends' Provident Institution, 45 Darley Street, Bradford, Yorkshire.*

Date of Admission.

1885* Cameron, George, *Actuary, Standard Life Assurance Company, 3 George Street, Edinburgh.*

1904* Cameron, John Campbell, M.A., *Scottish Metropolitan Life Assurance Company, Ltd., 25 St. Andrew Square, Edinburgh.*

1893* Campbell, Neil, *Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.*

1871* Carment, David, F.I.A., F.A.S., *Assistant Actuary, Australian Mutual Provident Society, Sydney.*

1890* Carment, David Malcolm, *Assistant Manager, Law Guarantee and Trust Society, 49 Chancery Lane, London, W.C.*

1904* Cathles, Lawrence Maclagan, A.I.A., *Actuary, Franklin Life Insurance Company, Springfield, Illinois, U.S.A.*

1886* Chatham, James, F.I.A. (Honorary Treasurer), *Manager, North British and Mercantile Insurance Company, 64 Princes Street, Edinburgh.*

1882* Chisholm, James, F.I.A., F.A.S., *(Late Manager and Actuary, Imperial Life Insurance Company), Crossfield, Alberta, N.W.T., Canada.*

1900* Clapperton, David Alexander, C.A., *6A George Street, Edinburgh.*

1874* Cockburn, Henry, F.I.A., F.A.S., *Manager of Life Dept. and Actuary, North British and Mercantile Insurance Company, 61 Threadneedle Street, London, E.C.*

1907* Colam, Stanley d'Eyncourt, *Yorkshire Insurance Company, York.*

FELLOWS

*Those marked thus * have passed the Faculty's Examinations.*

Date of Admission.

1878* Cousin, John William, (*Late Assistant Actuary, Edinburgh Life Assurance Company*), 58 *Newbattle Terrace, Edinburgh.*

1906* Cowan, Hugh Francis, A.I.A., *Edinburgh Life Assurance Company*, 22 *George Street, Edinburgh.*

1902* Cumming, Stuart Ferrier Macdonald, *Life Association of Scotland*, 82 *Princes Street, Edinburgh.*

1907* Davidson, William, *Scottish Amicable Life Assurance Society*, 35 *St. Vincent Place, Glasgow.*

1901* Denham, Walter, F.I.A., *City of Glasgow Life Assurance Company*, 30 *Renfield Street, Glasgow.*

1905* Denmead, John Charles, M.A. (Cantab.), F.I.A., *Estate - Duty Office, Somerset House, London, W.C.*

1876* Deuchar, John James Walker, F.I.A., *General Manager and Actuary, Norwich Union Life Insurance Society, Norwich.*

1901* Donald, Alexander Graham, M.A., A.I.A., *Scottish Provident Institution*, 6 *St. Andrew Square, Edinburgh.*

1907* Douglas, David Lawson Scott, *Standard Life Assurance Company*, 3 *George Street, Edinburgh.*

1884* Douglas, Gordon, F.I.A. (Honorary Secretary), *Actuary, Life Association of Scotland*, 82 *Princes Street, Edinburgh.*

1879* Dovey, William Roadley, A.I.A., F.A.S., 62 *Weston Park, Crouch End, London, N.*

1889* Duff, Campbell Kerr, C.A., 65 *Bath Street, Glasgow.*

Date of Admission.

1891 Duncan, Alexander (*Late General Manager, Scottish Union and National Insurance Company*), *Edinburgh.*

1900* Edgar, John, *Edinburgh Life Assurance Company*, 22 *George Street, Edinburgh.*

1877* Evans, William, A. I. A., F.R.S.E. (*Late Assistant Actuary, Scottish Widows' Fund and Life Assurance Society*), 38 *Morningside Park, Edinburgh.*

1902* Falconer, Herbert Francis, 21 *West Mayfield, Edinburgh.*

1896* Fenton, Francis Kerr, *Scottish Amicable Life Assurance Society*, 1 *Threadneedle Street, London, E.C.*

1891* Fenton, James, *North British and Mercantile Insurance Company*, 64 *Princes Street, Edinburgh.*

1907* Fenwick, William, *Life Association of Scotland*, 82 *Princes Street, Edinburgh.*

1898* Ferguson, John, *Assistant Actuary, Standard Life Assurance Company*, 3 *George Street, Edinburgh.*

1906* Forrest, James Nairne, *Scottish Equitable Life Assurance Society*, 28 *St. Andrew Square, Edinburgh.*

1888* Fox, Charles Edward, A.I.A., *Standard Life Assurance Company*, 83 *King William Street, London, E.C.*

1897* Fraser, Alexander, Jun., F.I.A., *Actuary, Scottish Life Assurance Company, Ltd.*, 19 *St. Andrew Square, Edinburgh.*

1907* Frazer, Thomas, Junior, *Scottish Union and National Insurance Company*, 35 *St. Andrew Square, Edinburgh.*

FELLOWS

*Those marked thus * have passed the Faculty's Examinations.*

Date of Admission.

1904* Fyfe, Austyn James Claude, *Northern Assurance Company, 1 Union Terrace, Aberdeen.*

1897* Gaff, William Robertson, C.A., A.I.A. (Messrs. Gaff, Harper & Company), *53 New Broad Street, London, E.C.*

1899* Gibb, James Burnett, A.I.A., *Penn: Mutual Life Insurance Company, 923 Chestnut Street, Philadelphia, U.S.A.*

1891* Gibson, William, *Assistant Actuary, Caledonian Insurance Company, 19 George Street, Edinburgh.*

1906* Gillhespy, William Edward, *Yorkshire Coalowners Mutual Indemnity Company, Foster's Buildings, Sheffield.*

1884* Gillison, John Brotch, F.I.A., *Manager for the United Kingdom, National Mutual Life Association of Australasia, Limited, 5 Cheapside, London, E.C.*

1891* Glen, Ninian, M.A., B.Sc., C.A., *107 St. Vincent Street, Glasgow.*

1875* Gordon, Charles, F.I.A., *Actuary, South African Mutual Life Assurance Society, Darling Street, Cape Town.*

1892* Govan, John, *Norwich Union Life Insurance Society, Norwich.*

1905* Graham, George, Junior. *Provident Savings Life Assurance Society, 346 Broadway, New York, U.S.A.*

1883* Graham, James, F.I.A., *Actuary, Australian Widows' Fund Life Assurance Society, Melbourne.*

1907* Granger, Charles Keith, *City of Glasgow Life Assurance Company, 30 Renfield Street, Glasgow.*

Date of Admission.

1871* Gunn, Niel Ballingal, F.I.A. (Vice-President), *Manager and Actuary, Scottish Widows' Fund and Life Assurance Society, 9 St. Andrew Square, Edinburgh.*

1895* Guthrie, Charles, *Secretary, Scottish Amicable Life Assurance Society, 35 St. Vincent Place, Glasgow.*

1892* Halliday, William Ross, *4 Broad Street Place, London, E.C.*

1905* Hamilton, William, *North British and Mercantile Insurance Company, 64 Princes Street, Edinburgh.*

1904* Harrison, Edward John, C.A., *1 Broad Street Place, London, E.C.*

1877* Hewat, Archibald, F.I.A. (President), *Manager and Actuary, Edinburgh Life Assurance Company, 22 George Street, Edinburgh.*

1907* Hope, Francis Moffat, *Caledonian Insurance Company, 19 George Street, Edinburgh.*

1856† Howden, James, C.A., *8 York Place, Edinburgh.*

1893* Hunter, Arthur, A.I.A., F.A.S., *Actuary, New York Life Insurance Company, 346 and 348 Broadway, New York, U.S.A.*

1906* Hunter, Robert Marshall, *Scottish Provident Institution, 6 St. Andrew Square, Edinburgh.*

1895* Hutcheson, William Anderson, F.I.A., F.A.S., *Associate Actuary, Mutual Life Insurance Company of New York, 32 Nassau Street, New York, U.S.A.*

† Original Member.

FELLOWS

*Those marked thus * have passed the Faculty's Examinations.*

Date of Admission.

1895* Hutton, William, F.I.A., *Manager and Actuary, Scottish Amicable Life Assurance Society, 35 St. Vincent Place, Glasgow.*

1904* Imrie, John Hamilton, M.A., A.I.A., *Life Association of Scotland, 82 Princes Street, Edinburgh.*

1905* Jackson, Samuel, *Resident Secretary, Scottish Widows' Fund and Life Assurance Society, 48 Castle Street, Liverpool.*

1903* Jamieson, Charles William Steele, A.I.A., *Scottish Amicable Life Assurance Society, 1 Threadneedle Street, London, E.C.*

1899* Kilpatrick, Hugh Grainger, *Liverpool and London and Globe Insurance Company, 1 Dale Street, Liverpool.*

1885 King, George, F.I.A., F.A.S., *15 Walbrook, London, E.C.*

1907* Kyd, James Gray, *Northern Assurance Company, 1 Union Terrace, Aberdeen.*

1877* Kyd, Thomas, F.I.A., *Resident Manager, Northern Assurance Company, 1 Union Terrace, Aberdeen.*

1905* Laing, James Murray, F.I.A., *National Mutual Life Association of Australasia, 5 Cheapside, London, E.C.*

1885* Lamb, John, C.A., *Secretary, Scottish Provident Institution, 6 St. Andrew Square, Edinburgh.*

1896* Latta, Alexander, A.I.A., *Guardian Assurance Company, Ltd., 28 King Street, Covent Garden, London, W.C.*

1896* Laughton, Alexander Millar, F.I.A., *National Mutual Life Association of Australasia, Limited, Melbourne.*

Date of Admission.

1898* Lees, Mungo Mackenzie, *Assistant Secretary, Norwich Union Life Insurance Society, Norwich.*

1895* Lewis, John Norman, F.I.A., *London Assurance Corporation, 7 Royal Exchange, London, E.C.*

1887* Lisle, George, C.A., *5 North St. David Street, Edinburgh.*

1874* Low, George Macritchie, F.I.A., F.R.S.E. (Vice-President), *Manager and Actuary, Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.*

1888* Lugton, Hugh, A.I.A., *Assistant Actuary, North British and Mercantile Insurance Company, 61 Threadneedle Street, London, E.C.*

1856 Macandrew, James M'Lean, C.A., *The Pulteney Hotel, Pulteney Street, Bath.*

1870* Macdonald, William Rae, A.I.A., *Secretary and Actuary, Scottish Metropolitan Life Assurance Company, Ltd., 25 St. Andrew Square, Edinburgh.*

1904* M'Ilvenna, William Richard, *Scottish Equitable Life Assurance Society, 14 Westmoreland Street, Dublin.*

1903* M'Intosh, Archibald, *Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.*

1906* M'Intosh, James, *North British and Mercantile Insurance Company, 64 Princes Street, Edinburgh.*

1903* Mackay, Robert, *Secretary, Northern Assurance Company, 1 Union Terrace, Aberdeen.*

1902* M'Kean, Philip Campbell, *Scottish Widows' Fund and Life Assurance Society, 9 St. Andrew Square, Edinburgh.*

FELLOWS

*Those marked thus * have passed the Faculty's Examinations.*

Date of Admission.

1879* Mackenzie, Alexander George, F.I.A., *29 Chester Terrace, Regent's Park, London, N.W.*

1905* M'Laren, George Cunningham, *South African Mutual Life Assurance Society, Darling Street, Cape Town.*

1877* M'Lauchlan, James John, *Secretary, Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.*

1893* Macphail, Donald, F.I.A., *Yorkshire Insurance Company, York.*

1889* Manson, John, M.A., *Scottish Widows' Fund and Life Assurance Society, 9 St. Andrew Square, Edinburgh.*

1898* Marr, Vyvyan, F.I.A., *Edinburgh Life Assurance Company, 22 George Street, Edinburgh.*

1899* Maxwell, Benjamin Bell, A.I.A., *Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.*

1897* Meikle, Henry George Watson, *Actuary, Oriental Government Security Life Assurance Company, Bombay.*

1892* Miller, Robert Duncan, *Actuary, Mutual Life Association of Australasia, George and Wynyard Streets, Sydney.*

1876* Milligan, James Macmillan, *Messrs. J. & J. Milligan, W.S., 15 York Place, Edinburgh.*

1893* Moir, Henry, F.I.A., F.A.S., *Actuary, Provident Savings Life Assurance Society, 346 Broadway, New York, U.S.A.*

1884* Murrie, Robert, *Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.*

Date of Admission.

1906* Nicholl, Charles Carlyon, B.A. (Cantab.), A.I.A., *Royal Exchange Assurance Corporation, Royal Exchange, London, E.C.*

1890* Nicol, William Smith, *General Manager, City of Glasgow Life Assurance Company, 30 Renfield Street, Glasgow.*

1886* Nicoll, John, A.I.A., *Life Association of Scotland, 82 Princes Street, Edinburgh.*

1885* Nisbet, Robert James, *North British and Mercantile Insurance Company, 64 Princes Street, Edinburgh.*

1880* Notman, John, *Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.*

1888* Orr, Lewis Potter, A.I.A., *Secretary, Scottish Life Assurance Company, Ltd., 19 St. Andrew Square, Edinburgh.*

1899* Padday, Percy King, A.I.A., *Actuary, Scottish Metropolitan Life Assurance Company, Ltd., 8 King Street, Cheapside, London, E.C.*

1873* Park, David Francis, C.A., A.I.A., *Crédit Foncier of Mauritius, 39 Lombard Street, London, E.C.*

1893 Paulin, David, F.R.S.E., *Manager, Scottish Life Assurance Company, Ltd., 19 St. Andrew Square, Edinburgh.*

1897* Pearce, Henry John, Jun., A.I.A., *Secretary, Scottish Amicable Life Assurance Society, 1 Threadneedle Street, London, E.C.*

1862* Pearson, David, C.A., *64 Queen Street, Edinburgh.*

1894* Prentice, Alexander, *Secretary, Life Association of Scotland, 32 Mosley Street, Newcastle-on-Tyne.*

FELLOWS

*Those marked thus * have passed the Faculty's Examinations.*

Date of Admission.

1904* Price, Edmund Forbes, *Standard Life Assurance Company, 3 George Street, Edinburgh.*

1874* Pringle, James, C.A., A.I.A., *42 Drumsheugh Gardens, Edinburgh.*

1886* Pullar, James, A.I.A., *Secretary and Actuary, Colonial Mutual Life Assurance Society, 419 Collins Street, Melbourne.*

1906* Ramsay, William Livingstone, M.A., *Edinburgh Life Assurance Company, 11 King William Street, London, E.C.*

1903* Rankin, John Adam, A.I.A., *Edinburgh Life Assurance Company, 22 George Street, Edinburgh.*

1886* Reid, Walter Alexander, C.A., *Messrs. James Meston & Co., 6 Golden Square, Aberdeen.*

1896* Rennie, William John Martin, *North British and Mercantile Insurance Company, 64 Princes Street, Edinburgh.*

1891 Richardson, Josephus Hargreaves, A.I.A., F.A.S., *Government Insurance Commissioner, Wellington, N.Z.*

1892* Robertson, Archibald, *Secretary, Caledonian Insurance Company, Caledonian Buildings, Tithebarn Street, Liverpool.*

1904* Robertson, Frederick William, A.I.A., *Caledonian Insurance Company, 19 George Street, Edinburgh.*

1904* Robertson, James Leask, A.I.A., *Edinburgh Life Assurance Company, 22 George Street, Edinburgh.*

1889* Robertson, William, A.I.A., *29 Stafford Street, Edinburgh.*

Date of Admission.

1900* Robertson, William Alexander, *Scottish Union and National Insurance Company, 35 St. Andrew Square, Edinburgh.*

1901* Robertson-Durham, Alexander Weir, C.A., *Messrs. A. & J. Robertson, C.A., 33 Charlotte Square, Edinburgh.*

1874* Robertson-Durham, James Alexander, C.A., *Manager, Scottish Reversionary Company, Ltd., 33 Charlotte Square, Edinburgh.*

1898* Roddick, Robert Murray M'Cheyne, M.A., *Actuary, Century Insurance Company, Limited, 18 Charlotte Square, Edinburgh.*

1884 Rolland, John Henry William, C.A., *General Secretary for England, Standard Life Assurance Company, 83 King William Street, London, E.C.*

1872* Ronaldson, Peter, C.A., *Actuary, Edinburgh Reversion Company, Limited, 3A North St. David Street, Edinburgh.*

1903* Ross, Frederick Alexander, C.A., *Messrs. Moncreiff & Horsbrugh, C.A., 46 Castle Street, Edinburgh.*

1902 Sanderson, Frank, M.A., A.I.A., F.A.S., *Actuary, Canada Life Assurance Company, Toronto, Ontario.*

1889* Sewell, Richard, C.A. (C. F. Burton, Sewell & Co., C.A., 35 Copthall Avenue, London), *General Manager and Actuary, British Widows' Assurance Company, Limited, 1 Old Street, London, E.C.*

1891* Shearer, Gilbert Edward, A.I.A., *Scottish Provident Institution, 3 Lombard Street, London, E.C.*

FELLOWS

*Those marked thus * have passed the Faculty's Examinations.*

Date of Admission.

1896* Sim, William Abernethy, F.I.A., *Scottish Union and National Insurance Company, 35 St. Andrew Square, Edinburgh.*

1891* Slimon, William James, *North British and Mercantile Insurance Company, 64 Princes Street, Edinburgh.*

1907* Smith, John Tasker, *Britannic Assurance Company, Limited, 178 Avenue Parade, Accrington, Lancashire.*

1885 Smith, John Turnbull, LL.D., C.A., *Manager, Life Association of Scotland, 82 Princes Street, Edinburgh.*

1886* Smith, Joseph Brown, C.A., *Prell's Buildings, 60 to 70 Queen Street, Melbourne.*

1899* Smith, Randolph Gordon, F.I.A., *Assistant Actuary, Scottish Amicable Life Assurance Society, 35 St. Vincent Place, Glasgow.*

1884 Smith, Walter Arthur, *Manager and Secretary, English and Scottish Law Life Assurance Association, 41 Charlotte Square, Edinburgh.*

1877* Sorley, James, F.I.A., C.A., F.R.S.E., *32 Onslow Square, London, S.W.*

1894* Sprague, Alfred Ernest, F.I.A., D.Sc. (Edinburgh), M.A. (Cantab.), (Honorary Librarian), *Assistant Secretary, Edinburgh Life Assurance Company, 22 George Street, Edinburgh.*

1905* Steell, Graham, *Standard Life Assurance Company, 3 George Street, Edinburgh.*

1886* Stenhouse, George Crighton, *Assistant Actuary, Scottish Widows' Fund and Life Assurance Society, 9 St. Andrew Square, Edinburgh.*

Date of Admission.

1887* Stewart, John, F.I.A., *Actuary, City of Glasgow Life Assurance Company, 30 Renfield Street, Glasgow.*

1893* Stewart, Ralph Hill, *Secretary and Actuary, Caledonian Insurance Company, 19 George Street, Edinburgh.*

1892* Stirling, James, *Sub-Manager, Life Department, Law Union and Crown Insurance Company, 126 Chancery Lane, London, W.C.*

1890* Stirling, Robert, F.I.A., *Secretary, Rock Life Assurance Company, 15 New Bridge Street, London, E.C.*

1884* Stuart, John Moody, F.I.A., *Manager and Actuary, Leeds Permanent Benefit Building Society, 18 Park Lane, Leeds.*

1866* Surenne, David John, *(Late Secretary, Caledonian Insurance Company), 29 Inverleith Row, Edinburgh.*

1881* Tait, Edwin, *North British and Mercantile Insurance Company, 91 Union Street, Aberdeen.*

1893 Teece, Richard, F.I.A., F.A.S., *General Manager and Actuary, Australian Mutual Provident Society, Sydney.*

1893 Thiselton, Herbert Cecil, F.I.A., F.A.S., *Life Manager and Actuary, Commercial Union Assurance Company, Ltd., 24 Cornhill, London, E.C.*

1904* Thomson, Gordon William, *African Life Assurance Company, Johannesburg.*

1906* Thomson, James Allan (Secretary), *Scottish Widows' Fund and Life Assurance Society, 9 St. Andrew Square, Edinburgh.*

FELLOWS

*Those marked thus * have passed the Faculty's Examinations.*

Date of Admission.

1904* Thomson, John Walter, A.I.A., *Scottish Life Assurance Company, Ltd.*, 19 *St. Andrew Square, Edinburgh.*

1895* Turnbull, Andrew Dalzel Lindsay, A.I.A., C.A., *Assistant Secretary, Scottish Widows' Fund and Life Assurance Society*, 9 *St. Andrew Square, Edinburgh.*

1862* Turnbull, Andrew Hugh, F.I.A., F.R.S.E. (VICE-PRESIDENT), (*Late Manager and Actuary, Scottish Widows' Fund and Life Assurance Society*), *The Elms, Whitehouse Loan, Edinburgh.*

1902* Turnbull, Phipps, 1 *Royal Circus, Edinburgh.*

1885* Walker, Davidson, *Secretary, Norwich Union Life Insurance Society, Norwich.*

1897* Wallace, David Eckford, C.A., 105 *Hanover Street, Edinburgh.*

1869* Wallace, Thomas, F.I.A., *Actuary, North British and Mercantile Insurance Company*, 64 *Princes Street, Edinburgh.*

1904* Wallace, Thomas, Jun., *Guardian Assurance Company, Ltd.*, 27 *Cross Street, Manchester.*

1881* Walton, William Gandy, A.I.A., *Actuary, Scottish Provident Institution*, 6 *St. Andrew Square, Edinburgh.*

Date of Admission.

1884* Warden, John Mabon, *Assistant Actuary, Scottish Equitable Life Assurance Society*, 28 *St. Andrew Square, Edinburgh.*

1891 Watson, James Graham, C.A., *Manager, Scottish Provident Institution*, 6 *St. Andrew Square, Edinburgh.*

1897* Watt, James, A.I.A., W.S., 28 *Charlotte Square, Edinburgh.*

1896* Webster, George Rae, C.A., *Messrs. Price, Waterhouse & Company, Monadnock Building, San Francisco, U.S.A.*

1898* Whigham, Charles Frederick, A.I.A., C.A., 46 *Castle Street, Edinburgh.*

1899* Wilkie, William Thom, *North British and Mercantile Insurance Company*, 64 *Princes Street, Edinburgh.*

1905* Wilson, William Maxwell Gunn, *Scottish Widows' Fund and Life Assurance Society*, 9 *St. Andrew Square, Edinburgh.*

1904* Wink, John Whyte, *Scottish Metropolitan Life Assurance Company, Ltd.*, 25 *St. Andrew Square, Edinburgh.*

1888* Wood, William Alexander, C.A., 4 *Melville Street, Edinburgh.*

1898* Young, William, F.A.S., *New York Life Insurance Company*, 346 *and* 348 *Broadway, New York, U.S.A.*

ASSOCIATES

All of whom have passed the Faculty's Examinations.

Date of Admission.

1906 Borland, Walter, *Scottish Temperance Life Assurance Company, Ltd.*, 105 *St. Vincent Street, Glasgow.*

Date of Admission.

1906 Catchlove, Charles Hamilton Leyland, F.I.A., *Australian Mutual Provident Society, Sydney, New South Wales.*

STUDENTS

The figures indicate the number of Examinations to be passed for qualification as Associate.

Date of Enrolment.

1907 [2]Adam, Colin Hay,
64 Forest Road, Aberdeen.

1904 [1]Adams, James Napier,
Caledonian Insurance Company, 19 George Street, Edinburgh.

1906 [3]Allin, Henry Philip,
66 St. Barnabas Road, Cambridge.

1900 [2]Andrew, Alexander,
Scottish Accident Life and General Insurance Company, Ltd., 115 George Street, Edinburgh.

1902 [1]Barnett, John,
Messrs. Lindsay, Jamieson & Haldane, C.A., 24 St. Andrew Square, Edinburgh.

1906 [3]Barron, Albert Gibson,
68 Spottiswoode Street, Edinburgh.

1906 [3]Begg, David,
Messrs. Hardie & Grieve, C.A., 157 St. Vincent Street, Glasgow.

1902 [1]Bews, Alfred William,
City of Glasgow Life Assurance Company, 30 Renfield Street, Glasgow.

1905 [1]Blyth, Robert Oswald,
Scottish Amicable Life Assurance Society, 35 St. Vincent Place, Glasgow.

1905 [3]Bolton, James Butler,
Messrs. Niven & Sutherland, C.A., 16 Young Street, Edinburgh.

1905 [3]Bonnar, James Norman,
Scottish Life Assurance Company, Ltd., 19 St. Andrew Square, Edinburgh.

1907 [2]Brown, Archibald Gray Robertson,
Schoolhouse, Blackridge, Linlithgowshire.

1885 [1]Brown, George,
Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.

Date of Enrolment.

1907 [3]Campbell, Malcolm Norman,
Messrs. Smith & Williamson, C.A., 142 St. Vincent Street, Glasgow.

1906 [3]Campbell, Samuel Wilson,
Scottish Provident Institution, 6 St. Andrew Square, Edinburgh.

1904 [1]Clark, George Hutchison,
Scottish Union and National Insurance Company, 35 St. Andrew Square, Edinburgh.

1906 [1]Cockburn, Arthur,
Scottish Metropolitan Life Assurance Company, Ltd., 25 St. Andrew Square, Edinburgh.

1898 [1]Collier, Charles St. John,
1 Cornhill, London, E.C.

1897 [1]Cownie, Thomas,
Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.

1899 [1]Crawford, William Lindsay,
National Mutual Life Association of Australasia, Limited, 5 Cheapside, London, E.C.

1887 [1]Crow, John Reid,
Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.

1896 [2]Cumming, Robert, B.A.,
Assistant District Manager, Commercial Union Assurance Company, Ltd., 10 North St. David Street, Edinburgh.

1903 [1]Davidson, John George,
Scottish Metropolitan Life Insurance Company, Ltd., 25 St. Andrew Square, Edinburgh.

1895 [1]Dick, Alexander,
Scottish Accident Life and General Insurance Company, Limited, 115 George Street, Edinburgh.

1905 [3]Dick, Alexander Blaikie,
217 Albert Road, Pollokshields, Glasgow.

STUDENTS

The figures indicate the number of Examinations to be passed for qualification as Associate.

Date of Enrolment.

1895 [1]Dick, James,
Scottish Accident Life and General Insurance Company, Ltd., 115 *George Street, Edinburgh.*

1905 [2]Drake, John William, Jun.,
Britannic Life Assurance Company, Ltd., Fern Bank, Wisewood, Sheffield.

1899 [2]Elgin, George Eric,
Scottish Equitable Life Assurance Society, 28 *St. Andrew Square, Edinburgh.*

1901 [1]Forrest, Douglas,
Scottish Equitable Life Assurance Society, 28 *St. Andrew Square, Edinburgh.*

1907 [2]Fraser, David Spence,
Commercial Union Assurance Company, Ltd., 150 *Hope Street, Glasgow.*

1904 [1]Fyfe, Alexander Adair,
Messrs. J. and C. Logan and Steven, C.A., 82 *Gordon Street, Glasgow.*

1900 [1]Geoghegan, John Michael, C.A.,
Messrs. Barstow & Millar, C.A., 30 *York Place, Edinburgh.*

1905 [1]Gerrish, Frank Wilfrid, B.A. (Cantab.),
Minerva Villa, Albert Road South, Buckhurst Hill, Essex.

1906 [3]Gibb, John Francis Pillans,
North British and Mercantile Insurance Company, 64 *Princes Street, Edinburgh.*

1907 [2]Gibson, George Vallance,
Royal Exchange Assurance Corporation, 68 *George Street, Edinburgh.*

1901 [2]Gordon, James Alison,
Messrs. John Mann & Son, C.A., 142 *St. Vincent Street, Glasgow.*

1907 [2]Govan, Thomas Darling,
Messrs. Watson & Morrison, C.A., 16 *South Charlotte Street, Edinburgh.*

Date of Enrolment.

1905 [3]Grant, Frederick John,
Edinburgh Life Assurance Company, 12 *King Street, Manchester.*

1907 [3]Grant, James,
Messrs. Scott & Paterson, C.A., 2 *York Buildings, Edinburgh.*

1907 [2]Gray, Joseph Train,
Messrs. Lindsay, Jamieson & Haldane, C.A., 24 *St. Andrew Square, Edinburgh.*

1906 [1]Green, Alexander John,
Scottish Provident Institution, 6 *St. Andrew Square, Edinburgh.*

1907 [2]Gunn, Grahame Oncken,
Edinburgh Life Assurance Company, 22 *George Street, Edinburgh.*

1905 [2]Halcrow, Malcolm,
Scottish Union and National Insurance Company, 35 *St. Andrew Square, Edinburgh.*

1906 [3]Harcus, James Logie,
Messrs. Hope, Todd & Kirk, 19 *Charlotte Square, Edinburgh.*

1902 [1]Hardie, Steven James Lindsay,
Messrs. Hardie & Grieve, C.A., 157 *St. Vincent Street, Glasgow.*

1903 [1]Harrower, Archibald,
Northern Assurance Company, 20 *St. Andrew Square, Edinburgh.*

1899 [1]Hart, William Frederick,
Atlas Assurance Company, Ltd., 180 *West George Street, Glasgow.*

1905 [2]Heron, David, M.A.,
Viewbank, New Scone, Perth.

1906 [1]Hogg, Alexander Mackintosh,
Scottish Amicable Life Assurance Society, 35 *St. Vincent Place, Glasgow.*

1907 [3]Hutton, George Clark,
1 *Stow Street, Paisley.*

STUDENTS

The figures indicate the number of Examinations to be passed for qualification as Associate.

Date of Enrolment.

1905 [1]Jackson, John Bell,
North British and Mercantile Insurance Company, 64 *Princes Street, Edinburgh.*

1904 [1]Jeffrey, Robert, Jun.,
Scottish Provident Institution, 6 *St. Andrew Square, Edinburgh.*

1907 [3]Johnston, William James,
North British and Mercantile Insurance Company, 64 *Princes Street, Edinburgh.*

1906 [2]Jones, Henry Edwin,
Scottish Union and National Insurance Company, 35 *St. Andrew Square, Edinburgh.*

1887 [1]Kay, Thomas Chalmers,
English and Scottish Law Life Assurance Association, 41 *Charlotte Square, Edinburgh.*

1904 [2]Knox, George Cruden,
Northern Assurance Company, 1 *Union Terrace, Aberdeen.*

1907 [3]Lakeman, Harold Leslie,
Messrs. John Wilson & Stirling, C.A., 186 *St. Vincent Street, Glasgow.*

1900 [1]Lang, John Charles,
Assistant Secretary, City of Glasgow Life Assurance Company, 5 *Lyons Range, Calcutta.*

1893 [1]Little, John Robert,
Assistant Secretary, Century Insurance Company, Limited, 18 *Charlotte Square, Edinburgh.*

1907 [3]Lodge, Humphrey Francis,
Scottish Widows' Fund and Life Assurance Society, 9 *St. Andrew Square, Edinburgh.*

1906 [3]Lord, Wilfrid,
Messrs. Ronaldson & Gordon, C.A., 3A *North St. David Street, Edinburgh.*

1896 [1]Lorimer, Norman William,
North British and Mercantile Insurance Company, 64 *Princes Street, Edinburgh.*

Date of Enrolment.

1903 [1]Macfarlane, John Miller,
North British and Mercantile Insurance Company, 102 *St. Vincent Street, Glasgow.*

1906 [1]Macfarlane, William,
City of Glasgow Life Assurance Company, 30 *Renfield Street, Glasgow.*

1891 [1]Macgibbon, James Morrison,
Alliance Assurance Company, Bartholomew Lane, London, E.C.

1905 [1]Maclean, Alexander Tweedie,
City of Glasgow Life Assurance Company, 30 *Renfield Street, Glasgow.*

1907 [2]Maclean, Joseph Brotherton,
Scottish Temperance Life Assurance Company, Limited, 105 *St. Vincent Street, Glasgow.*

1901 [3]Macnair, Donald John,
Edinburgh Life Assurance Company, 22 *George Street, Edinburgh.*

1904 [1]M'Naught, James Provan,
Scottish Temperance Life Assurance Company, Ltd., 105 *St. Vincent Street, Glasgow.*

1903 [1]MacPhail, Angus Donald,
Scottish Life Assurance Company, Ltd., 19 *St. Andrew Square, Edinburgh.*

1903 [2]MacPherson, John Reid,
English and Scottish Law Life Association, 41 *Charlotte Square, Edinburgh.*

1907 [2]M'Rae, Charles,
Scottish Union and National Insurance Company, 35 *St. Andrew Square, Edinburgh.*

1907 [2]Mathieson, Hugh Cameron,
General Accident Fire and Life Assurance Corporation, Limited, General Buildings, Perth.

1905 [1]Meiklem, William Halliday,
City of Glasgow Life Assurance Company, 30 *Renfield Street, Glasgow.*

STUDENTS

The figures indicate the number of Examinations to be passed for qualification as Associate.

Date of Enrolment.

1907 [3]Melrose, James M'Pherson, *Scottish Widows' Fund and Life Assurance Society, 9 St. Andrew Square, Edinburgh.*

1905 [3]Millar, Alexander Mitchell, *Caledonian Insurance Company, 19 George Street, Edinburgh.*

1905 [1]Moodie, John Robert, *City of Glasgow Life Assurance Company, 30 Renfield Street, Glasgow.*

1899 [1]Morham, Malcolm, *Liverpool and London and Globe Insurance Company, 1 Dale Street, Liverpool.*

1907 [2]Morris, William Macgregor, *Edinburgh Life Assurance Company, 22 George Street, Edinburgh.*

1894 [1]Muir, Arthur Henry, C.A., *Scottish Provident Buildings, 2 Wellington Place, Belfast.*

1907 [3]O'Hanlon, Aubrey, *c/o William Dilly, C.A., 74 George Street, Edinburgh.*

1907 [3]Oliver, Christopher, *Liverpool and London and Globe Insurance Company, 23 St. Andrew Square, Edinburgh.*

1902 [1]Oliver, John Pringle Pattison, *North British and Mercantile Insurance Company, 64 Princes Street, Edinburgh.*

1902 [1]Paterson, George Watson, *General Accident Fire and Life Assurance Corporation, General Buildings, Perth.*

1906 [3]Patrick, Walter Sydney, *Britannic Assurance Company, Limited, Broad Street Corner, Birmingham.*

1904 [3]Pulford, William Henry Edward, *16 Spottiswoode Road, Edinburgh.*

Date of Enrolment.

1907 [3]Rae, Sydney Carson, *Edinburgh Life Assurance Company, 124 St. Vincent Street, Glasgow.*

1906 [2]Ranken, George, *Scottish Equitable Life Assurance Society, 28 St. Andrew Square, Edinburgh.*

1899 [1]Robertson, David M'Intosh, *Scottish Metropolitan Life Assurance Company, Ltd., 25 St. Andrew Square, Edinburgh.*

1901 [1]Robertson, John Howard, M.A., *University Life Assurance Society, 219 St. Vincent Street, Glasgow.*

1907 [3]Ross, William, Junior, *Scottish Temperance Life Assurance Company, Limited, 105 St. Vincent Street, Glasgow.*

1901 [1]Russell, James Wemyss, *Liverpool and London and Globe Insurance Company, 1 Dale Street, Liverpool.*

1901 [1]Sellar, Alexander Smith, M.A., *3 Windsor Street, Glasgow.*

1906 [3]Simpson, James Dyer, *Liverpool and London and Globe Insurance Company, 119 St. Vincent Street, Glasgow.*

1893 [2]Sinclair, Thomas Ord, C.A., *Finnart Road, Greenock.*

1907 [2]Smellie, Kennedy, *c/o D. N. Cotton, C.A., 9 North St. David Street, Edinburgh.*

1904 [2]Spence, John Dick White, *Messrs. Dawson & Macgregor, C.A., 34 York Place, Edinburgh.*

1907 [3]Stoddart, Charles Templeman, *4A York Place, Edinburgh.*

1906 [3]Strachan, Benjamin, *15 Wellington Street, Edinburgh.*

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The figures indicate the number of Examinations to be passed for qualification as Associate.

Date of Enrolment.

1903 [1]Strathie, David Norman,
8 *Bowmont Terrace, Kelvinside, Glasgow.*

1902 [2]Sutherland, William Robertson,
Life Association of Scotland, 82 *Princes Street, Edinburgh.*

1905 [2]Tavender, Sydney John,
Scottish Amicable Life Assurance Society, 13 *St. Andrew Square, Edinburgh.*

1903 [1]Walker, William Gray,
Scottish Amicable Life Assurance Society, 35 *St. Vincent Place, Glasgow.*

1907 [3]Watson, Patrick Campbell MacDougall,
Scottish Provident Institution, 6 *St. Andrew Square, Edinburgh.*

1894 [1]Watson, Thomas Jackson,
Messrs. Chalmers & Watson, C.A., 3 *North St. David Street, Edinburgh.*

1892 [1]Webster, Thomas,
617 *Birkbeck Bank Chambers, London, W.C.*

1893 [1]Wilson, James Hugh Elwes, A.S.A.A.
P.O. Box 771, *Cape Town, South Africa.*

Date of Enrolment.

1905 [1]Wilson, John Stark Benson,
49 *Balgreen Road, Murrayfield.*

1897 [1]Wink, Frank Adam,
Scottish Equitable Life Assurance Society, 28 *St. Andrew Square, Edinburgh.*

1896 [3]Winning, James, Junior,
Messrs. John Wilson & Stirling, 186 *St. Vincent Street, Glasgow.*

1905 [2]Wishart, William Muir,
Scottish Metropolitan Life Assurance Company, Ltd., 25 *St. Andrew Square, Edinburgh.*

1899 [1]Wood, Robert,
Northern Assurance Company, 1 *Union Terrace, Aberdeen.*

1898 [1]Wybar, John,
Scottish Amicable Life Assurance Society, 35 *St. Vincent Place, Glasgow.*

1906 [2]Wyse, William Edwin,
Edinburgh Life Assurance Company, 22 *George Street, Edinburgh.*

1906 [1]Young, David Goldie,
Scottish Life Assurance Company, Limited, 19 *St. Andrew Square, Edinburgh.*

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